2007 YR 11 ACCELERANTS HALF YEARLY

QUESTION 1.

a) $\sqrt[3]{\dfrac{625}{76+41}} = \underline{\underline{1.75\ (2dp)}}$

b) $5x^2 - 4x(1-x)$

$= 5x^2 - 4x + 4x^2$

$= \underline{\underline{9x^2 - 4x}}$

c). $\dfrac{2x-1}{4} = 1 - \dfrac{x}{3}$

$\dfrac{2x-1}{4} = \dfrac{3-x}{3}$

$\dfrac{3(2x-1)}{12} = \dfrac{4(3-x)}{12}$

$6x - 3 = 12 - 4x$

$10x = 15$

$x = 3/2$

$\underline{\underline{x = 1\tfrac{1}{2}}}$

d). $3x^2 = 6x$

$3x^2 - 6x = 0$

$3x(x-2) = 0$

$\underline{\underline{x = 0 \text{ or } 2}}$

e) $\dfrac{3\pi^c}{5} \Rightarrow 108°$

f) $0.\dot{3}\dot{7} = \dfrac{37}{99}$

g) $\tan 225° - \cos 120°$

$= 1 - (-0.5)$

$= \underline{\underline{1.5}}$

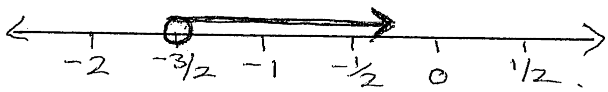
h). $4x^2 - 9$

$= \underline{\underline{(2x-3)(2x+3)}}$

i) $5 - 2x < 8$

$-2x < 3$

$\underline{\underline{x > \dfrac{-3}{2}}}$



j). $2x - 3y = 5$ ——— ①

$3x + 4y = -1$ ——— ②

$4 \times ① \Rightarrow \quad 8x - 12y = 20$ —— ③

$3 \times ② \Rightarrow \quad 9x + 12y = -3$ —— ④

③+④ $\quad 17x = 17$

$\underline{\underline{x = 1}}$

Sub $x=1$ into ①.

$2(1) - 3y = 5$

$-3y = 3$

$\underline{\underline{y = -1}}$

Solutions are $y = -1, x = 1$.

Year 11 Accelerated
Question 2

(a) 1.495×10^{8} km
(b) (i)

$$f(2)=\frac{2^2+2^{-2}}{2}$$
$$=\frac{17}{8}$$

(ii)

$$f(-x)=\frac{2^{-x}+2^{-(-x)}}{-x}$$
$$=-\frac{2^{-x}+2^{x}}{x}$$
$$=-f(x)$$

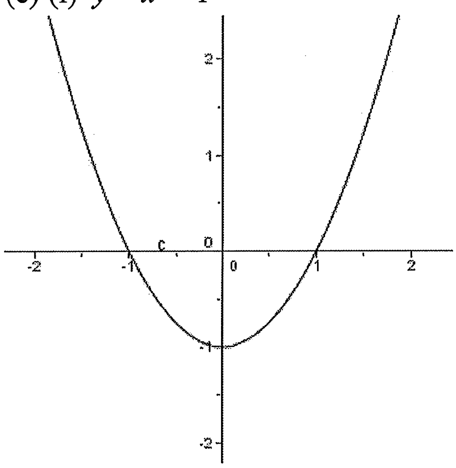
(c) $x^3+y^3=(x+y)(x^2-xy+y^2)$

(d) $|x-1|=4$

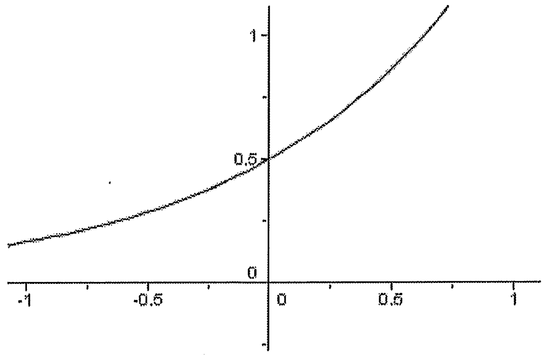
$x-1=4$ or $-(x-1)=4$

$x=5$ $x=-3$

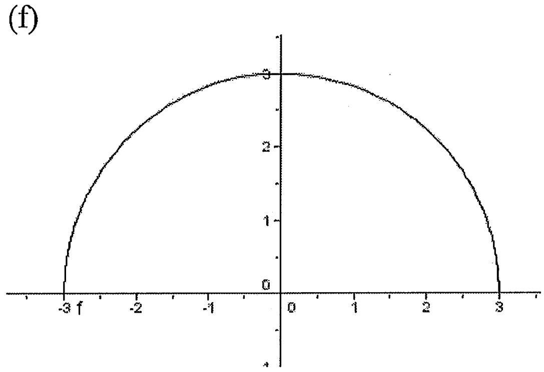
(e) (i) $y=x^2-1$



(ii) $y=\frac{1}{2}\left(3^x\right)$



(f)



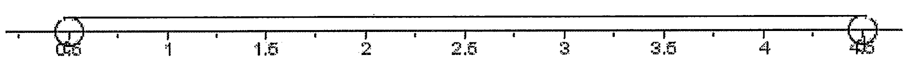
Domain: $-3 \le x \le 3$
Range: $0 \le f(x) \le 3$

(g) $|2x-5|<4$

$2x-5<4$ or $-(2x-5)<4$

$x<\frac{9}{2}$ or $x>\frac{1}{2}$

$\frac{1}{2}<x<\frac{9}{2}$



(h)

$$2\log_5 3=\log_5 x-\log_5 6$$

$$\log_5 9=\log_5 \frac{x}{6}$$

$$9=\frac{x}{6}$$

$$x=54$$

Question 3 (15 Marks)

(a) For the parabola $4y = x^2 + 2x - 7$ write down the

(i) equation of the axis of symmetry **[1]**

> **Solution:**
> $$4y = x^2 + 2x + 1 - 8,$$
> $$4(y+2) = (x+1)^2.$$
> $\therefore$ The axis of symmetry is $x = -1$.

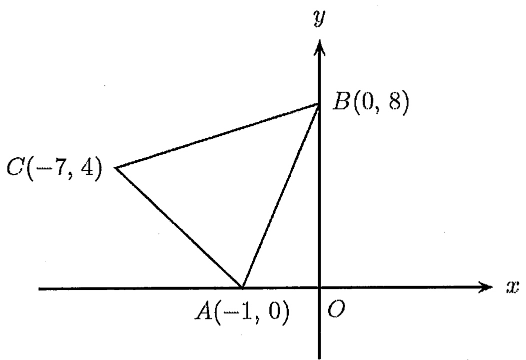
(ii) coordinates of the vertex **[1]**

> **Solution:** Vertex is $(-1,\ -2)$.

(iii) equation of the directrix and coordinates of the focus. **[2]**

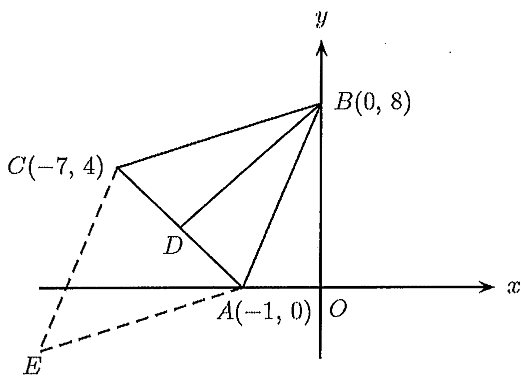
> **Solution:** Focal length is 1.
> $\therefore$ Directrix is $y = -3$.
> Focus is $(-1,\ -1)$.

(b)



The diagram above shows the points $A(-1,\ 0)$, $B(0,\ 8)$, and $C(-7,\ 4)$.
Copy the diagram to your answer booklet.

Solution:



(i) Find the gradient of the line AC. [1]

Solution:
$$\begin{aligned} m_{AC} &= \frac{0-4}{-1 - {}^-7}, \\ &= \frac{-4}{6}, \\ &= -\frac{2}{3}. \end{aligned}$$

(ii) Calculate the size of the angle CAO to the nearest degree. [1]

Solution:
$$\begin{aligned} \tan C\widehat{A}O &= -\tfrac{2}{3}. \\ \therefore\ C\widehat{A}O &= \tan^{-1}\left(-\tfrac{2}{3}\right) \text{ in } 2^{\text{nd}} \text{ quadrant}, \\ &\approx -33.69^\circ + 180^\circ, \\ &\approx 146^\circ. \end{aligned}$$

(iii) Find the equation of the line AC. [1]

Solution:
$$\begin{aligned} y - 0 &= -\tfrac{2}{3}(x+1), \\ 2x + 3y + 2 &= 0. \end{aligned}$$

(iv) Find the coordinates of D, the midpoint of AC. [1]

Solution: $\left(\dfrac{-7-1}{2}, \dfrac{4+0}{2}\right) = (-4, 2).$

(v) Show that AC is perpendicular to BD. [2]

Solution:

$$\begin{aligned} m_{BD} &= \frac{2-8}{-4-0}, \\ &= \frac{-6}{-4}, \\ &= \frac{3}{2}. \\ m_{AC} \times m_{BD} &= -\tfrac{2}{3} \times \tfrac{3}{2}, \\ &= -1. \\ \therefore\ AC &\perp BD. \end{aligned}$$

(vi) What does part (V) show about $\triangle ABC$? [1]

Solution: $\triangle ABC$ is isosceles and $BC = BA$.

(vii) Find the area of $\triangle ABC$. [3]

Solution:

$$\begin{aligned} \text{Length of } AC &= \sqrt{(-1+7)^2 + (0-4)^2}, \\ &= \sqrt{16+36}, \\ &= 2\sqrt{13}. \\ \text{Length of } BD &= \sqrt{(0+4)^2 + (8-2)^2}, \\ &= \sqrt{36+16}, \\ &= 2\sqrt{13}. \\ \therefore\ \text{Area of } \triangle\ ABC &= \tfrac{1}{2} \times 2\sqrt{13} \times 2\sqrt{13}, \\ &= 26. \end{aligned}$$

(viii) Write down the coordinates of the point E such that $ABCE$ is a rhombus. [1]

Solution: $\big(-4 + (-4-0),\, 2 + (2-8)\big) = (-8, -4)$.

4) a) i) $y = x+1$ ——— (1)

$y = (x-1)^2$ ——— (2)

sub (1) into (2)

$x+1 = x^2 - 2x + 1$

$x^2 - 3x = 0$

$x(x-3) = 0$

$x = 0, \quad x = 3$

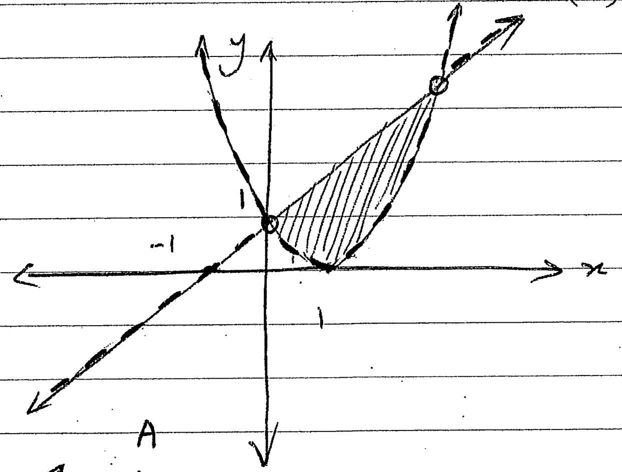
sub into (1)

$y = (0)+1 \qquad y = (3)+1$

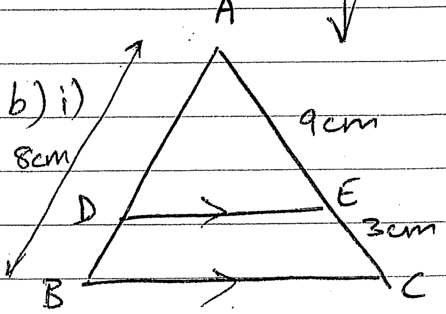
$y = 1 \qquad y = 4$

points of intersection (0,1) & (3,4)

ii)



b) i)



In Δ's ADE & ABC

∠BAC is common

∠ADE = ∠ABC (corresponding angles DE ∥ BC)

∴ ΔADE ||| ΔABC (equiangular)

ii) $\dfrac{AB}{AD} = \dfrac{AC}{AE}$ (corresponding sides of similar triangles in same ratio)

$\dfrac{8}{AD} = \dfrac{12}{9}$

$AD = 6$

since $AB = AD + DB$

$8 = 6 + DB$

$DB = 2\text{cm}$

c) i) $(x+2)^2 = 9x^2$

$(x+2)^2 - 9x^2 = 0$

$[(x+2)+3x][(x+2)-3x] = 0$

$[4x+2][-2x+2] = 0$

$4(2x+1)(-x+1) = 0$

$x = -\frac{1}{2}, \quad x = 1$

ii) $4^x - 20(2^x) + 64 = 0$

$2^{2x} - 20(2^x) + 64 = 0$

let $m = 2^x$

$m^2 - 20m + 64 = 0$

$(m-16)(m-4) = 0$ (× 64, + −20: −16, −4)

$m = 16, \quad m = 4$

$2^x = 16 \qquad 2^x = 4$

$2^x = 2^4 \qquad 2^x = 2^2$

$\therefore x = 4, \quad x = 2$

d) $1 - 6x - x^2 = -(x^2 + 6x - 1)$

$= -(x^2 + 6x + 9 - 10)$

$= -(x^2 + 6x + 9) + 10$

$= -(x+3)^2 + 10$

$(x+3)^2 \geq 0$

$-(x+3)^2 \leq 0$

$-(x+3)^2 + 10 \leq 10$

∴ greatest value is 10 when $x = -3$

e) $f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ 1 - x & \text{for } x \geq 1 \end{cases}$

i) $f(-1) + f(2) = (-1)^2 - 1 + 1 - (2)$

$= -1$

ii) $f(a^2+1) = 1 - (a^2+1)$, since $a^2 + 1 \geq 1$

$= -a^2$

(YR11 ACCEL'N.)

QUESTION 5

a (i) $y = 2x^3 - x^2 + 6$

$y' = \boxed{6x^2 - 2x}$

(ii) $y = (x+2)^{\frac{1}{3}}$

$y' = \boxed{\frac{1}{3}(x+2)^{-\frac{2}{3}}}$

(iii) $y = \frac{1}{3} x^{-2}$

$y' = \boxed{-\frac{2}{3} x^{-3}}$ OR $\boxed{\frac{-2}{3x^3}}$

b (i) $y = (x+1)(x-2)^7$

$y' = 1.(x-2)^7 + 7(x+1)(x-2)^6$

$= (x-2)^6 [x-2 + 7(x+1)]$

$= \boxed{(x-2)^6 (8x+5)}$

(ii) $y = \frac{x}{3x+1}$

$y' = \frac{(3x+1).1 - x.3}{(3x+1)^2}$

$= \boxed{\frac{1}{(3x+1)^2}}$

(iii) $f(x) = x + x^{\frac{1}{2}}$

$f'(x) = \boxed{1 + \frac{1}{2} x^{-\frac{1}{2}}}$

(α) $f'(1) = 1 + \frac{1}{2}$

$\boxed{= \frac{3}{2}}$

(β) $f(-2) = -2 + \sqrt{-2}$

$\boxed{\text{NOT REAL.}}$

(c) (i) $\Delta = 4 - 12k.$

(ii) For real roots $\Delta \geq 0.$

$4 - 12k \geq 0$

$12k \leq 4$

$k \leq \frac{1}{3}$

(d) $y = x^3 - 2x - 1$

$y' = 3x^2 - 2$

$\therefore m_T = 3 \times 2^2 - 2.$

$= 12 - 2$

$m_T = 10$

$m_N = -\frac{1}{10}$

$\therefore N: \frac{y-3}{x-2} = -\frac{1}{10}.$

$10(y-3) = -(x-2)$

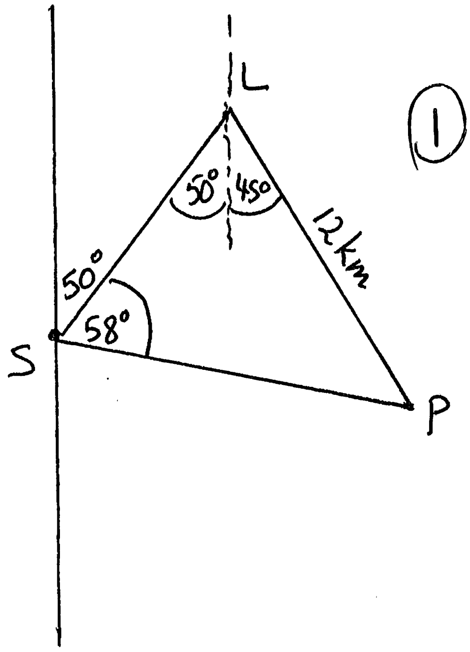
$10y - 30 = -x + 2.$

$x + 10y - 32 = 0.$

Question 6

(a)

(i) (1)



(ii) $\frac{SP}{\sin 95^\circ} = \frac{12}{\sin 58^\circ}$ (2)

$SP = \frac{12 \sin 95^\circ}{\sin 58^\circ}$

$SP = 14.1 \text{ km}$

(b) $RHS = \frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta}$

(2)

$= \frac{(1+\cos\theta) - (1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$

$= \frac{2\cos\theta}{\sin^2\theta}$

$LHS = 2\cot\theta \operatorname{cosec}\theta$

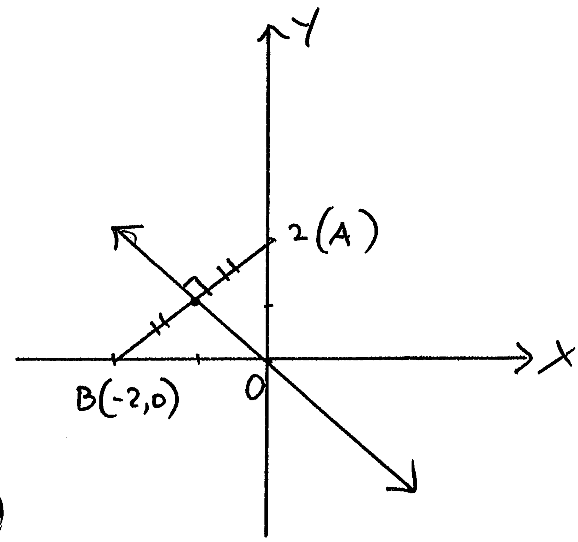
$= 2\frac{\cos\theta}{\sin\theta} \cdot \frac{1}{\sin\theta}$

$= \frac{2\cos\theta}{\sin^2\theta}$

$LHS = RHS$

(c)

(i) (1)



(1)

(ii) $y = -x$

(d) (3) let the roots be α, $\alpha - 1$

Sum of roots $2\alpha - 1 = -b$ — (1)

Prod. of roots $\alpha(\alpha - 1) = c$ — (2)

From (1) $\alpha = \frac{1-b}{2}$

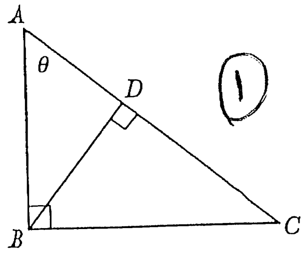
Subst into (2)

$\Rightarrow \left(\frac{1-b}{2}\right)\left(\frac{1-b}{2} - 1\right) = c$

on rearranging

$b^2 = 4c + 1$

(i)



(ii) $\cos\theta = \frac{AD}{AB}$

$\therefore \quad AD = AB\cos\theta$

$\tan\theta = \frac{BC}{AB}$

$\therefore \quad BC = AB\tan\theta$

$\sec\theta = \frac{AC}{AB}$

$\therefore \quad AC = AB\sec\theta.$

Since $\quad 6AD + BC = 5AC,$

$6AB\cos\theta + AB\tan\theta = 5AB\sec\theta$

$\therefore \quad 6\cos\theta + \tan\theta = 5\sec\theta.$

(iii) $$6\cos\theta + \frac{\sin\theta}{\cos\theta} = \frac{5}{\cos\theta}$$

$$6\cos^2\theta + \sin\theta = 5$$

$$\therefore \quad 6(1-\sin^2\theta) + \sin\theta = 5$$

$$6\sin^2\theta - \sin\theta - 1 = 0.$$

(iv) $(2\sin\theta - 1)(3\sin\theta + 1) = 0$

$\sin\theta = \frac{1}{2}$ or $-\frac{1}{3}$

and $0° \le \theta \le 90°$ because θ is a right-angled triangle,

$\therefore \quad \theta = 30°.$

SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2007
YEAR 11 ACCELERATED
ASSESSMENT TASK #1

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each question is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 90

- Attempt questions 1 – 6
- All questions are of equal value.

Examiner: *AM Gainford*

Question 1. (15 Marks) **(Start a new booklet.)**

(a) Calculate $\sqrt[3]{\frac{625}{76+41}}$ correct to two decimal places. **1**

(b) Simplify $5x^2 - 4x(1-x)$. **1**

(c) Solve the equation $\frac{2x-1}{4} = 1 - \frac{x}{3}$. **2**

(d) Solve the equation $3x^2 = 6x$. **2**

(e) Convert $\frac{3\pi}{5}$ radians to degrees. **1**

(f) Express $0\cdot\dot{3}\dot{7}$ as a common fraction. **1**

(g) Find the exact value of $\tan 225^0 - \cos 120^0$. **1**

(h) Factorise $4x^2 - 9$. **2**

(i) Graph on a number line the solution of the inequality $5 - 2x < 8$. **2**

(j) Find the point of intersection of the lines: **2**

$$2x - 3y = 5$$
$$3x + 4y = -1$$

Question 2. (15 Marks) **(Start a new Booklet)**

(a) The distance from the Earth to the Sun is approximately 149 492 000 km. Write this number in scientific notation, correct to four significant figures. **1**

(b) Given that $f\left(x\right)=\dfrac{2^{x}+2^{-x}}{x}$: **2**

(i) Find $f(2)$.

(ii) Show that $f(x)$ is an odd function.

(c) Factorise $x^{3}+y^{3}$. **1**

(d) Solve $\left|x-1\right|=4$. **1**

(e) Sketch the graphs of the following, showing their principal features: **4**

(i) $y=x^{2}-1$

(ii) $y=\frac{1}{2}\left(3^{x}\right)$

(f) State the natural domain and range of the function $f\left(x\right)=\sqrt{9-x^{2}}$. **2**

(g) Solve $\left|2x-5\right|<4$ and graph the solution on the number line. **2**

(h) Solve $2\log_{5}3=\log_{5}x-\log_{5}6$. **2**

Question 3. (15 Marks) **(Start a new booklet.)**

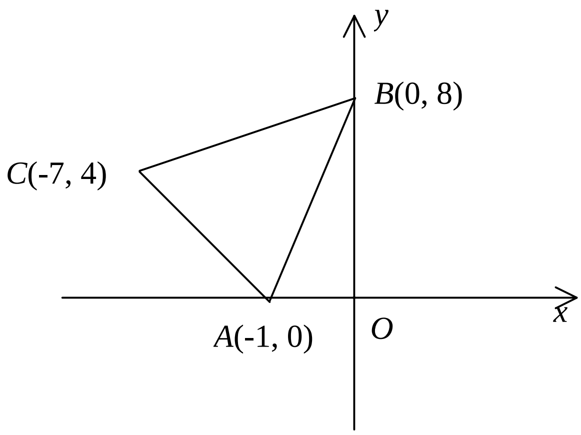
(a) For the parabola $4y = x^2 + 2x - 7$ write down the **4**

(i) equation of the axis of symmetry

(ii) coordinates of the vertex

(iii) equation of the directrix and coordinates of the focus.

(b) **11**



The diagram above shows the points $A(-1,0)$, $B(0,8)$, and $C(-7,4)$.

Copy the diagram to your answer booklet.

(i) Find the gradient of the line AC.

(ii) Calculate the size of the angle CAO to the nearest degree.

(iii) Find the equation of the line AC.

(iv) Find the coordinates of D, the midpoint of AC.

(v) Show that AC is perpendicular to BD.

(vi) What does part (v) show about ΔABC?

(vii) Find the area of ΔABC.

(viii) Write down the coordinates of the point E such that $ABCE$ is a rhombus.

Question 4. (15 Marks) **(Start a new booklet)**

(a) Given the line $y = x + 1$ and the parabola $y = (x-1)^2$ **3**

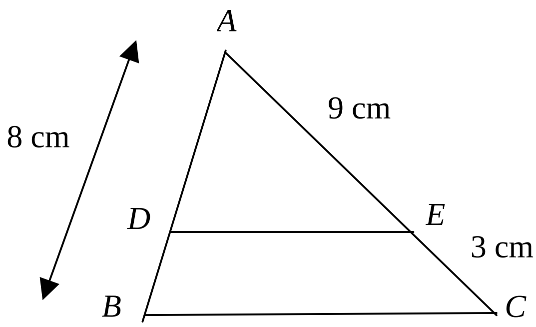
(i) Find the points of intersection of the line and the parabola.

(ii) Hence sketch the region where $y \le x + 1$ and $y > (x-1)^2$ hold simultaneously.

(b) In the diagram *AB* = 8 cm, *AE* = 9 cm, and *EC* = 3 cm. $DE \parallel BC$. **4**

(i) Prove that ΔADE is similar to ΔABC.

(ii) Find the length of *DB*.



(c) Solve the following equations: **4**

(i) $(x+2)^2 = 9x^2$

(ii) $4^x - 20(2^x) + 64 = 0$

(d) By completing the square, find the greatest value of the expression $1 - 6x - x^2$, state the *x* value for which it occurs. **2**

(e) Given that $f(x)$ is defined as below **2**

$$f(x) = \begin{cases} x^2 - 1 & \text{for } x < 1 \\ 1 - x & \text{for } x \ge 1 \end{cases}$$

(i) Find the value of $f(-1) + f(2)$.

(ii) Find $f(a^2 + 1)$.

Question 5 (15 Marks) **(Start a new booklet.)**

(a) Differentiate the following with respect to x: **3**

(i) $2x^3 - x^2 + 6$

(ii) $\sqrt[3]{x+2}$

(iii) $\frac{1}{3x^2}$

(b) **6**

(i) Use the product rule to find $\frac{dy}{dx}$ if $y = (x+1)(x-2)^7$.

(ii) Differentiate $y = \frac{x}{3x+1}$ by using the quotient rule.

(iii) If $f(x) = x + \sqrt{x}$, find

(α) $f'(1)$

(β) $f'(-2)$

(c) **3**

(i) Write down the discriminant of $3x^2 + 2x + k$.

(ii) For what values of k does $3x^2 + 2x + k = 0$ have real roots.

(d) For the curve $y = x^3 - 2x - 1$, find the gradient of the tangent to the curve at the point on the curve where $x = 2$. Also find the equation of the normal to the curve at this point. **3**

Question 6 (15 Marks) **(Start a new booklet)**

(a) The bearing of a lighthouse (*L*) from a ship (*S*) is 50^0T. A port (*P*) is 12 km due South-East of the lighthouse, and is on a bearing of 108^0T from the ship. **3**

(i) Draw a neat diagram to represent this situation.

(ii) Find the distance (*SP*) of the ship from the port.

(b) Prove the trigonometric identity **2**

$$2\cot\theta\operatorname{cosec}\theta = \frac{1}{1-\cos\theta} - \frac{1}{1+\cos\theta}$$

(c) (i) On a number plane diagram sketch the locus of all points equidistant from the points *A* (0,2) and *B* (-2,0). **2**

(ii) Write down an equation to describe this locus.

(d) The roots of the quadratic equation $x^2 + bx + c = 0$ differ by 1. **3**
Show that $b^2 = 4c + 1$.

(e) A triangle *ABC* is right-angled at *B*. *D* is the point on *AC* such that *BD* is perpendicular to *AC*. Let $\angle BAC = \theta$. **5** *A*

(i) Draw a diagram showing this information.

It is given that $6AD + BC = 5AC$.

(ii) Show that $6\cos\theta + \tan\theta = 5\sec\theta$.

(iii) Deduce that $6\sin^2\theta - \sin\theta - 1 = 0$.

(iv) Find θ.

This is the end of the paper.