SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008

YEAR 11

ASSESSMENT TASK #1

Mathematics Accelerated

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Each Question is to be returned in a separate bundle.
- All necessary working should be shown in every question.

Total Marks – 96

- Attempt questions 1 – 5
- All questions are **NOT** of equal value

Examiner: *A. Fuller*

Total marks 96

Attempt questions 1 to 5

Answer each **Question** in a **Separate** writing booklet

(Use a SEPARATE writing booklet)

Question 1 (20 marks)

(a) Evaluate $\sqrt{5^2 + 12^2}$. **1**

(b) Convert 80° to radians. **1**

(c) Express $\dfrac{\sqrt{2}}{3+\sqrt{2}}$ as a fraction with a rational denominator. **2**

(d) $x^{-1} = 3^{-1} + 4^{-1}$, what is the value of x? **1**

(e) If $f(x) = 2x + 1$; **2**

i. Evaluate $f(-1)$.

ii. Find $f(2x+1)$.

(f) If A is a negative number, write down the value of $|A|$. **1**

(g) Write $0.1\dot{5}$ as a fraction in its lowest terms. **2**

(h) Factorise the following; **6**

i. $1 - 8a^3$

ii. $6a^2 - a - 2$

iii. $x(x-1) - y(y-1)$

(i) Express $\dfrac{x-5}{2} - \dfrac{x-1}{6}$ as a single fraction in its simplest form. [2]

(j) State the natural domain of the following; [2]

i. $y = \dfrac{1}{x+1}$

ii. $y = \sqrt{x-3}$

(Use a SEPARATE writing booklet)

Question 2 (19 marks)

(a) Solve $x^2 < 4x$ and graph the solution on a number line. 3

(b) Give the exact value of the following; 5

i. $\sin 150°$

ii. $\sec \dfrac{7\pi}{4}$

iii. $\cos A$ if $\sin A = \dfrac{6}{7}$, and $\tan A < 0$.

(c) The function $f(x)$ is odd, and it is known that $f(3) = 7$. 2

i. What is the value of $f(-3)$?

ii. Sketch a possible graph of $y = f(x)$.

(d) A parabola has its vertex at $(-1, 6)$ and its directrix is $y = 7$. 2

i. Write down the coordinates of the focus.

ii. What is the equation of the parabola?

(e) Differentiate the following; 7

i. $x^2 + x + 1$

ii. $\dfrac{x^2 + 1}{x}$

iii. $x\sqrt{x}$

iv. $(x^2 + 1)^5$

v. $\dfrac{x}{x^2 + 1}$

(Use a SEPARATE writing booklet)

Question 3 (20 marks)

(a) If $2^{2x-4} = 8$, what is the value of x? 2

(b) Shade the region defined by $x^2 + y^2 < 4$ and $x + y \geq 1$. 2

(c) Find the value of k for which the equation $3x^2 + 10x + k = 0$ has; 3

i. one root equal to 4.

ii. one root which is the reciprocal of the other.

(d) Solve $2\cos x + \sqrt{3} = 0$ for $0° \leq x \leq 360°$. 2

(e) If $\log_a 5 = x$ and $\log_a 2 = y$, find $\log_a 400$ in terms of x and y. 2

(f) The lines $y = 2x + 3$, $y = 8x + 15$, and $y = 5x + b$ are concurrent. 3

What is the value of b?

(g) Solve $2(4)^x - 3(2)^x + 1 = 0$. 3

(h) Differentiate $f(x) = 2x + x^2$ from first principles. 3

Question 4 (19 marks)

(a) a, b and c are positive, consecutive terms of a geometric series. Show that the graph of $y = ax^2 + bx + c$ is entirely above the x-axis. **[2]**

(b) The nth term of an arithmetic series is given by $T_n = 555 - 7n$. **[3]**

If $S_n = T_1 + T_2 + ... + T_n$, determine the smallest value of n for which $S_n < 0$.

(c) The line $\mathcal{L}$ passes through the points $B(7, -1)$ and $C(-1, 7)$ **[7]**

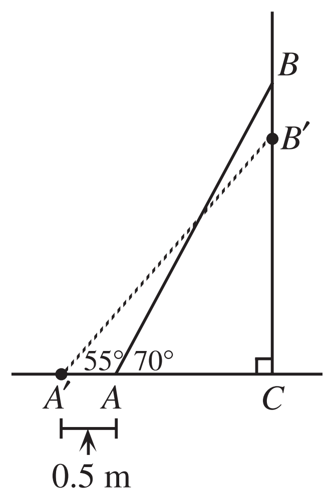
i. Show that the equation of line $\mathcal{L}$ is $y = -x + 6$.

ii. Determine the coordinates of the point P on the line $\mathcal{L}$ so that P is equidistant from the points $A(10, -10)$ and $O(0, 0)$.

iii. Determine the coordinates of all points Q on the line $\mathcal{L}$ so that $\angle OQA = 90°$.

(d) A ladder, AB, is positioned so that its bottom sits on horizontal ground and its top rests against a vertical wall, as shown below. In this initial position, the ladder makes an angle of 70° with the horizontal. The bottom of the ladder is then pushed 0.5m away from the wall, moving the ladder to position $A'B'$. In this new position, the ladder makes an angle of 55° with the horizontal. 4

Calculate, to the nearest centimetre, the distance that the ladder slides down the wall (that is, the length of BB').



(e) For each value of x, $f(x)$ is defined to be the minimum value of the three numbers $2x + 2$, $\frac{1}{2}x + 1$ and $-\frac{3}{4}x + 7$. What is the range of $y = f(x)$? 3

(Use a SEPARATE writing booklet)

Question 5 (18 marks)

(a) i. Sketch the graph of $y = |2x - 3|$ and $y = x$ on the same axes. 7

ii. Find any solutions to $|2x - 3| = x$.

iii. Hence, find the solution to the inequality $|2x - 3| > x$.

iv. The equation $|2x - 3| = mx$ has only one solution. What values can m take?

(b) A fund is established to provide prizes for a cricket team's annual Awards night. $10 000 is placed in the fund one year before the first Awards night. It is decided that $450 will be withdrawn from the fund each year to purchase the annual prizes. The money in the fund is invested at 3%p.a. compounded annually with the interest paid into the fund before each annual Awards night. 11

i. Show that the fund contains $9695.50 after the second awards night.

ii. If A_n is the amount in dollars remaining in the fund after the nth Awards night, prove that $A_n = 5000(3 - 1.03^n)$.

iii. Find the amount of money in the fund after the 25th Awards night. Give your answer correct to the nearest dollar.

iv. Find the maximum number of Awards nights that can be financed using this fund.

v. For the fund described above it is decided to increase the amount of money withdrawn for each Awards night by 2% each year.

Show that $A_n = 45000(1.02)^n - 35000(1.03)^n$.

End of paper

Yr 11 Ass#1 Accelerated 2008

Q3 (a) $2^{2x-4} = 8$

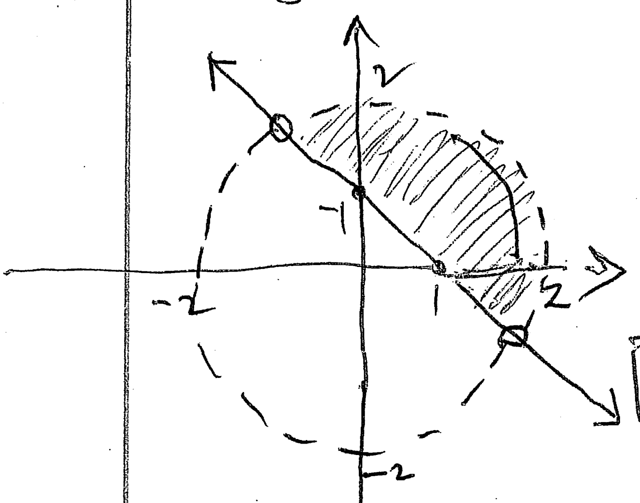
$= 2^3$

$\therefore 2x - 4 = 3$

$2x = 7$

$x = \frac{7}{2}$ [2]

(b) $x^2 + y^2 < 4 \quad x + y \geq 1$



[2]

(c) $3x^2 + 10x + k = 0$

(i) $\alpha + \beta = \frac{-10}{3} \quad \alpha\beta = \frac{k}{3}$

Let $\beta = 4$

$\alpha + 4 = \frac{-10}{3}$

$3\alpha + 12 = -10$

$3\alpha = -22$

$\alpha = -\frac{22}{3}$

$\therefore -\frac{22}{3} \times 4 = \frac{k}{3}$ [2]

$k = -88$

(ii) Let roots be $\alpha, \frac{1}{\alpha}$

$\alpha \cdot \frac{1}{\alpha} = \frac{k}{3}$

$\therefore k = 3$ [1]

(d) $2\cos x + \sqrt{3} = 0$

$\cos x = -\frac{\sqrt{3}}{2}$ [2]

$\therefore x = 150°, 210°$

(e) $\log 400 = \log 2^4 5^2$

$= 4\log 2 + 2\log 5$

$= 4y + 2x$ [2]

(f) By substitution

$2x + 3 = 8x + 15$

$-12 = 6x$

$x = -2$

$y = -4 + 3$

$= -1$

Concurrent at $(-2, -1)$

$\therefore y = 5x + b$

$-1 = -10 + b$

$b = 9$ [3]

(g) $2(4)^x - 3(2)^x + 1 = 0$

Let $u = 2^x$

$2u^2 - 3u + 1 = 0$

$(2u-1)(u-1) = 0$

$\therefore u = 1$ or $\frac{1}{2}$

$2^x = 1$ or $\frac{1}{2}$ [3]

$\therefore x = 0$ or -1

(h) $f(x) = 2x + x^2$

$$f'(x) = \lim_{h \to 0} \frac{2(x+h) + (x+h)^2 - (2x + x^2)}{h}$$

$$= \lim_{h \to 0} \frac{2x + 2h + x^2 + 2xh + h^2 - 2x - x^2}{h}$$

$$= \lim_{h \to 0} \frac{2h + 2xh + h^2}{h}$$

$$= \lim_{h \to 0} \frac{h(2 + 2x + h)}{h}$$

$$= \lim_{h \to 0} (2 + 2x + h)$$

$= 2x + 2$ [3]

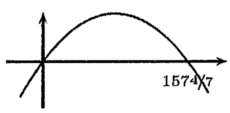
Year 11 Accelerated Mathematics: **Solutions Assessment Task #1**

Question 4 (19 Marks)

(a) a, b and c are positive, consecutive terms of a geometric series. Show that the graph of $y = ax^2 + bx + c$ is entirely above the x-axis. [2]

> **Solution:** $a > 0$, so the parabola is concave upwards.
>
> $$\begin{aligned} \text{Now } \frac{c}{a} &= \frac{b}{a}, \text{ as the common ratios are equal.} \\ \therefore\ ac &= b^2, \\ \triangle &= b^2 - 4ac, \\ &= -3ac, \\ &< 0. \end{aligned}$$
>
> $\therefore$ The graph does not cut the x-axis but is completely above it.

(b) The nth term of an arithmetic series is given by $T_n = 555 - 7n$.
If $S_n = T_1 + T_2 + ... + T_n$, determine the smallest value of n for which $S_n < 0$. [3]

> **Solution:** The common difference, $d = -7$.
>
> $$\begin{aligned} T_1 &= 548. \\ \therefore\ S_n &= \frac{n}{2}(548 + 555 - 7n). \\ \textit{i.e.},\ n(1103 - 7n) &< 0, \\ n &> 157 4/7. \\ \therefore\ n &= 158. \end{aligned}$$
>
> 
>

(c) The line ℓ passes through the points $B(7, -1)$ and $C(-1, 7)$ [7]

i. Show that the equation of line ℓ is $y = -x + 6$.

> **Solution:**
>
> $$\begin{aligned} m &= \frac{7 - {}^{-}1}{-1 - 7}, \\ &= -1. \\ y + 1 &= -1(x - 7), \\ y &= -y + 6. \end{aligned}$$

ii. Determine the coordinates of the point P on the line ℓ so that P is equidistant from the points $A(10, -10)$ and $O(0, 0)$.

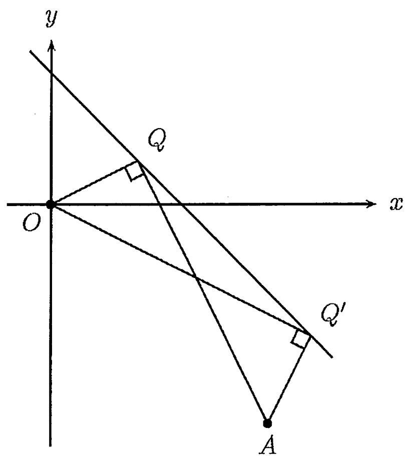
Solution: Let P be the point $(x,\ 6-x)$, then

$$\begin{aligned} x^2+(6-x)^2 &= (x-10)^2+(6-x+10)^2, \\ 36-12x &= -20x+100+256-32x, \\ 40x &= 320, \\ x &= 8. \end{aligned}$$

$\therefore\ P(8,\ -2)$.

iii. Determine the coordinates of all points Q on the line ℓ so that $\angle OQA = 90°$.

Solution:

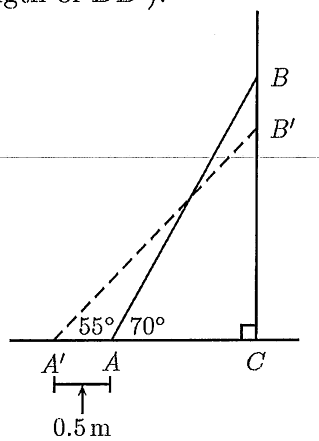


Again, let Q be the point $(x,\ 6-x)$,

$$\begin{aligned} m_{OQ} &= \frac{6-x}{x}. \\ m_{QA} &= \frac{-10-(6-x)}{10-x}. \\ \text{Now } m_{OQ}\times m_{QA} &= -1, \\ \text{i.e. } \frac{6-x}{x} &= -\left(\frac{10-x}{x-16}\right), \\ 6x-96-x^2+16x &= -10x+x^2, \\ 2x^2-32x+96 &= 0, \\ x^2-16x+48 &= 0, \\ (x-4)(x-12) &= 0, \\ x &= 4,\ 12. \end{aligned}$$

$\therefore\ Q$ is at $(4,\ 2)$ and $(12,\ -6)$.

(d) A ladder, AB, is positioned so that its bottom sits on horizontal ground and its top rests against a vertical wall, as shown below. In this initial position, the ladder makes an angle of 70° with the horizontal. The bottom of the ladder is then pushed 0.5 m away from the wall, moving the ladder to position $A'B'$. In this new position, the ladder makes an angle of 55° with the horizontal. Calculate, to the nearest centimetre, the distance that the ladder slides down the wall (that is, the length of BB'). 4



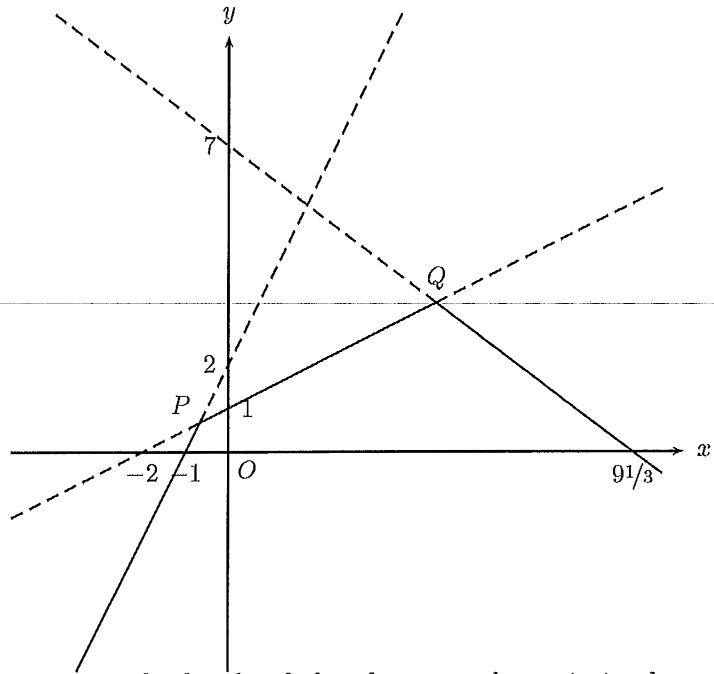
Solution: Let the ladder be l m long.

$$\frac{AC}{l} = \cos 70^\circ, \qquad \frac{AC'}{l} = \cos 55^\circ,$$

$$\begin{aligned}
\frac{AC}{AC'} &= \frac{\cos 70^\circ}{\cos 55^\circ}, \\
AC\cos 55^\circ &= (AC + 0.5)\cos 70^\circ, \\
AC &= 0.5 \times \cos 70^\circ \div (\cos 55^\circ - \cos 70^\circ), \\
&\approx 0.7385\,\text{m}. \\
CB &= AC\tan 70^\circ, \\
&\approx 2.02908\,\text{m}. \\
CB' &= (AC + 0.5)\tan 55^\circ, \\
&\approx 1.76880\,\text{m}. \\
\therefore\ BB' &\approx 0.26026\,\text{m} \\
&\approx 26\,\text{cm}.
\end{aligned}$$

(e) For each value of x, $f(x)$ is defined to be the minimum value of the three numbers $2x+2$, $\frac{1}{2}x+1$ and $-\frac{3}{4}x+7$. What is the range of $y=f(x)$? [3]

Solution:



From a rough sketch of the three numbers, it is clear that we only need locate point Q.

$$\begin{aligned} \tfrac{1}{2}x+1 &= -\tfrac{3}{4}x+7, \\ 2x+4 &= -3x+28, \\ 5x &= 24, \\ x &= 4\tfrac{4}{5}, \\ y &= 3\tfrac{2}{5}. \end{aligned}$$

Thus the range of y is $f(x) \leqslant 3\tfrac{2}{5}$.

QUESTION 1.

a) $\sqrt{5^2+12^2} = 13.$

b) $80° \Rightarrow \frac{4}{9}\pi$ radians

c) $\frac{\sqrt{2}}{3+\sqrt{2}} \times \frac{3-\sqrt{2}}{3-\sqrt{2}} = \frac{3\sqrt{2}-2}{7}$

d) $\frac{1}{x} = \frac{1}{3} + \frac{1}{4} = \frac{7}{12}$

$x = \frac{12}{7}$

e) $f(-1) = 2(-1)+1$
$= -1$

$f(2x+1) = 2(2x+1)+1$
$= 4x+3$

f) $A<0 \quad |A| = -A$

g) $x = 0.1555\ldots$
$10x = 1.555\ldots$
$100x = 15.555\ldots$
$90x = 14$
$x = \frac{14}{90} = \frac{7}{45}$

h) i) $1-8a^3 = 1-(2a)^3$
$= (1-2a)(1+2a+4a^2)$

ii) $6a^2-a-2$
$= (3a-2)(2a+1)$

h) iii) x^2-x-y^2+y
$= x^2-y^2-x+y$
$= (x-y)(x+y)-(x-y)$
$= (x-y)(x+y-1)$

i) $\frac{x-5}{2} - \frac{x-1}{6}$

$= \frac{3(x-5)}{6} - \frac{x-1}{6}$

$= \frac{3x-15-x+1}{6}$

$= \frac{2x-14}{6} = \frac{x-7}{3}$

j) i) $x \in \mathbb{R}: x \neq -1$

ii) $x \in \mathbb{R}: x \geq 3$

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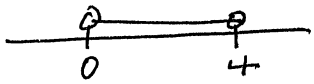
QUESTION 2.

a. $x^2 < 4x$

$\therefore x^2 - 4x < 0$

$x(x-4) < 0$

$0 < x < 4$



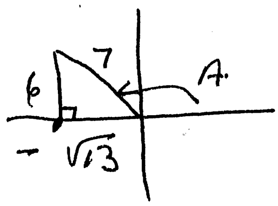
✓✓✓

b. (i) $\sin 150^\circ = \sin 30^\circ$

$= \frac{1}{2}$ ✓

(ii) $\sec \frac{7\pi}{4} = \frac{1}{\cos \frac{7\pi}{4}} = \frac{1}{\cos \frac{\pi}{4}} = \frac{1}{1/\sqrt{2}} = \sqrt{2}$ ✓✓

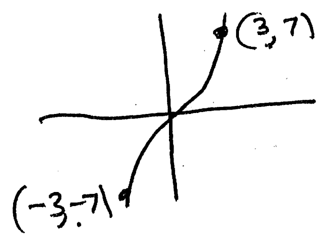
(iii)



$\cos A = \frac{-\sqrt{13}}{7}$ ✓✓

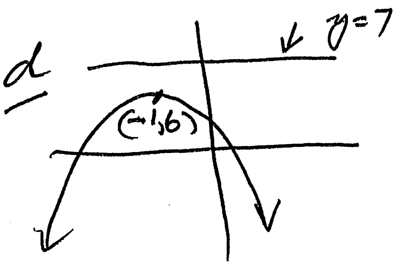
c. (i) $f(-3) = -7$ ✓

(ii)



✓

d.



(i) Focus $(-1, 5)$ ✓

(ii) $(x+1)^2 = -4(y-6)$ ✓

e. (i) $y = x^2 + x + 1$

$y' = 2x + 1$ ✓

(ii) $y = \frac{x^2 + 1}{x}$

$= x + x^{-1}$ ✓

$y' = 1 - \frac{1}{x^2}$

(iii) $y = x\sqrt{x}$

$= x^{3/2}$

$y' = \frac{3}{2} x^{\frac{1}{2}}$ ✓

$= \frac{3\sqrt{x}}{2}$

(iv) $y = (x^2+1)^5$

$y' = 5(x^2+1)^4 \times 2x$

$= 10x(x^2+1)^4$ ✓✓

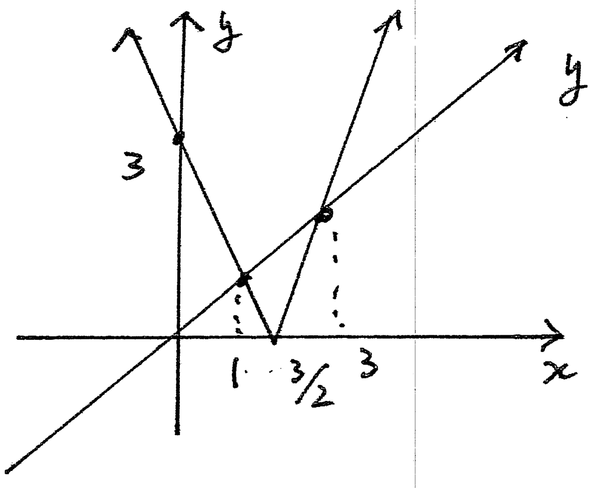
(v) $y = \frac{x}{x^2+1}$

$y' = \frac{(x^2+1).1 - x.2x}{(x^2+1)^2}$

$= \frac{1 - x^2}{(x^2+1)^2}$ ✓✓

Question 5

(i)



(ii)

$2x - 3 = x$

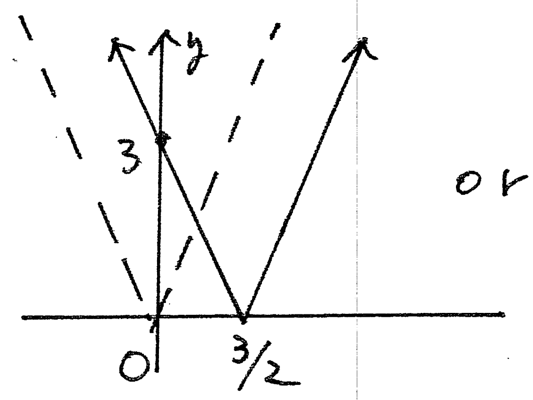
$\boxed{x = 3.}$

$2x - 3 = -x$

$\boxed{x = 1}$

(iii) $x > 3$ or $x < 1$.

(iv)



$m > 2$ or $m < -2$

(b) $P = 10000.$

$M = 450.$

$R = 1.03$ Let A_n be the amount left in the fund after the nth award.

(i) $A_1 = 10000(1.03) - 450$

$A_2 = A_1(1.03) - 450$

$= [10000(1.03) - 450](1.03) - 450$

$= 10000(1.03)^2 - 450(1 + 1.03).$

$= 9695.50$

$\vdots$

$A_n = 10000(1.03)^n - 450(1 + 1.03 + 1.03^2 + \cdots + 1.03^{n-1})$

(ii) $A_n = 10000(1.03)^n - 450\left(\frac{(1.03)^n - 1}{0.03}\right).$

$= 10000(1.03)^n - 15000[(1.03)^n - 1]$

$= 15000 - 5000(1.03)^n$

$= 5000[3 - (1.03)^n].$