SYDNEY BOYS HIGH
MOORE PARK, SURRY HILLS

MAY 2009
HALF-YEARLY EXAMINATION
YEAR 11 Accelerants

Mathematics

General Instructions:

- Reading time—5 minutes.
- Working time—90 minutes.
- Write using black or blue pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may NOT be awarded for messy or badly arranged work.
- Answer in simplest exact form unless otherwise stated.

Total marks—88 Marks

- Attempt ALL questions.
- The mark-value of each question is boxed in the right margin.
- Start each NEW question in a separate answer booklet.
- Hand in your answer booklets in 8 sections: Q1, Q2, Q3, Q4, Q5, Q6, Q7, and Q8.

Examiner: Mr D. Hespe

This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.

Marks

Question 1 (11 marks)

(a) Write down the next two terms of the following series:

(i) $2 + 4 + 6 + \ldots$ $\boxed{1}$

(ii) $2 + 4 + 8 + \ldots$ $\boxed{1}$

(iii) $1 + 3 + 6 + 10 + \ldots$ $\boxed{1}$

(b) Express $0.1\dot{2}\dot{3}$ as a fraction. $\boxed{2}$

(c) Write down the exact value of $\sec 30°$. $\boxed{1}$

(d) Sketch $y = |x - 2|$ for $-1 \leqslant x \leqslant 4$, showing all significant features. $\boxed{3}$

(e) Factorise $54p^3 - 16$. $\boxed{2}$

Marks

Question 2 (11 marks)
(Use a separate writing booklet.)

(a) Sketch the function $y = \dfrac{1}{x-2}$, showing all significant features. $\boxed{2}$

(b) Tom and Jerri are throwing a party. During the party the door-bell rang 9 times. The first time the bell rang, only one guest arrived. Each time the bell rang after that, twice as many guests arrived as had arrived on the previous ring. How many guests arrived at the party altogether? $\boxed{3}$

(c) Solve for x:

(i) $6^x . \left(6^2\right)^3 = 1$, $\boxed{1}$

(ii) $8^x = 4^{1-2x}$. $\boxed{2}$

(d) Find the distance between the lines $2x + 3y - 6 = 0$ and $4x + 6y + 14 = 0$. $\boxed{3}$

Marks

Question 3 (11 marks)

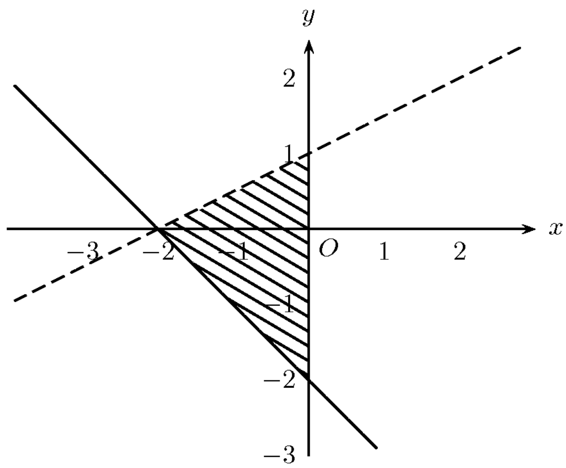
(Use a separate writing booklet.)

(a) Evaluate: $\displaystyle\sum_{n=3}^{5}\big(2 \times 3^n - 2n\big)$. $\boxed{3}$

(b) Graph the solution set of $|2x + 5| < 3$ on a number line. $\boxed{2}$

(c) Find the equation of the line that passes through the point $A(4, -2)$ and the point of intersection, B, of the lines with equations $4x + 2y + 2 = 0$ and $3x + 5y - 9 = 0$. $\boxed{3}$

(d) Using inequalities, describe the region shaded in the graph below: $\boxed{3}$



Marks

Question 4 (11 marks)
(Use a separate writing booklet.)

(a) For the arithmetic series

$$5 + 8 + 11 + \dots ,$$

find the

(i) n^{th} term, 1

(ii) 50^{th} term, 1

(iii) sum of the first 50 terms. 1

(b) For the geometric series

$$1 + {}^1\!/_2 + {}^1\!/_4 + \dots ,$$

find the

(i) general term, 1

(ii) sum of the first 20 terms (leave your answer in index form). 1

(c) Solve $|x - 3| = 2 + |2x + 2|$. 3

(d) Consider the parabola with equation $y^2 = 4(x - 3)$.

(i) Find the coördinates of the vertex of the parabola. 1

(ii) Find the coördinates of the focus of the parabola. 2

Marks

Question 5 (11 marks)

(Use a separate writing booklet.)

(a) Find the value of x if three successive terms of an arithmetic series are $\boxed{2}$

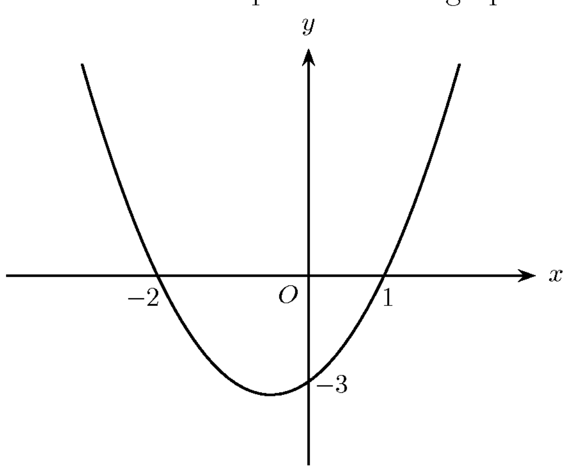
$$x + 1,\ 2x,\ 5x - 13.$$

(b) Find the simultaneous solution to the equations $\boxed{3}$

$$\begin{aligned} 2x + 2y + z &= 2, \\ 6x - y - 2z &= 7, \\ -4x + 7y + 3z &= 3. \end{aligned}$$

(c) A point moves such that it is always twice as far from the x-axis as it is from the y-axis and it passes through the point $(2, -1)$. $\boxed{2}$
Find the equation of its locus.

(d) Write down the equation of the graph shown below. $\boxed{4}$



Marks

Question 6 (11 marks)
(Use a separate writing booklet.)

(a) Is it possible to have an infinite geometric series with a limiting sum of $\dfrac{3}{8}$ and a first term of 1? [Give clear reasons.] 3

(b) (i) Find the locus of a point which moves such that it is always more than twice as far from (4, 2) as it is from (0, 1). 3

(ii) Make a careful sketch of the locus above, showing all important features. 2

(c) (i) Simplify $\log_b b^b$. 1

(ii) Make x the subject of $y = ae^{4x}$. 2

Marks

Question 7 (11 marks)
(Use a separate writing booklet.)

(a) The line $y = x + b$ meets the parabola $y = x^2$ at A and B.

(i) Show that the x-coördinates of A and B satisfy $x^2 - x - b = 0$. $\boxed{1}$

(ii) Find the sum of the roots of this quadratic, and hence show that the midpoint M of AB lies on the vertical line $x = 1/2$. Sketch the situation. $\boxed{4}$

(b) (i) The fifth term of an arithmetic series is 18 and the nineteenth term is 39. Calculate the value of the first term. $\boxed{3}$

(ii) The fifth term of an geometric series is 162 and the eighth term is 4374. Find the first term of the series. $\boxed{3}$

Marks

Question 8 (11 marks)

(Use a separate writing booklet.)

(a) Prove by consideration of series, or otherwise, that **3**

$$\sum_{r=1}^{n}(2r-1)=n^2.$$

(b) A man invests \$200 per month in an investment fund which promises 9% p.a., compounded monthly.

(i) Show that the value of the first investment will be $\$200 \times (1.0075)^{120}$ when the fund matures at the end of ten years. **1**

(ii) Find the value of the third investment when the fund matures. **1**

(iii) How much in total will he have at the end of the ten years? **2**

(c) (i) How many positive integer powers of $\frac{1}{5}$ are greater than 10^{-10}? **2**

(ii) Simplify $p^{q\log_p q}$. **2**

End of Paper

Question 1.

a) i) 8, 10 1

ii) 16, 32 1

iii) 15, 21 1

b) $x = 0.1\dot{2}\dot{3}$

$10x = 1.\dot{2}\dot{3}$

$1000x = 123.\dot{2}\dot{3}$

2

$990x = 122$

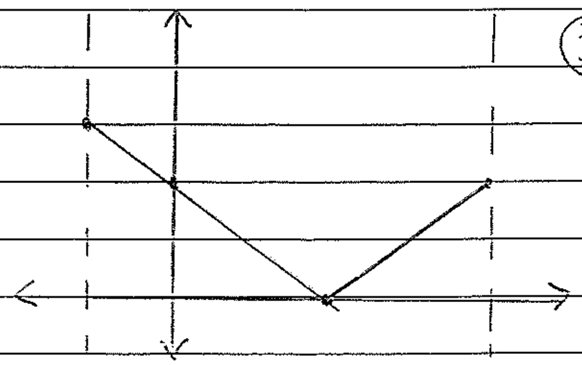
$x = \frac{122}{990} = \frac{61}{495}$

c) $\sec 30^\circ = \frac{1}{\cos 30^\circ}$ 1

$= \frac{1}{\sqrt{3}/2}$

$= \frac{2}{\sqrt{3}}$

d) 3



$y = |x-2|$

$y = |-1-2|$
$= |-3|$
$= 3$

$y = |4-2|$
$= |2|$
$= 2$

$y = |0-2|$
$= |-2|$
$= 2$

$0 = |x-2|$
$x = 2$

e) $54p^3 - 16$ 2

$= 2(27p^3 - 8)$

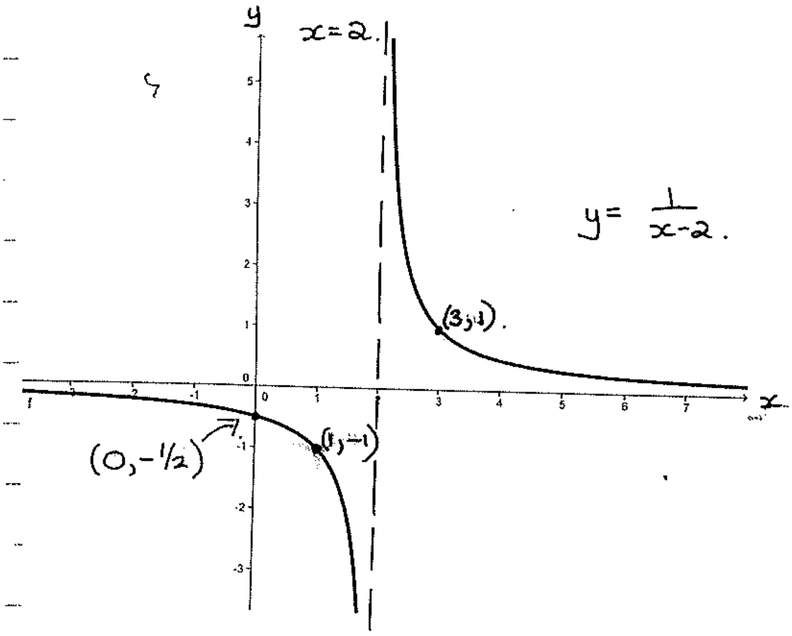
$= 2((3p)^3 - (2)^3)$

$= 2(3p-2)(9p^2+6p+4)$

YEAR 11 2009 ACCELERANTS HALF YEARLY EXAMINATION.

QUESTION TWO.

a)



b) $\sum_{n=0}^{8} 2^n$

$= 1^0 + 2 + 4 + 8 + 16 + 32 + 64 + 128 + 256$

$= 511.$

OR.

$a = 1 \quad n = 9 \qquad S_n = \frac{a(r^n - 1)}{r-1}$

$r = 2.$

$S_9 = \frac{1(2^9 - 1)}{2-1}$

$= 511$

c) i) $6^x \times (6^2)^3 = 1.$

$6^x = \frac{1}{6^6} = 6^{-6}$

$x = -6$

ii) $8^x = 4^{1-2x}.$

$(2^3)^x = 2^{2(1-2x)}.$

$3x = 2 - 4x.$

$7x = 2$

$x = 2/7$

d) $2x + 3y - 6 = 0$ —— ①

$4x + 6y + 14 = 0$ —— ②

① + ② are parallel $m = -2/3$

Take a point P on line ① P(0,2)

Perp dist from P(0,2) to line 2

$d = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \frac{|4(0) + 6(2) + 14|}{\sqrt{16 + 36}}$

$= \frac{26}{\sqrt{52}} = \frac{26}{2\sqrt{13}} = \frac{13}{\sqrt{13}} = \sqrt{13}$

Question (3) Solution's

11

(a) $\sum_{n=3}^{5}(2\times3^n - 2n)$

$= 2\left(\sum_{n=3}^{5}(3^n - n)\right)$ 1

$= 2[27+81+243-(12)]$

$= 2(351-12)$ 1

$= 2\times 339$

$= \boxed{678}$ 1

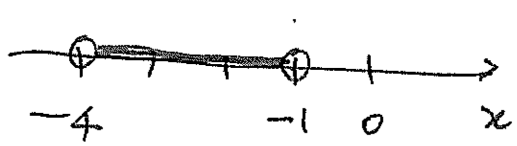
$\boxed{3}$

(b) $|2x+5| < 3$

$-3 < 2x+5 < 3$ 1

$-8 < 2x < -2$

$\boxed{-4 < x < -1}$



1

$\boxed{2}$

(c) Method ①

K — method

The line passes through $(4,-2)$ and the point of intersection of $4x+2y+2=0$ and $3x+5y-9=0$ has the form 1

$4x+2y+2+k(3x+5y-9)=0$

$\because$ it passes through $(4,-2)$

$\therefore 16-4+2+k(12-10-9)=0$

i.e $7k=14 \therefore k=2.$ 1

$\therefore$ The line is

$4x+2y+2+6x+10y-18=0$

$\boxed{5x+6y-8=0}$ 1

Method ②

$4x+2y = -2$ — ①

$3x+5y = 9$ — ②

① × 5, ② × 2.

$20x+10y = -10$

$6x+10y = 18$

$\therefore 14x = -28 \Rightarrow x=-2.$

$\therefore y = 3$

$(-2,3)$ The equation passes through $(-2,3)$ $(4,-2)$

is $\frac{y-3}{x+2} = \frac{5}{-6}$ $y = \frac{-5}{6}x+\frac{4}{3}$

i.e $5x+6y-8=0$

(d) choose $(0,0)$ to test the "equations" of lines

1
1
1

link and int.

$$\left\{\begin{array}{l} y < \frac{1}{2}x+1 \\ y \geq -x-2 \\ x \leq 0 \end{array}\right.$$

$\boxed{3}$

the region is the intersection of the above

$x-2y+2>0$
$x+y+2>0$
$x<0$

$\boxed{3}$

Question 4 YR11 MY. ACCEL.

a (i) $3n+2$ 1

(ii) 152. 1

(iii) $S_{50} = \frac{50}{2}(5+152)$ 1

$= 3925$

b (i) $\left(\frac{1}{2}\right)^{n-1}$ or 2^{1-n}. 1

(ii) $S_{20} = \frac{1-\left(\frac{1}{2}\right)^n}{1-\frac{1}{2}}$

$= \frac{2.2^n - 2}{2^n}$

$= 2 - 2^{1-n}$ 1

$= 2 - 2^{-19}$

(c) 4 cases

I	+	+
II	+	−
III	−	+
IV	−	−

I $x-3 = 2+2x+2$

$x = -7$.

II $x-3 = 2-2x-2$ 2

$x = 1$

III $-x+3 = 2+2x+2$

$x = -\frac{1}{3}$

IV $-x+3 = 2-2x-2$ 3

$x = -3$.

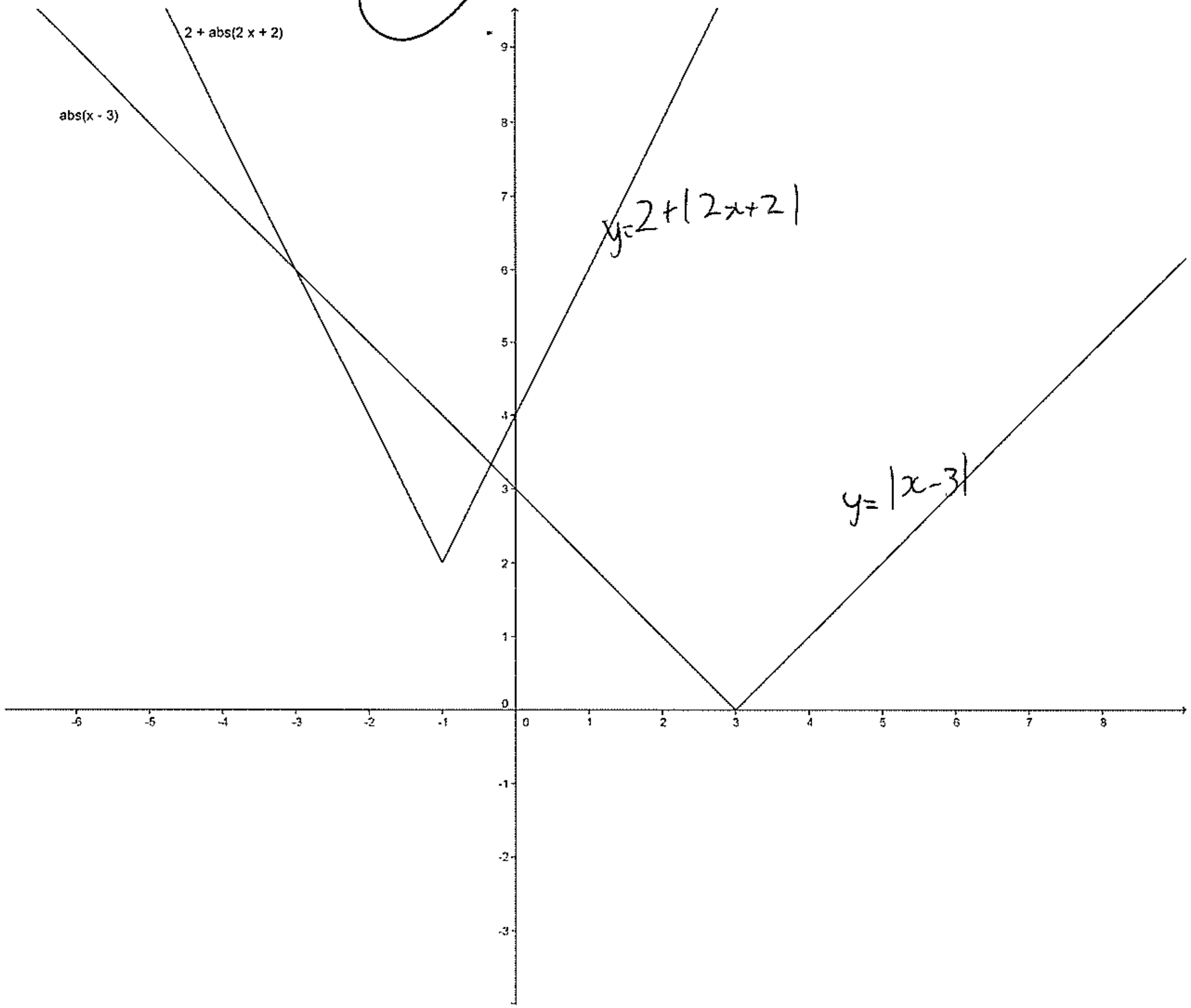
Only III and IV work. 1

$\therefore$ $x = -\frac{1}{3}, -3$.

d (i) $(3,0)$ 1

(ii) $(4,0)$. 2.

(c)



5) a) $T_2 - T_1 = T_3 - T_2$ for arithmetic series

$$2x - (x+1) = 5x - 13 - 2x$$
$$x - 1 = 3x - 13$$
$$12 = 2x$$
$$\underline{\underline{x = 6}}$$

b)
$$2x + 2y + z = 2 \quad \text{—— (1)}$$
$$6x - y - 2z = 7 \quad \text{—— (2)}$$
$$-4x + 7y + 3z = 3 \quad \text{—— (3)}$$

Rearrange (1)

$$z = 2 - 2x - 2y \quad \text{—— (1a)}$$

sub (1a) into (2)

$$6x - y - 2(2 - 2x - 2y) = 7$$
$$10x + 3y = 11 \quad \text{—— (4)}$$

sub (1a) into (3)

$$-4x + 7y + 3(2 - 2x - 2y) = 3$$
$$-10x + y = -3 \quad \text{—— (5)}$$

(4) + (5)

$$4y = 8$$
$$y = 2$$

sub into (4)

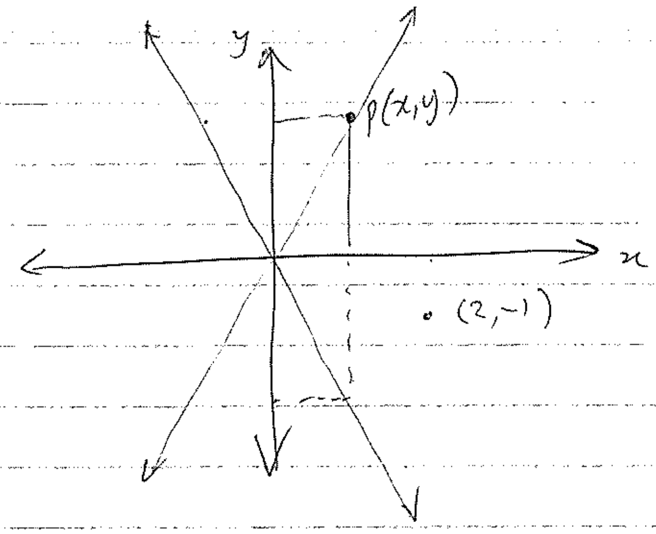
$$10x + 3(2) = 11$$
$$10x = 5$$
$$x = \frac{1}{2}$$

sub into (1a)

$$z = 2 - 2(\tfrac{1}{2}) - 2(2)$$
$$z = -3$$

$\therefore\ x = \frac{1}{2},\ y = 2,\ z = -3$

c)



P must lie on the line $y = 2x$ or $y = -2x$

<u>Note:</u> Neither line passes through $(2, -1)$

If the point was $(-1, 2)$ or $(1, -2)$ then the equation of its locus would be $y = -2x$.

d) $y = a(x+2)(x-1)$

when $x = 0$, $y = -3$

$-3 = a(2)(-1)$

$-2a = -3$

$a = \frac{3}{2}$

$\therefore y = \frac{3}{2}(x+2)(x-1)$

Question 6 Year 11 Half Yearly 2009 Accelerants.

(a) $S_\infty = \dfrac{a}{1-r}$

$\dfrac{3}{8} = \dfrac{1}{1-r}$

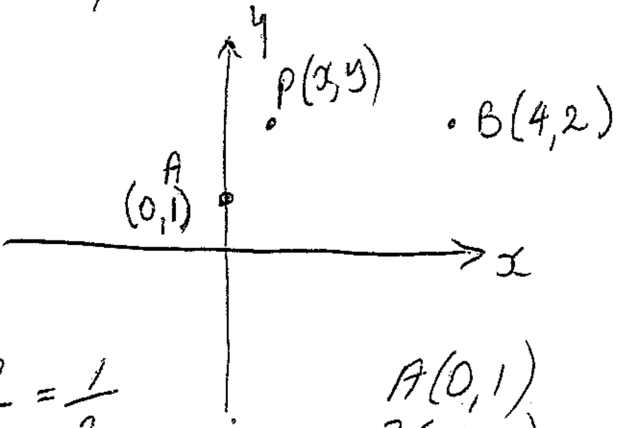
$3 - 3r = 8$

$-3r = 5$

$r = -\dfrac{5}{3}$ (3)

no! $-1 < r < 1$ is the range needed for S_∞ to exist.

(b)



let

$\dfrac{AP}{PB} = \dfrac{1}{2}$ A(0,1)

P(x,y)

$2AP = 1PB$ B(4,2)

$2\sqrt{(x-0)^2+(y-1)^2} = \sqrt{(x-4)^2+(y-2)^2}$

$4\left[x^2+y^2-2y+1\right] = x^2-8x+16+y^2-4y+4$

$4x^2+4y^2-8y+4 = x^2-8x+y^2-4y+20$

$3x^2+3y^2+8x-4y-16=0$ (3)

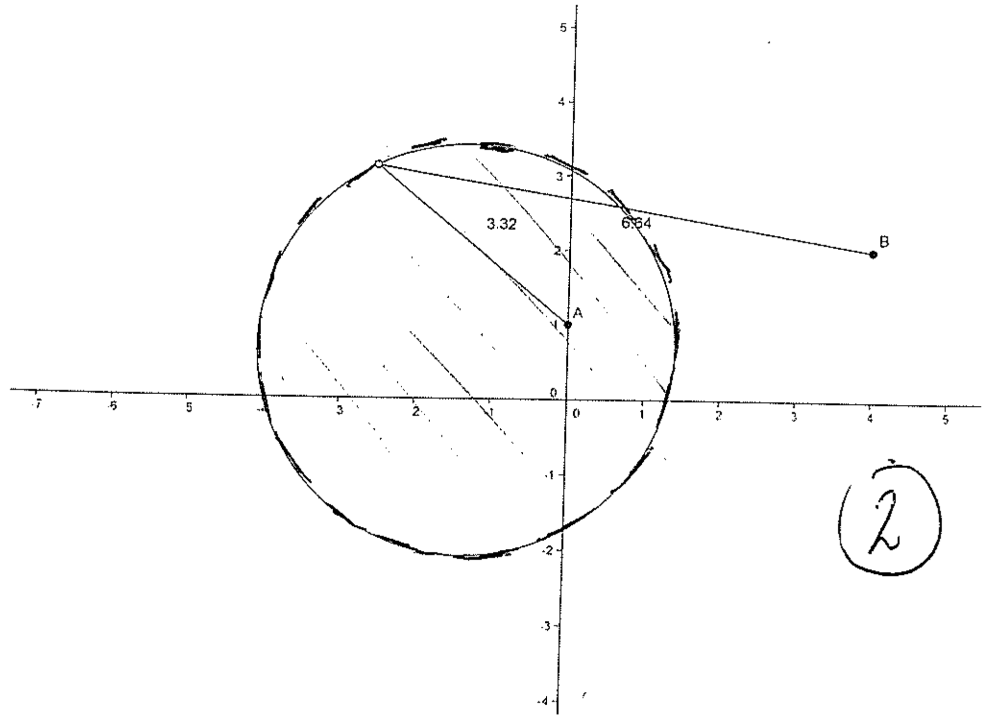
(ii) $3x^2+8x+3y^2-4y=16$

$\div 3$ $x^2+\frac{8}{3}x+\frac{16}{9}+y^2-\frac{4}{3}y+\frac{4}{9} = \frac{16}{3}+\frac{16}{9}+\frac{4}{9}$

$\left(x+\frac{4}{3}\right)^2+\left(y-\frac{2}{3}\right)^2 = \frac{68}{9}$ $C\left(-\frac{4}{3}, \frac{2}{3}\right)$

$r \doteq 2.7$.

now establish the region which reflects the question given



2

(c) (i) $\log_b b^b = b \log_b b = b$ 1

(ii) $y = ae^{4x}$

$\frac{y}{a} = e^{4x}$

$\log_e\left(\frac{y}{a}\right) = \log_e e^{4x}$

$\log_e\left(\frac{y}{a}\right) = 4x \log_e e$

$4x = \log_e\left(\frac{y}{a}\right)$

$x = \frac{1}{4}\log_e\left(\frac{y}{a}\right)$ 2

OR// answer to any legitimate base, say $\log_{10}$

So $y = ae^{4x}$

$\frac{y}{a} = e^{4x}$

$\log_{10}\left(\frac{y}{a}\right) = \log_{10} e^{4x}$

$\log_{10}\left(\frac{y}{a}\right) = 4x \log_{10} e$

so $x = \frac{\log_{10}\left(\frac{y}{a}\right)}{4\log_{10} e}$

OR//

$y = ae^{4x}$

$\log_{ae} y = \log_{ae} ae^{4x}$

$\log_{ae} y = 4x$

$x = \frac{\log_{ae} y}{4}$

Or//

$\log_{10} y = \log_{10} ae^{4x}$

$\log_{10} y = 4x \log_{10} ae$

$x = \frac{\log_{10} y}{4\log_{10} ae}$ if $a=1$

OR// $\log y = \log a + \log e^{4x}$

$\frac{\log y - \log a}{4\log e} = x$

OR//

$$y = ae^{4x}$$

$$e^{4x} = \frac{y}{a}$$

$$e^{4} \times e^{x} = \frac{y}{a}$$

$$e^{x} = \frac{y}{ae^{4}}$$

$$\log e^{x} = \log\left(\frac{y}{ae^{4}}\right)$$

$$x \log e = \log y - \log(ae^{4})$$

$$x = \frac{\log y - \log(ae^{4})}{\log e}$$

Question 7

(a) (i) let $x^2 = x + b \Rightarrow x^2 - x - b = 0$

& solving simultaneously gives the x coords of the intersection pts of the two graphs. (1)

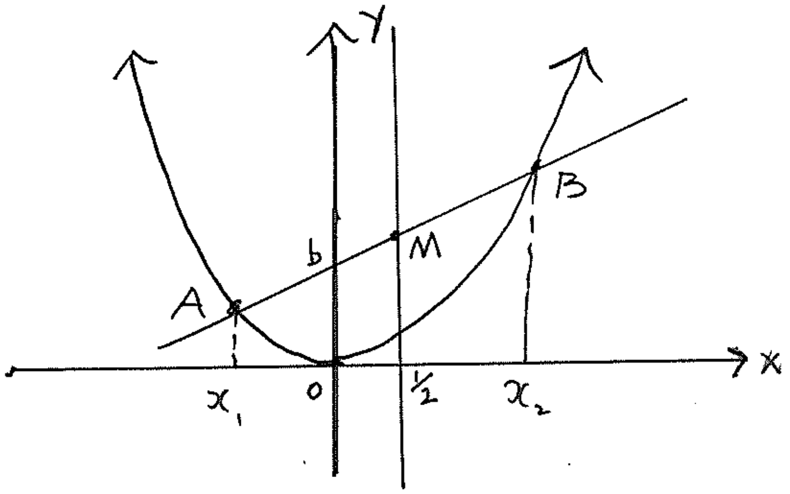
(ii) sum of roots $= \dfrac{-(-1)}{1} = 1$ (1)

$\therefore$ x Coords of A and B can be denoted by x_1, x_2 where $x_1 + x_2 = 1$ (1)

$\Rightarrow$ x coord. of the midpt of AB is $\dfrac{x_1 + x_2}{2} = \dfrac{1}{2}$.

$\therefore$ Coords of M are $\left(\frac{1}{2}, \frac{1}{2} + b\right)$ since M lies on $y = x + 2$ (1)

M lies on the vertical line $x = \frac{1}{2}$.



(1)

OR x Coords of M: Solve $x^2 - x - b = 0 \Rightarrow x = \dfrac{1 \pm \sqrt{1+4b}}{2}$

let $x_1 = \dfrac{1 - \sqrt{\ }}{2}$ $\quad x_2 = \dfrac{1 + \sqrt{\ }}{2}$ $\quad \therefore \dfrac{x_1 + x_2}{2} = \dfrac{1}{2}$

etc. Note $1 + 4b \geq 0 \Rightarrow \boxed{b \geq -\frac{1}{4}}$

(b) (i) $T_5 = 18 \Rightarrow a + 4d = 18$ —— ① ($\frac{1}{2}$)

$T_{19} = 39 \Rightarrow a + 18d = 39$ —— ② ($\frac{1}{2}$)

② − ① $\Rightarrow 14d = 21$ ie $\boxed{d = \frac{3}{2}}$ (1)

Subst. into ① $\Rightarrow a + 4\left(\frac{3}{2}\right) = 18$ (1)

$\boxed{a = 12}$

(ii) $T_5 = 162 \Rightarrow ar^4 = 162$ —— ① ($\frac{1}{2}$)

$T_8 = 4374 \Rightarrow ar^7 = 4374$ —— ② ($\frac{1}{2}$)

② ÷ ① $\Rightarrow r^3 = \frac{4374}{162} = 27$ (1)

$\therefore \boxed{r = 3}$

Subst into ① $\Rightarrow a(3)^4 = 162$

$81a = 162$

$\boxed{a = 2}$ (1)

QUESTION 8

(a) $\sum_{r=1}^{n}(2r-1) = 1+3+5+\cdots+(2n-1)$ ie ARITHMETIC SERIES with n terms

$$S_n = \frac{n(a+l)}{2}$$
$$= \frac{n(1+(2n-1))}{2}$$
$$= \frac{n \times 2n}{2}$$
$$= n^2.$$

(b) (i) 9% p.a $= 0.75\%$ p.m.
$= 0.0075$

& 10 years = 120 months.

$\therefore$ Using $A = P(1+r)^n$

$$A = \$200 \times (1+0.0075)^{120}$$
$$= \$\,200(1.0075)^{120}.$$

(ii) $I_3 = 200\,(1.0075)^{118}$
$= \$\,483.00.$

(iii) $\text{TOTAL} = 200(1.0075)^{120} + 200(1.0075)^{119} + \cdots$
$\cdots + 200(1.0075)^{1}$

$$= 200\left[1.0075^1 + \cdots + 1.0075^{119} + 1.0075^{120}\right]$$
$$= 200 \times \frac{1.0075 \times (1.0075^{120}-1)}{1.0075-1}$$

[using $S_n = \frac{a(r^n-1)}{r-1}$.]

$= \$38933.13$

(c) (i) Let $\left(\frac{1}{5}\right)^n > 10^{-10}$.

Taking reciprocals

$$5^n < 10^{10}$$

Taking logs of both sides.

$$\log_{10} 5^n < \log_{10} 10^{10}$$

$$n \log_{10} 5 < 10 \log_{10} 10$$

$$n \log_{10} 5 < 10.$$

$$n < \frac{10}{\log_{10} 5}$$

$$n < 14.3 \quad \text{(calculator)}$$

$\therefore$ n is 14

(NB NO MARKS IF NO WORKING SHOWN)

(ii) Let $N = p^{q \log_p q}$.

now $N = p^{\log_p q^q}$.

Take logs base p of both sides.

$$\therefore \log_p N = \log_p p^{\log_p q^q}$$

$$\log_p N = \log_p q^q \times \log_p p \quad \left[\text{NB } \log_p p = 1\right]$$

$$\log_p N = \log_p q^q$$

$$\therefore \boxed{N = q^q}.$$

[NB WORKING NEEDS TO BE SHOWN FOR ANY MARKS].