

## Question 1

Yr. 11 - Half yearly 2003

(a) Simplify -

(i).  $|2| - |-5|$

(ii).  $(3x - 6) - 2(5 - 4x)$

(iii).  $\sqrt{27} - \sqrt{3} + 3\sqrt{18}$

(b) (i). Evaluate  $\frac{143.95 - 11.61}{\sqrt{3.61}}$ , giving your answer correct to two decimal places.

(ii). Evaluate  $\frac{8.2 \times 10^6}{(3.5 \times 10^8)^2}$ , giving your answer in scientific notation correct to two significant figures.

(c) The temperature of a body in  $F$  degrees Fahrenheit or  $C$  degrees Celsius is related by the formula  $F = \frac{9C}{5} + 32$ .

(i). Find the value of  $F$  when  $C = 25$ .

(ii). Make  $C$  the subject of the formula.

## Question 2

(a) Expand and simplify -

(i).  $(y + 4)^2$

(ii).  $(4x - 3)(2x - 3)$

(b)  $g(x) = x^2 + 1$

(i). Evaluate  $g(-3)$ .

(ii). For what values of  $x$  is  $g(x) = 2$ ?

## Question 2 (continued)

(c) Solve -

(i).  $\frac{x}{3} - 2 = \frac{4x}{5}$

(ii).  $x^2 = 3x$

(iii).  $2 - 3x < 7$

(d) Factorise completely -

(i).  $1 - 8x^3$

(ii).  $48x - 3x^3$

(iii).  $ab - a + bx - x$

(iv).  $5x^2 - 2x - 3$

## Question 3

(a) Solve simultaneously -

$$\begin{cases} 3x - 2y = 29 \\ 4x + 5y = 8 \end{cases}$$

(b) Rationalise the denominator and simplify  $\frac{8}{\sqrt{5}-1}$ .

(c) Sketch, showing all major features -

(i).  $y = 2^{-x}$

(ii).  $y = x^3 + 1$

(iii).  $y = 25 - x^2$

(d) Solve  $|2x + 3| \geq 2$  and graph the solution on the number line.

## Question 4

(a). Simplify –

(i).  $\frac{x^2 - y^2}{y - x}$

(ii).  $\frac{3}{x^2 + 2x - 8} - \frac{2}{x^2 + x - 6}$

(iii).  $\frac{3x - 3y}{xy} \div \frac{x^2 - xy}{x^2}$

(b). If  $(2 + 5\sqrt{3})^2 = a + \sqrt{b}$ , find the values of  $a$  and  $b$ .

(c). Simplify –

(i).  $(-2a^4b)^3$

(ii).  $\frac{3c \times 4c^2 \times 5c^3}{3c^2 + 4c^2 + 5c^2}$

(d). Sketch  $f(x)$  given that  $f(x) = \begin{cases} x^2 & \text{for } x \leq 2 \\ 2x - 5 & \text{for } x > 2 \end{cases}$

## Question 5

(a). Write an equation of a straight line which is NOT a function.

(b). Solve  $|x - 1| = 2x - 1$ .

(c). Express  $0.1\overline{24}$  as a simple fraction. Show working.

## Question 5 (continued)

(d). State the domain for the following –

(i).  $y = x + 2$

(ii).  $y = \frac{3}{x}$

(iii).  $x^2 + y^2 = 100$

(e). State the range for the following –

(i).  $y = |x|$

(ii).  $y = -\sqrt{9 - x^2}$

(iii).  $y = 2^x - 8$

## Question 6

(a). Sketch the region satisfied by all three inequalities –

$$x^2 + y^2 \leq 25, 2x - y > 0 \text{ and } y \geq 0.$$

(b). A cyclist rides for 5 hours at a certain speed and then for 4 hours at a speed 6 km/h greater than her original speed. If she rides 655.8 km altogether, what was –

(i). her average speed?

(ii). her initial speed?

(c). For the graph  $y = 16 - 2(x + 5)^3$ , find –

(i). the  $x$ -intercept;

(ii). the  $y$ -intercept.

## Question 6 (continued)

- (d). The lines  $x = 3$ ,  $y = -2$  and  $y = 10$  are tangents to a circle which never crosses the  $y$ -axis. Find its --
- centre;
  - radius.
- (e). With full working to support your answer, determine whether the function  $f(x) = \frac{4x}{x^2 + 4}$  is odd, even or neither.
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## Question 7

- (a). Solve  $3x^2 = 7x - 1$ , leaving your answer in exact form.
- (b). Sketch, showing all major features --
- $y = -\sqrt{4 - x^2}$
  - $y = 5 - \frac{3}{x + 2}$
  - $y = |3x - 4|$
- (c). Express  $\frac{1 - x^{-1}}{x^{\frac{1}{2}}}$  in simplest form (without fractional/negative indices) and with rational denominator. [Assume  $x$  is an integer.]
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## Question 8

- (a). Find values for  $A$ ,  $B$  and  $C$  if  $y^2 + 16y + 94 - 6x$  is expressed in the form  $(y + C)^2 - B(x + A)$ .
- (b). If  $x + y = 1$  and  $x^3 + y^3 = 19$ , find the value of  $x^2 + y^2$ .
- (c). Sketch  $x \geq \frac{1}{y}$ .
- (d). For every pair of numbers  $a$  and  $b$ , the function  $f(x)$  satisfies  $b^2 f(a) = a^2 f(b)$ . If  $f(2) \neq 0$ , find the value of  $\frac{f(5) - f(1)}{f(2)}$ .
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Total = 113

Question 1

- (a) (i)  $-3$  ①  
 (ii)  $3x - 6 - 10 + 8x = 11x - 16$  ②  
 (iii)  $3\sqrt{3} - \sqrt{3} + 9\sqrt{2} = 2\sqrt{3} + 9\sqrt{2}$  ②
- (b) (i)  $69.65$  ② [1 for answer, 1 for 2 d.p.]  
 (ii)  $6 \cdot 7 \times 10^{-11}$  ② [1 for answer in SH, 1 for sig. fig.]
- (c) (i)  $F = 77$  ①  
 (ii)  $F = \frac{9c}{5} + 32$   
 $5F = 9c + 160$   
 $c = \frac{5F - 160}{9}$  ②

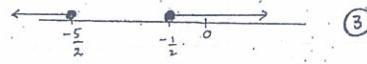
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Question 2

- (a) (i)  $y^2 + 8y + 16$  ①  
 (ii)  $8x^2 - 6x - 12x + 9 = 8x^2 - 18x + 9$  ②
- (b) (i)  $g(-3) = 10$  ①  
 (ii)  $x^2 + 1 = 2$   
 $x = \pm 1$  ②
- (c) (i)  $\frac{x-6}{3} = \frac{4x}{5}$   
 $5x - 30 = 12x$   
 $7x = -30 \therefore x = -4\frac{2}{7}$  ③
- (ii)  $x^2 - 3x = 0$   
 $x(x-3) = 0 \therefore x = 0 \text{ or } 3$  ②
- (iii)  $-3x < 5$   
 $x > -\frac{5}{3}$  ②

3. (d).

$2x + 3 \geq 2$  or  $2x + 3 \leq -2$   
 $x \geq -\frac{1}{2}$  or  $x \leq -\frac{5}{2}$



Total 17

QUESTION 4

- (a) (i)  $\frac{(x+y)(x-y)}{-(x-y)} = -x-y$  ②
- (ii)  $\frac{3}{(x+4)(x-2)} - \frac{2}{(x+3)(x-2)}$   
 $= \frac{3x+9 - (2x+8)}{(x+4)(x-2)(x+3)}$   
 $= \frac{x+1}{(x+4)(x-2)(x+3)}$  ③
- (iii)  $\frac{3(x-y)}{xy} \times \frac{x^2}{x(x-y)}$   
 $= \frac{3}{y}$  ②
- (b)  $4 + 20\sqrt{3} + 75 = 79 + \sqrt{1200}$   
 $\therefore a = 79 \quad b = 1200$  ②

2 (continued)

- (a) (i)  $(1-2x)(1+2x+4x^2)$  ①  
 (ii)  $3x(16-x^2) = 3x(4+x)(4-x)$  ②  
 (iii)  $a(b-1) + x(b-1) = (a+x)(b-1)$  ②  
 (iv)  $(5x+3)(x-1)$  ②

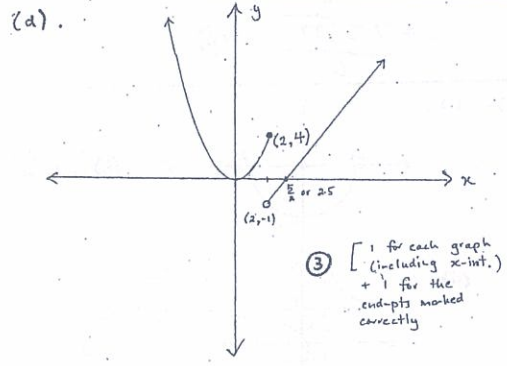
Total 20

Question 3

- (a)  $15x - 10y = 145$  }  $23x = 161$   
 $8x + 10y = 16$  }  $\therefore x = 7, y = -4$  ③
- (b)  $\frac{8}{\sqrt{5}-1} \times \frac{\sqrt{5}+1}{\sqrt{5}+1} = \frac{8\sqrt{5}+8}{5-1} = \frac{8\sqrt{5}+8}{4} = 2\sqrt{5}+2$  ③
- (c) (i) ② [1 for shape, 1 for y-int.]  
 (ii) ② [2 for both intercepts]  
 (iii) ③ [1 for shape, 1 for all x/y intercepts]

4. (c) (i)  $-8a^{12}b^5$  ①

(ii)  $\frac{60c^6}{12c^2} = 5c^4$  ②



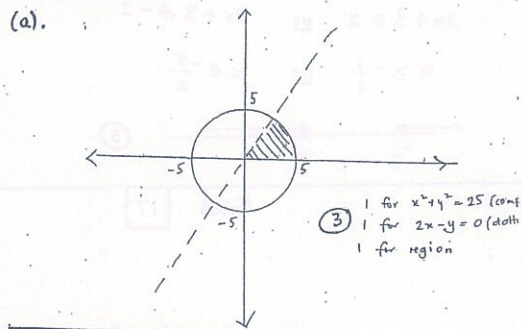
Total 15

Question 5

- (a)  $x = 2$  or  $x = -2$  etc. ①
- (b)  $x-1 = 2x-1$  or  $x-1 = 1-2x$   
 $0 = x$   $3x = 2$   
 $\therefore x = 0$  or  $\frac{2}{3}$
- Check  $x = 0$   
 $LHS = |0-1| = 1$   
 $RHS = 0-1 = -1 \quad \times$
- Check  $x = \frac{2}{3}$   
 $LHS = |\frac{2}{3}-1| = \frac{1}{3}$   
 $RHS = 2 \times \frac{2}{3} - 1 = \frac{1}{3} \quad \checkmark$
- $\therefore x = \frac{2}{3}$  only solution ③
- (c) let  $x = 0.1242424 \dots$   
 $100x = 12.4242424 \dots$   
 $99x = 12.3 \quad x = \frac{41}{330}$  ③

5. (d). (i).  $x \in \mathbb{R}$   
 (ii).  $x \in \mathbb{R}$  except  $x=0$   
 (iii).  $-10 \leq x \leq 10$  (3)
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- (e). (i).  $y \geq 0$   
 (ii).  $-3 \leq y \leq 0$   
 (iii).  $y > -8$  (3)
- Total 13

Question 6



- (b). (i). 72.87 km/h (1)  
 (ii). let  $x = \text{initial speed}$   
 $5x + 4(x+6) = 655.8$   
 $9x = 631.8$  (2)  
 $\therefore x = 70.2 \text{ km/h}$  ← initial speed
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- (c). (i).  $0 = 16 - 2(x+5)^3$   $(x+5)^3 = 8$   
 $\therefore x = -3$  (2)  
 $(-3, 0)$   
 (ii).  $y = -234$  (1)  $(0, -234)$

7. (c).  $1 - \frac{1}{x}$   
 $\frac{1}{\sqrt{x}}$   
 $= \frac{x-1}{x\sqrt{x}} \times \frac{\sqrt{x}}{\sqrt{x}}$   
 $= \frac{x\sqrt{x} - \sqrt{x}}{x^2}$  (3)

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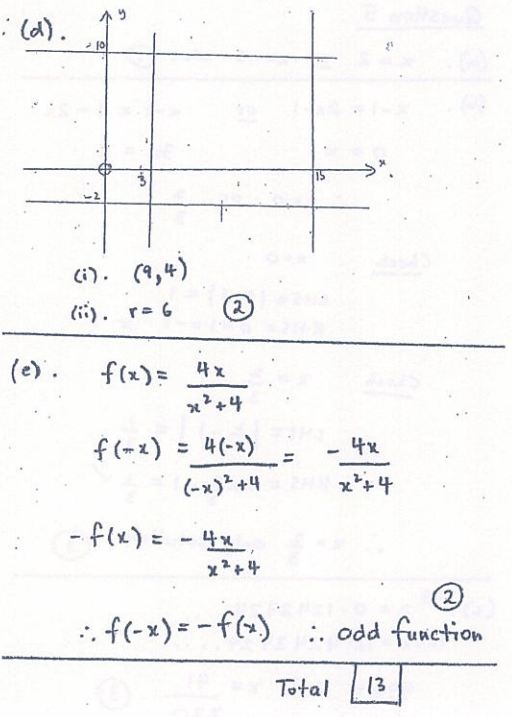
Total 13

Question 8

(a).  $y^2 + 16y + 94 - 6x$   
 $y^2 + 16y + 64 + 30 - 6x$   
 $= (y+8)^2 - 6(x-5)$   
 $A = -5$   $B = 6$   $C = 8$  (3)

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(b).  $x^3 + y^3 = (x+y)(x^2 - xy + y^2)$   
 $19 = x^2 + y^2 - xy$   
 $\therefore 19 = x^2 + (1-x)^2 - x(1-x)$   
 $19 = 3x^2 - 3x + 1$   
 $3x^2 - 3x - 18 = 0$   
 $x^2 - x - 6 = 0$   
 $(x-3)(x+2) = 0$   
 $x = 3$  or  $-2$   
 $y = -2$  or  $3$   
 $\therefore x^2 + y^2 = 9 + 4 = 13$  (3)



Question 7

