TEACHER'S NAME: _____

STUDENT'S NAME: _____

BAULKHAM HILLS HIGH SCHOOL

YEAR 11

HALF YEARLY EXAMINATION

2008

MATHEMATICS

Time allowed · Two hours
(Plus five minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions. Marks shown are a guide only.
- Start each of the 9 questions on a new page.
- All necessary working should be shown.
- Write your teacher's name and your name on the cover sheet provided.
- At the end of the exam, staple your answers in order behind the cover sheet provided.

Question 1 — Marks

a) Evaluate, correct to 3 significant figures

$$\frac{6.7^2 - 4.1^3}{3 \times 9.7 \times 5.8}$$ 2

b) Sona and Jess paid $97.40 for a meal at a restaurant. This includes a 13.5% tip. What was the cost of the meal without the tip? 1

c) Factorise the following 6

(i) $3x^2 - 2x - 1$

(ii) $24x^3 - 3$

(iii) $a^2 - b^2 - 3a - 3b$

d) Solve the following equation, leaving the answers in the simplest exact form.

$4x^2 - 2x - 1 = 0$ 3

Question 2 (Start a new page)

a) Solve the following pair of simultaneous equations.

$$2x + y = 1$$
$$5x + 3y = 1$$ 2

b) Solve the following equation,

$|x + 4| = 2x - 1$ 3

c) Given θ is an obtuse angle and $\sin\theta = \frac{5}{13}$, without finding θ, find the value of $\cos\theta$. 2

d) Simplify, leaving your answer in the simplest surd form

(i) $\sqrt{27} + \sqrt{48} - \sqrt{12}$ 2

(ii) $\frac{1}{\sqrt{5}-1} + \frac{1}{\sqrt{5}+1}$ 2

Question 3 (Start a new page)

a) Solve $3x-5(x-2)+8(2x+3)=0$ 3

b) Solve $\tan\theta=-\frac{1}{\sqrt{3}}$ for $0^\circ\le\theta\le360^\circ$ 2

c) Find x if $\cos(x+29)^\circ=\sin(3x+41)^\circ$ and angles are acute. 2

d) Find a and b if $\frac{2\sqrt{5}}{3\sqrt{5}+4}=a+b\sqrt{5}$ 3

e) Express $1.4\dot{5}$ as a fraction in its simplest form. 2

Question 4 (Start a new page)

a) Show that $\cos210^\circ\times\operatorname{cosec}150^\circ=-\sqrt{3}$ 2

b) Simplify the following

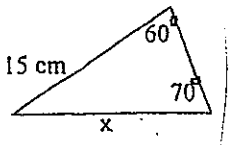
(i) $\frac{2m-n}{3}-\frac{m-3n}{6}$ 2

(ii) $\frac{x}{x^2-4}+\frac{2}{x-2}$ 2

c) Solve $-13<3x+2<8$ and plot the solution on the number line. 3

d) (i) Find the length of the side marked x. 4

(ii) Find the area of the triangle.



Question5 (Start a new page)

a) Draw a neat sketch of the following, showing all the important features. 6

(i) $y=x^2-3x-4$

(ii) $y=|2x+1|-2$

b) (i) Show that $\frac{x-3}{x+5}=1-\frac{8}{x+5}$ 1

(ii) Find the range and domain of $y=\frac{x-3}{x+5}$ 2

(iii) Hence or otherwise draw the graph of $y=\frac{x-3}{x+5}$ 2

Question 6 (Start a new page)

a) If $f(x)=\begin{cases}3-x & \text{for } x\le1\\ x^2-1 & \text{for } x>1\end{cases}$

(i) Evaluate $f(2)+f(-2)$ 2

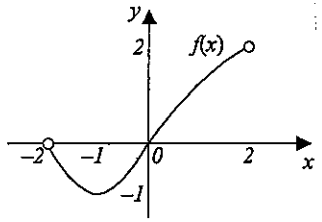
(ii) Sketch the graph of $f(x)$ 3

b) Find the range and domain of the following

(i) $y=4-3^{-x}$ 2

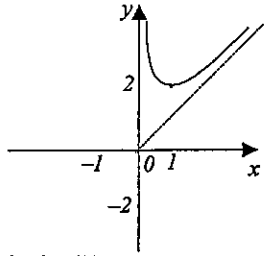
(ii) $y=\sqrt{4-x^2}$ 2

(iii) 2



Question 7(Start a new page)

a) Complete the following graph on your answer sheet if it is an odd function. 1



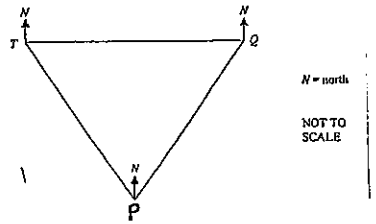
b) Simplify, leaving your answer in simplified index form:

$\frac{16}{2^{3x}\times8^{1-x}}$ 2

c) Determine if the function is odd, even or neither

$$f(x)=\frac{x^2+1}{x}$$ 2

d) The diagram represents the flight path of a plane. The plane flies from a town T to a property P, which is 83 km away on a bearing of 129°. The plane then flies to another property Q, which is 91 km away from P and due east of T.



(i) Copy the diagram into your answer sheet and mark all the given information. 1

(ii) Find $\angle TQP$ to the nearest degree. 2

(iii) What is the bearing of Q from P?

e) Solve the inequality $|5(x-3)| \geq 2$ and plot the solution on the number line. 3

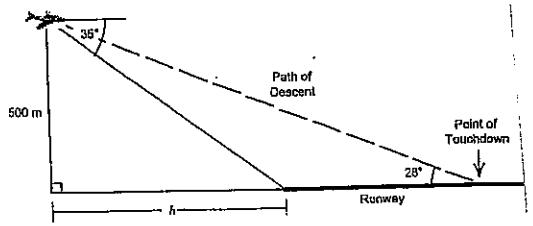
Question 8 (Start a new page)

a) On a number plane shade in the region given by the two conditions

$x^2+y^2 \leq 4$ and $x+y>1$ 3

b) If $(2-\sqrt{3})^2(1+2\sqrt{3}) = A+B\sqrt{C}$, Find the value of A, B and C. 3

c) A plane is coming in to land at an airport. When the plane is at an altitude of 500 metres, the angle of depression from the plane to the start of the runway is 35°. The plane's path of descent is along a straight line which makes an angle of 28° with the horizontal.



(i) What is the horizontal distance, h, of the plane from the start of the runway? 1

(ii) How far along the runway does the plane touch down? 2

Question 9 (Start a new page)

a) Simplify, leaving your answer without negative indices

$$\frac{x^{-2}-y^{-2}}{x^{-1}+y^{-1}}$$ 3

b) A rectangular area can be tiled with 180 square tiles. If the side of each tile was increased by 1cm, it would only take 125 tiles to tile the area. Find the side length of the smaller tile. 3

c) An airplane leaves an aircraft carrier and flies due south at 400km/h.The carrier proceeds 60° west of north at 32km/h. If the plane has enough fuel for 5 hours of flying, what is the maximum distance south the pilot can travel, so that the fuel remaining will allow a safe return to the carrier? 3

END OF EXAMINATION

Solutions Yr 11 2 unit

Ques1

a) $-0.1423\ldots$ —①

$= -0.142$ —②

b) $\text{Cost} = \frac{97.40}{1.135}$

$= \$85.81$ —①

c) (i) $(3x+1)(x-1)$ ① ①

(ii) $3(8x^3-1)$ —①

$3(2x-1)(4x^2+2x+1)$ ①

(iii) $(a+b)(a-b) - 3(a+b)$ —①

$(a+b)(a-b-3)$ —①

d) $4x^2 - 2x - 1 = 0$

$x = \frac{2 \pm \sqrt{4+16}}{8}$ —①

$x = \frac{2 \pm \sqrt{20}}{8}$

$= \frac{2 \pm 2\sqrt{5}}{8}$

$x = \frac{1 \pm \sqrt{5}}{4}$ —②

(12)

b) $|x+4| = 2x-1$

$x+4 = 2x-1$ | $x+4 = -2x+1$

$x = 5$ | $3x = -3$

① | $x = -1$ ①

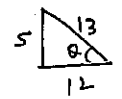
Test

$x=5$ $|9| = 2\times5-1$

$9 = 9$ ✓

$x=-1$ $|3| = -3$ ✗

$\therefore$ soln is $x=5$ ①

c) 

$\cos\theta = -\frac{12}{13}$ —①

d) (i) $3\sqrt{3} + 4\sqrt{3} - 2\sqrt{3}$ —①

$= 5\sqrt{3}$ —①

(ii) $\frac{1}{\sqrt{5}-1} + \frac{1}{\sqrt{5}+1}$

$\frac{\sqrt{5}+1+\sqrt{5}-1}{(\sqrt{5}-1)(\sqrt{5}+1)}$ —①

$\frac{2\sqrt{5}}{(5)^2-1} = \frac{2\sqrt{5}}{4} = \frac{\sqrt{5}}{2}$ —①

(11)

c) $\cos(x+29) = \sin(3x+41)$

$\therefore x+29+3x+41 = 90$ —①

$4x+70 = 90$

$4x = 20$

$x = 5^\circ$ —①

d) $\frac{2\sqrt{5}}{3\sqrt{5}+4} \times \frac{3\sqrt{5}-4}{3\sqrt{5}-4}$ —①

$\frac{6\times5 - 8\sqrt{5}}{(3\sqrt{5})^2 - 4^2}$

$= \frac{30-8\sqrt{5}}{9\times5-16}$

$= \frac{30-8\sqrt{5}}{29} = a + b\sqrt{5}$

$\therefore a = \frac{30}{29}$, $b = \frac{-8}{29}$ —①

① ①

e) $x = 1.4555\ldots$

$10x = 14.555\ldots$ (A) ①

$\therefore 100x = 145.555\ldots$ (B)

Sub. (B)-(A)

$90x = 131$

$x = \frac{131}{90}$ —①

(12)

d) $\frac{4m-2n}{6} - \frac{m-3n}{6}$ —①

$\frac{4m-2n-m+3n}{6}$

$= \frac{3m+n}{6}$ —①

c) $\frac{x}{x^2-4} + \frac{2(x+2)}{(x+2)(x-2)}$ —①

$\frac{x+2x+4}{x^2-4}$

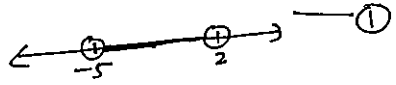
$= \frac{3x+2}{x^2-4}$ —①

d) $-13 < 3x+2 < 8$

$-15 < 3x < 6$

$-5 < x < 2$ —①

① ①



d) (i) $\frac{x}{\sin60} = \frac{15}{\sin70}$ —①

$x = \frac{15\times\sin60}{\sin70}$

$x = 13.824$ —②

or 13.8 cm

(ii) $A = \frac{1}{2}ab\sin C$

$C = 180-60-70$

$= 50^\circ$ —①

$= \frac{1}{2}\times15\times13.8\times\sin50$ —①

$= 79.285$

(or 79.42)

(13)

12 a) $2x+y = 1$ — 1

$5x+3y = 1$ — 2

multiply 1 × 3

$6x+3y = 3$ — ①

$5x+3y = 1$

$x = 2$ ①

Q3 a) $3x - 5x + 10 + 16x + 24 = 0$ —①

$14x + 34 = 0$ —①

$x = -\frac{34}{14}$ or $-\frac{17}{7}$ —①

b) $\tan\theta = -\frac{1}{\sqrt{3}}$ ✓ ✓

acute angle $= 30^\circ$

$\theta = 180-30,\ 360-30$

$= 150^\circ,\ 330^\circ$

Q4 a) LHS $= -\cos30 \times \frac{1}{\sin30}$

① ①

$= -\frac{\sqrt{3}}{2} \times \frac{2}{1}$

$= -\sqrt{3} = \text{RHS}$

Q5 a (i) $y = (x-4)(x+1)$

x int $= 4, -1$

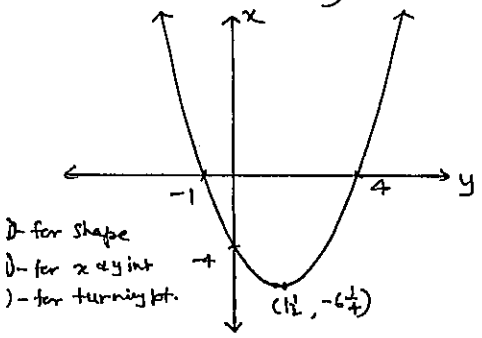
y int $= -4$

Axis of sym $x = \frac{3}{2}$

$y = \frac{9}{4} - \frac{9}{2} - 4$

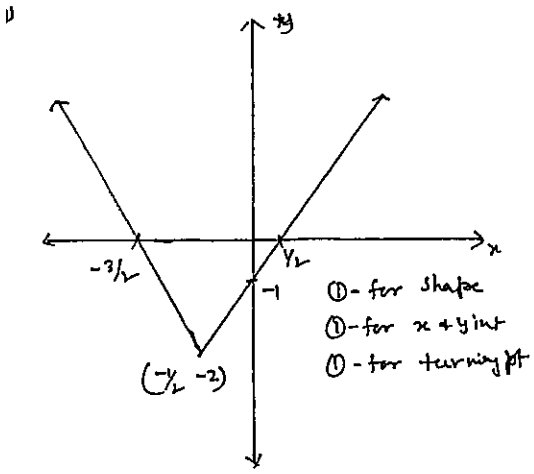
$= -6\frac{1}{4}$

Vertex $(\frac{3}{2}, -6\frac{1}{4})$



①- for shape
①- for x & y int
①- for turning pt.

(ii)



①- for shape
①- for x & y int
①- for turning pt

(ii) $\frac{x-3}{x+5} = \frac{x+5-8}{x+5}$

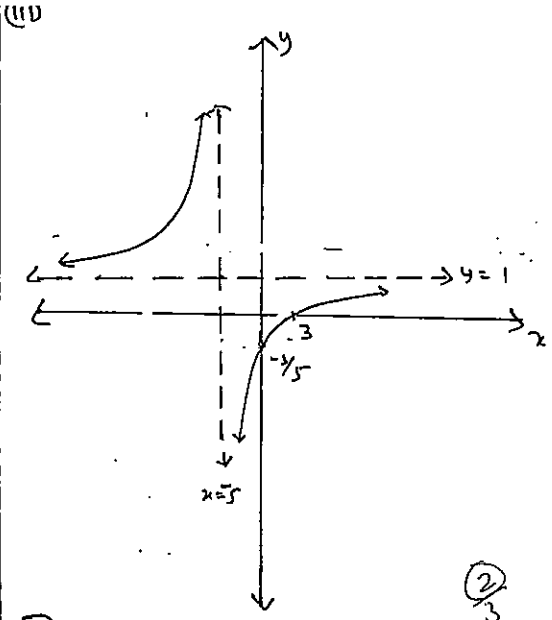
$= 1 - \frac{8}{x+5}$ —①

(need to show working)

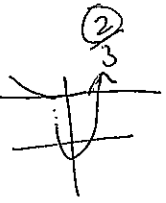
(iii) D: all real x, $x \neq -5$ —①

R: all real y, $y \neq 1$ —①

(iii)



(11)

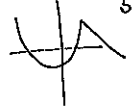


Q6 a)(i) $f(2) = 2^2 - 1$

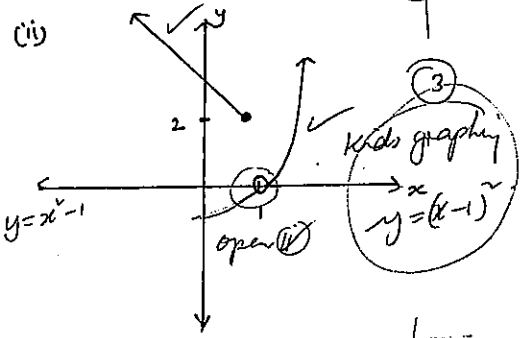
$= 3$ —①

$f(-2) = 3 + 2 = 5$ —① ①

$\therefore f(2) + f(-2) = 8$



(ii)



b) (i) D: all real x —①

R: $y \leq 4$ —①

(ii) D: $-2 \leq x \leq 2$ —①

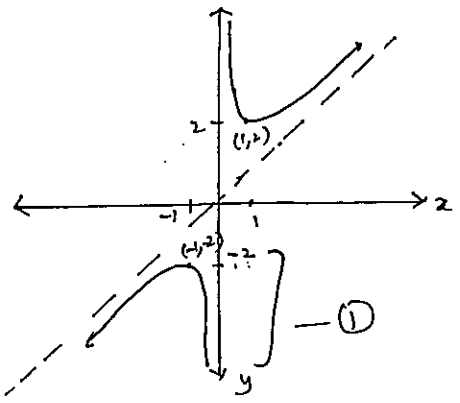
R: $0 \leq y \leq 2$ —①

(iii) D: $-2 < x < 2$ —①

R: $-1 \leq y < 2$ —①

(11)

Q7



—①

b) $\frac{16}{2^{3x} \times 8^{1-x}}$

$= \frac{2^4}{2^{3x} \times 2^{3-3x}}$ —①

$= \frac{2^4}{2^3} = 2$ —①

c) $f(x) = \frac{(-x)^2+1}{(-x)}$ —①

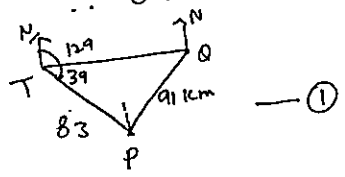
$= \frac{x^2+1}{-x}$

or $\frac{-x^2-1}{x}$

$= -f(x)$

$\therefore$ odd —①

d)



—①

(ii) $T\hat{Q}P$

$\frac{\sin\theta}{83} = \frac{\sin 39}{91}$ —①

$\sin\theta = \frac{83 \times \sin 39}{91}$

$= 0.5739$

$\theta = 35.029$

$\doteq 35°$ —①

(iii) $T\hat{P}Q = 180 - (39 + 35)$

$= 106$

$\therefore$ Bearing $= 55°T$ —①

e) $|5(x-3)| \geq 2$

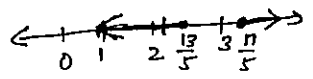
$5x - 15 \geq 2$, $5x - 15 \leq -2$

$5x \geq 17$ | $5x \leq 13$

$x \geq \frac{17}{5}$ | $x \leq \frac{13}{5}$

①

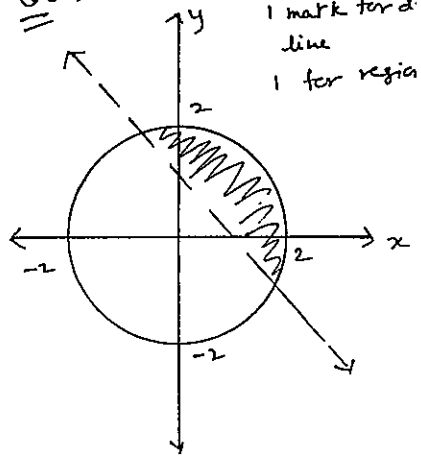
①



①

(12)

Q8 a)



1 mark for c
1 mark for d
line
1 for regio

a) $(2-\sqrt{3})^2(1+2\sqrt{3})$

$= (4+3-4\sqrt{3})(1+2\sqrt{3})$

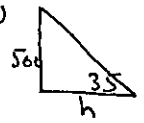
$= (7-4\sqrt{3})(1+2\sqrt{3})$

$= 7-4\sqrt{3}+14\sqrt{3}-8\times 3$

$= -17+10\sqrt{3}$

$\therefore A=-17 \quad B=10, \quad C=3.$ (1) (1) (1)

c) (i)

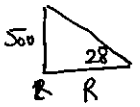


$\tan 35 = \frac{500}{h}$

$h = \frac{500}{\tan 35}$

$= 714.074..$ (1)

(ii)



$\tan 28 = \frac{500}{R}$

$R = \frac{500}{\tan 28}$ (1)

$= 940.36$

$\therefore \quad = R-h$

$= 226.289$ (1)

9a) $\dfrac{\frac{1}{x^2}-\frac{1}{y^2}}{\frac{1}{x}+\frac{1}{y}} = \dfrac{\frac{y^2-x^2}{x^2y^2}}{\frac{y+x}{xy}}$ —(1)

$\frac{(y+x)(y-x)}{x^2y^2} \times \frac{xy}{y+x} = \frac{y-x}{xy}$ —(1)

b)

$x^2 \times 180 = (x+1)^2 \times 125$ —(1)

$180x^2 = 125x^2 + 250x + 125$

$55x^2 - 250x - 125 = 0$

$11x^2 - 50x - 25 = 0$

$x = \frac{50 \pm \sqrt{50^2 + 4\times 25\times 11}}{22}$ —(1)

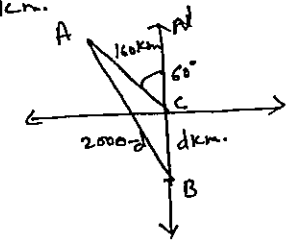
$= \frac{50 \pm 60}{22}$

$x = 5, -0.45$

$\therefore$ tile length $= 5$ cm. —(1)

c) let d km be the distance south that the pilot can travel.

In 5 hrs, the plane can travel 2000 km & the carrier travels 160 km.



$AC = 160$ km

$BC = d$ km, $AB = 2000-d$

$A\hat{C}B = 120°$ —(1)

$\therefore (2000-d)^2 = 160^2 + d^2 - 2(160)d\cos 120$ —(1)

$4000000 + d^2 - 4000d = 25600 + d^2 + 160d$

$4160d = 3974400$

$d \doteq 955.39$ —(1)

$\therefore$ Pilot travel 955km South before turning back for a safe return.

———x———