BAULKHAM HILLS HIGH SCHOOL

HALF YEARLY EXAMINATION

2009

YEAR 11

MATHEMATICS ADVANCED

Time Allowed:

Two hours
(plus 5 mins reading time)

Instructions:

- Write in the answer booklets provided
- Do not write on this question paper
- Show all working
- Use black or blue pens only
- Start a new page for each question
- Write your name and your teacher's name at the top of each page
- Board approved calculators may be used

Question 1 (10 marks) **Marks**

a) The retail price of a computer is $940.50. This includes 10% GST. What is the GST on the computer? **1**

b) Evaluate $\frac{2\left(1-\left(\frac{2}{3}\right)^7\right)}{1-\frac{2}{3}}$ correct to 2 decimal places **2**

c) If $1^3+2^3+3^3+\ldots\ldots\ldots+n^3=\frac{n^2(n+1)^2}{4}$

i) Find $1^3+2^3+\ldots\ldots\ldots+10^3$ **1**

ii) Find $11^3+12^3+\ldots\ldots\ldots+30^3$ **2**

d) Express $0.1\dot{7}$ as a fraction in its simplest form. **2**

e) Light travels at 300000 km/s. How far does light travel in 1 year? Give your answer in scientific notation, correct to 2 significant figures. **2**

Question 2 (12 Marks)

a) Simplify $\sqrt{8}-2\sqrt{18}$ **1**

b) If $\sqrt{A}=\frac{3\sqrt{6}}{\sqrt{3}}$ find the value of A **2**

c) If $\frac{3\sqrt{2}-1}{\sqrt{2}+1}=a+b\sqrt{2}$ find the values of a and b **2**

d) Expand $(x-2)^3$ **2**

e) Factorise

i) $x^2 - 5x - 6$ **1**

ii) $4x^2 - 100$ **2**

iii) $x^4 - x^3 - xy^3 + y^3$ **2**

Marks

Question 3 (11 Marks)

a) Simplify:

i) $\dfrac{x}{x^2-2xy}$ **1**

ii) $\dfrac{1}{a+2} + \dfrac{1}{a^2-4}$ **2**

b) Solve:

i) $1+\dfrac{6x}{5} = 4x$ **2**

ii) $\dfrac{2x+3}{2} - \dfrac{x+4}{4} = -3$ **3**

iii) $(1-2x)^2 = 9$ **3**

Question 4 (13 marks)

a) If $\cos\theta = \dfrac{2}{7}$ and $\sin\theta < 0$, find the exact value of $\tan\theta$ **3**

b) Find the exact value of $\sec 570^\circ$ **2**

c) Sketch the graph of the following, showing all essential features.

i) $y=2^x+1$ **2**

ii) $(x-2)^2 + (y+1)^2 = 4$ **2**

d) State the domain and range for each of the graphs in Question 4 c). **4**

Question 5 (10 marks)

a) If $f(x) = \begin{cases} x^2 & x \le 0 \\ \frac{x}{2} & 0 < x \le 1 \\ \frac{1}{x} & x > 1 \end{cases}$

i) find $f(-2)+f(1)+f(1\frac{1}{2})$ **2**

ii) find $f(a^2+2)$ **1**

b) Solve

i) $9-x^2 \le 4$ **2**

ii) $|x-2| = 2x+3$ **3**

iii) $|4-x| \le 2$ **2**

Question 6 (10 marks)

a) Solve $\tan^2\theta - 1 = 0$ for $0 \le \theta \le 360^0$ **3**

b) Prove

$$\frac{\cos\alpha}{1+\sin\alpha} + \frac{\cos\alpha}{1-\sin\alpha} = 2\sec\alpha$$ **2**

c) If $\frac{6^{2n-1}}{18^{1-4n}} = 2^a \times 3^b$ find expressions for a and b in terms of n **3**

d) Draw a function which has a domain of $x \neq 2$ and a range of $y < 0$ where x and y are real numbers **2**

Question 7 (11 marks)

Marks

a) Solve simultaneously

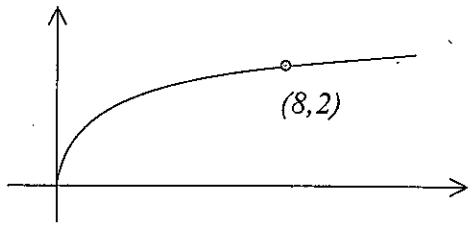
$y = \dfrac{10}{x+1}$ and $y = x - 2$ **2**

b) Simplify

$$\frac{a - a^{-1}}{1 - \dfrac{1}{a^{-2}}}$$ **3**

c) A person walks 10km west then 6km north. **3**
What is his bearing **back to** his starting point?

d) Below is a part of a function $y = f(x)$ which is odd.



i) Copy and complete the graph. **1**

ii) If $f(x) + 2 = 0$, find x **2**

Question 8 (10 marks) Marks

a) Sketch the region which satisfies both inequalities.

$xy \geq 1$ and $y \leq \sqrt{4-x^2}$ **3**

b) i) Sketch on the same set of axes

$y = 4 - x^2$ and $y = 2|x-2|$ **3**

ii) Hence, solve $4 - x^2 > 2|x-2|$ **1**

c) The parabola $y = ax^2 + bx + c$ has a vertex at (2,1) and passes through (0,0). Find a, b and c. **3**

Question 9 (12 marks)

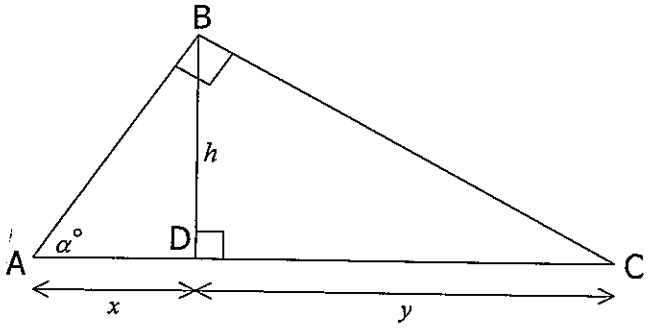
a) If $f(x) = x^2$ and $g(x) = \dfrac{2}{x+1}$

i) Show that if $g[f(x)] = f[g(x)]$ then $2x^2 - 4x + 2 = 0$ **2**

ii) Hence find the values of x for which $g[f(x)] = f[g(x)]$ **2**

b) If $x + y = 1$ and $x^3 + y^3 = 19$, find the value of $x^2 + y^2$ **3**

c)



i) By finding 2 expressions for h, show that $x \tan \alpha = y \tan(90 - \alpha)$ **2**

ii) Hence, show $\dfrac{y}{x} = \tan^2 \alpha$ **1**

iii) For what values of α is $y > x$? **2**

Question 1: — [10]

a) $110\% = \$940.50$

$10\% = 940.50 \div 11$

$= \$85.50$ (1)

(1)

b) $5.6488\ldots = 5.65$ (1)

c) (i) $1^3 + \ldots + 10^3 = \frac{10^2(11)^2}{4}$

$= 3025$ (1)

(ii) $11^3 + \ldots + 30^3 = S_{30} - S_{10}$

(1)

$= \frac{(30)^2(31)^2}{4} - 3025$

$= 216225 - 3025$

$= 213200$ (1)

d) let $x = 0.1777\ldots$

$10x = 1.777\ldots$

$100x = 17.777\ldots$ } (1)

$\therefore 90x = 16$

$x = \frac{16}{90}$

If $\frac{17}{99}$ (1) $= \frac{8}{45}$ (1)

Solutions: - Yr 11 2 Unit ½ Yearly.

(1)

e) Dist. $= 300000 \times 3600 \times 24 \times 365$

$= 9.5 \times 10^{12}$ (1)

Question 2. [12]

a) $\sqrt{8} - 2\sqrt{18} = 2\sqrt{2} - 6\sqrt{2}$

$= -4\sqrt{2}$ (1)

b) $\sqrt{A} = \frac{3\sqrt{6}}{\sqrt{3}}$

$= 3\sqrt{2}$

$= \sqrt{18}$ (1)

$\therefore A = 18$ (1)

c) $\frac{(3\sqrt{2}-1)}{(\sqrt{2}+1)} \times \frac{(\sqrt{2}-1)}{(\sqrt{2}-1)}$ (1)

$= \frac{6 - 3\sqrt{2} - \sqrt{2} + 1}{2-1}$

$= 7 - 4\sqrt{2}$

$\therefore a = 7 \quad b = 4$ (1)

d) $(x-2)^3 = (x-2)(x^2 - 4x + 4)$

1 off each error $= x^3 - 4x^2 + 4x - 2x^2 + 8x - 8$

$= x^3 - 6x^2 + 12x - 8$ (2)

e) (i) $x^2 - 5x - 6 = (x-6)(x+1)$

(ii) $4x^2 - 100 = 4(x^2 - 25)$

2 for 3 factors $= 4(x+5)(x-5)$

1 for 2 factors

(iii) $x^4 - x^3 - xy^3 + y^3$

$= x^3(x-1) - y^3(x-1)$

$= (x^3 - y^3)(x-1)$ (1)

$= (x-y)(x^2 + xy + y^2)(x-1)$ (1)

Question 3. [11]

a) (i) $\frac{x}{x^2 - 2xy} = \frac{1}{x(x-2y)}$

$= \frac{1}{x-2y}$ (1)

(ii) $\frac{1}{a+2} + \frac{1}{a^2-4}$

$= \frac{a-2+1}{(a+2)(a-2)}$ (1) $= \frac{a-1}{(a+2)(a-2)}$ (1)

3b) (i) $1 + \frac{6x}{5} = 4x$

$5 + 6x = 20x$ (1)

$14x = 5$

$x = \frac{5}{14}$ (1)

(ii) $\frac{2x+3}{2} - \frac{x+4}{4} = -3$

$\frac{4x+6-x-4}{4} = -3$ (1)

$3x + 2 = -12$ (1)

$3x = -14$

$x = -\frac{14}{3}$ (1)

(iii) $(1-2x)^2 = 9$

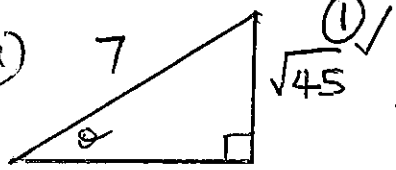
$1 - 2x = \pm 3$ (1)

$-2x = -1 \pm 3$

$x = \frac{-1 \pm 3}{-2}$

$x = -1, 2$ (1) (1)

Question 4. [15]

a) 

(1)✓

$\tan\theta = -\frac{\sqrt{45}}{2}$ (1) (1)

$= \left(-\frac{3\sqrt{5}}{2}\right)$

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b) $\sec 570° = \frac{1}{\cos 210°}$

$= \frac{1}{-\cos 30°}$

$= (1)\checkmark \frac{2}{\sqrt{3}}$ ← (1)

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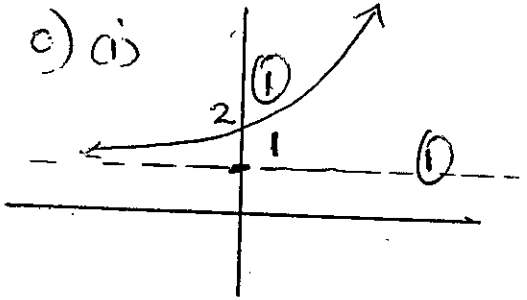
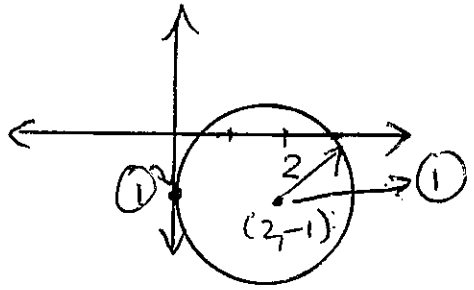
c) (i)



(ii)



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d) (i) D: all real x (1)

R: $y > 1$, y is real (1)

(ii) D: $0 \le x \le 4$, x is real (1)

R: $-3 \le y \le 1$, y is real (1)

Question 5. [10]

a) (i) $f(-2) + f(1) + f(1\frac{1}{2})$

$= (-2)^2 + \frac{1}{2} + \frac{1}{(\frac{3}{2})}$ −1 for each error

$= 5\frac{1}{6}$ (2)

(ii) $f(a^2+2) = \frac{1}{a^2+2}$ (1)

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b) (i) $9 - x^2 \le 4$

$-x^2 \le -5$

$x^2 \ge 5$ −1 for wrong sign

$x \le -\sqrt{5}, \quad x \ge \sqrt{5}$ (2)

(ii) $|x-2| = 2x+3$ −1 not checked

$x - 2 = 2x + 3$ $\quad -x + 2 = 2x + 3$

$-5 = x$ $\quad -1 = 3x$

$x = -\frac{1}{3}$

Check $|-7| = -7$ False $\quad |-2\frac{1}{3}| = 2\frac{1}{3}$ ✓

$\therefore x = -\frac{1}{3}$ (3)

ii) $|4-x| \leq 2$ 1 off each error.

$-4+x \leq 2$ $\quad 4-x \leq 2$

$x \leq 6$ $\quad -x \leq -2$

$x \geq 2$

$\therefore 2 \leq x \leq 6$ (2)

Question 6. [10] 45, 225 - 1

) $\tan^2\theta - 1 = 0$ 1 off each error.

$\tan\theta = \pm 1$

$\theta = 45, 135, 225, 315$ (3)

) $\frac{\cos\alpha}{1+\sin\alpha} + \frac{\cos\alpha}{1-\sin\alpha} = 2\sec\alpha$

LHS =

$\frac{\cos\alpha - \sin\alpha\cos\alpha + \cos\alpha + \cos\alpha\sin\alpha}{1-\sin^2\alpha}$ (1)

$= \frac{2\cos\alpha}{\cos^2\alpha}$ (1)

$= 2\sec\alpha$

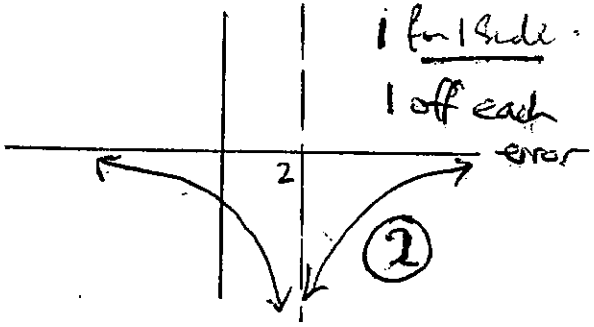
$= RHS.$

) $\frac{6^{2n-1}}{18^{1-4n}} = \frac{2^{2n-1}\cdot 3^{2n-1}}{2^{1-4n}\cdot(3^2)^{1-4n}}$ (1)

$= 2^{2n-1-(1-4n)}\cdot 3^{2n-1-(2-8n)}$ (1)

$= 2^{6n-2}\cdot 3^{10n-3}$ (1)

d) 1 for side. 1 off each error.



(2)

Question 7. [11]

a) $y = \frac{10}{x+1}$ b) $y = x-2$

$x-2 = \frac{10}{2x+1}$

$x^2 - 2x + x - 2 = 10$

$x^2 - x - 12 = 0$

$(x-4)(x+3) = 0$

$x = 4, -3$ (1)

$y = 2, -5$ (1)

$(4,2)$ $\quad (-3,-5)$

b) $\frac{a-a^{-1}}{1-\frac{1}{a^{-2}}} = \frac{a-\frac{1}{a}}{1-a^2}$

$= \frac{a^2-1}{a}$

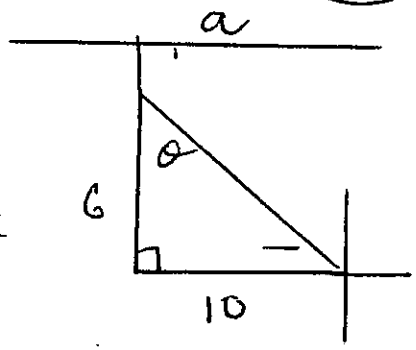
1 off each error $\frac{a}{1-a^2}$

2 for $\frac{a-1}{a(1-a)} = \frac{(a+1)(a-1)}{a(1-a)(1+a)}$

2 for answer a

$= \frac{-1}{a}$ (3)

c)



1 off each error

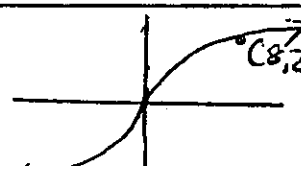
$\tan\theta = \frac{10}{6}$

$\theta = 59°2'$ (1)

$\therefore$ Bearing $= 180 - 59°2'$ (1)

$= 120°58'$ (1)

d) (i)



(1)

(d)(ii) If $f(x)+2=0$

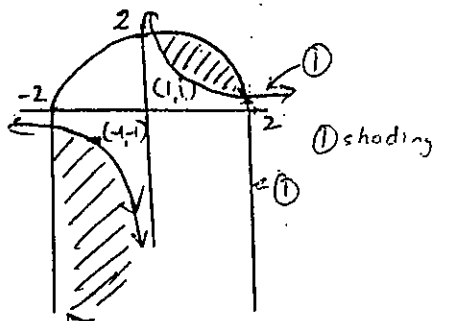
$f(x)=-2$ (1)

Since its odd. $f(-8)=-2$.

$\therefore x=-8$ (1)

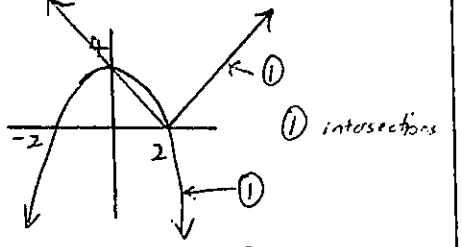
Question 8 [10] $xy \geq 1$ $y \leq \sqrt{4-x^2}$

a)



(1) shading (1) (1)

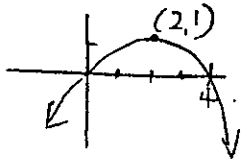
b)(i)



(1) (1) intersections (1)

ii) $0 < x < 2$ (1)

c)



$y = ax^2+bx+c$

(0,0) satisfies $\Rightarrow c=0$ (1)

(2,1) " $\therefore 1 = 4a+2b$ (1)

also (4,0) satisfies

$\therefore$ (1) since $\begin{cases} 0 = 16a+4b & (2) \\ 2 = 8a+4b & (3) \end{cases}$

(1) $\times 2$

(2) $-$ (3) $\quad -2 = 8a$

$a = -\frac{1}{4}$

$b = 1$

$\therefore a=-\frac{1}{4}$ $b=1$ $c=0$ (1)

Question 9. $f(x)=x^2$

$g(x) = \frac{2}{x+1}$

$g[f(x)] = g(x^2)$

$= \frac{2}{x^2+1}$

$f[g(x)] = f\left(\frac{2}{x+1}\right)$

$= \left(\frac{2}{x+1}\right)^2$

$= \frac{4}{(x+1)^2}$

(1) for both

$\frac{2}{x^2+1} = \frac{4}{(x+1)^2}$

$2(x+1)^2 = 4(x^2+1)$

$2x^2+4x+2 = 4x^2+4$ (1)

$2x^2-4x+2=0$

(ii) $2(x^2-2x+1)=0$

$2(x-1)^2=0$

$\therefore x=1.$

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b) $x+y=1$ $\quad x^3+y^3=19$

$x^3+y^3 = (x+y)(x^2-xy+y^2)$ (1)

$19 = 1(x^2-xy+y^2)$

$\therefore x^2-xy+y^2 = 19$ - (A)

now $(x+y)^2 = 1$

$\therefore x^2+2xy+y^2 = 1$ - (B)

(A) $-$ (B) $\quad -3xy = -18$

$xy = -6$ (1)

sub into (1)

$x^2+6+y^2 = 19$

$x^2+y^2 = 13.$ (1)

9(c)(i) $\tan\alpha = \frac{h}{x}$

$\therefore h = x\tan\alpha.$ (1)

$\angle BCD = 90-\alpha$

$\therefore \tan(90-\alpha) = \frac{h}{y}$

$h = y\tan(90-\alpha)$ (1)

(ii) $\therefore x\tan\alpha = y\tan(90-\alpha)$

$x\tan\alpha = y\cot\alpha$

$x\tan\alpha = \frac{y}{\tan\alpha}$ (1)

$\therefore \frac{y}{x} = \tan^2\alpha$

(iii) $y > x$

then $\frac{y}{x} > 1$

$\therefore \tan^2\alpha > 1$ (1)

α is acute

$\therefore 45 < \alpha < 90$ (1)