

Student's Name:.....

Teacher's Name:.....



Year 11

Half Yearly Examination

2010

**MATHEMATICS ADVANCED**

Time allowed – 2 hours  
(plus 5 minutes reading time)

**DIRECTIONS TO CANDIDATES:**

- Attempt ALL questions. Marks shown are a guide only.
- All necessary working must be shown.
- Write in blue or black pen and do not use liquid paper
- Write your teacher's name and your name on this cover sheet.

**Question 1. Start on a new page**

- a) Evaluate correct to two significant figures:  $\sqrt[3]{\frac{5.21+4.71}{3.94 \times 1.76}}$  2
- b) Factorise fully :  $10x + 2xy - 10y - 2y^2$  2
- c) Simplify :  $2\sqrt{3} - \sqrt{27} + \sqrt{48}$  2
- d) Expand and simplify:  $(x-3)(x^2 + 5x - 1)$  2
- e) If  $f(x) = 9 - 2x^2$ , find the value of  $f(-1)$  1

**Question 2. Start on a new page**

- a) Rationalise the denominator :  $\frac{5}{5-\sqrt{2}}$  2
- b) If  $T = 2\pi \sqrt{\frac{l}{g}}$
- i) Find the value of T, if  $l = 80$  and  $g = 9.8$  1
- ii) Make g the subject of the formula 2
- c) Solve for x:  $\frac{3x}{x+1} - \frac{2}{3} = 0$  3
- d) Express  $0.14\dot{2}$  as a fraction in its simplest terms. 1

**Question 3. Start on a new page**

- a) Factorise completely
- i)  $8x^3 - 1$  2
- ii)  $(2a - b + c)^2 - (a + 5b - c)^2$  2
- b) Find the domain and the range of  $y = \frac{3}{2x-1}$  2
- c) Evaluate  $\sin 60^\circ + \tan^2 30^\circ$ . Leave your answer in exact form. 2

Question 4. Start on a new page

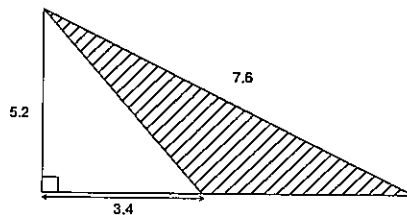
- a) Solve
- i.  $3x + y = 4$   
 $2x + 3y = 1$  3
  - ii.  $x^4 - 16 = 0$  2
  - iii.  $\frac{1-8x}{5} < 9$  3
- b) Find the value of  $a^3b^2$  in the index form if  $a = (\frac{1}{2})^3$  and  $b = (\frac{4}{5})^2$ , expressing your answer in the form of  $2^x5^y$  2
- c) Simplify  $\frac{1}{\sqrt{2}+1} + \frac{2}{\sqrt{2}-1}$  2

Question 5. Start on a new page

- a) Sketch the following and show the important features. Write down the domain and range of each function.
- i.  $y = x^2 - x - 6$  3
  - ii.  $y = (x-3)^3$  3
- b) Solve  $|2x + 1| \leq 7$  and graph the solution on a number line 3

Question 6. Start on a new page

- a) Simplify  $\frac{3}{x+1} + \frac{2}{x^2-1} - \frac{4}{x-1}$  3
- b) Solve  $|5x - 1| = 3x + 3$  3
- c) Find the area of the shaded region correct to two decimal places: 3

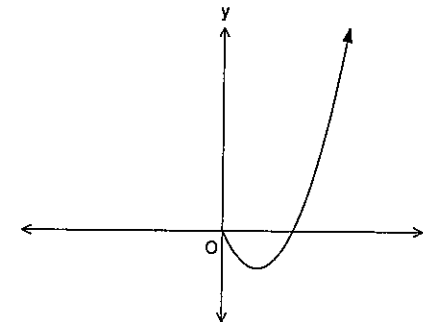


Question 7. Start on a new page

- a) If  $f(x) = \begin{cases} x^2 & \text{when } x \geq 2 \\ 5x & \text{when } 0 \leq x < 2 \\ -3 & \text{when } x < 0 \end{cases}$ ,
- sketch the graph of  $f(x)$  for  $-3 \leq x \leq 5$  4
- b) Find all possible values of  $A$  if  $2 \sin A = \cos A$  and  $0^\circ \leq A \leq 360^\circ$  2
- c) Solve  $9x^2 + 6x - 4 = 0$  by completing the square. Leave the answer in exact form 3

Question 8. Start on a new page

- a) Three points A, B and C lie on a horizontal ground. From B the bearing of A is  $047^\circ T$  and the bearing of C is  $081^\circ T$ . From A the bearing of C is  $149^\circ T$ . The distance between A and B is 10km.
- i. Draw a diagram and mark all the information given above. 3
  - ii. Find the distance from A to C. 2
- b) If  $y = f(x)$  is an odd function, copy and complete the graph below 2



Question 9. Start on a new page

- a) Find a value of  $\beta$  if  $\cos \beta = \sin 43^\circ 50'$  2
- b) i. Show that  $\frac{x+3}{x+1}$  is  $1 + \frac{2}{x+1}$  1
- ii. Hence or otherwise sketch the graph of  $y = \frac{x+3}{x+1}$  showing any intercepts and asymptotes 3
- iii. If  $f(x) = \frac{x^3 - 3x^2 + 1}{x(1-x)}$ , prove that  $f(\frac{1}{x}) = f(1-x)$  where  $x \neq 0, 1$  3

END OF EXAMINATION

1. a)  $1.1267 \dots \checkmark$   
 $1.1 \checkmark$  (2)

b)  $2\sqrt{(5x+xy-5y-y^2)}$   
 $= 2(5+y)(x-y)$  (2)

c)  $= 2\sqrt{3} - 3\sqrt{3} + 4\sqrt{3} \checkmark$   
 $= 3\sqrt{3} \checkmark$  (2)

d)  $= x(x^2+5x-1) - 3(x^2+5x-1) \checkmark$   
 $= x^3+5x^2-x-3x^2-15x+3$   
 $= x^3+2x^2-16x+3 \checkmark$  (2)

e)  $f(-1) = 9 - 2(-1)^2$   
 $= 7 \checkmark$  (1) (9)

3 a) (i)  $= (2x-1)(4x^2+2x+1)$  (2)

(ii)  $= [(2a-b+c) - (a+5b-c)] [2a-b+c+a]$   
 $= (a+4b+2c)(3a+4b)$  (2)

b)  $D = \{x : x \in \mathbb{R}, x \neq \frac{1}{2}\} \checkmark$   
 $R = \{y : y \in \mathbb{R}, y \neq 0\} \checkmark$  (2)

c)  $= \frac{\sqrt{3}}{2} + \frac{1}{3} \checkmark$   
 $= \frac{3\sqrt{3}+2}{6} \checkmark$  (2) (8)

4 a) (i)  $3x+y=4$  — ①  
 $2x+3y=1$  — ②

①  $\times 3$  — ②  $\Rightarrow 7x = 11 \checkmark$   
 $\therefore x = \frac{11}{7} \checkmark$   
 $\therefore y = 4 - 3 \times \frac{11}{7} = -\frac{5}{7} \checkmark$  (3)

(ii)  $(x^2-4)\sqrt{x^2+4} = 0$   
 $(x-2)(x+2)\sqrt{x^2+4} = 0$   
 $\therefore x = 2, -2 \checkmark$  (2)

(iii)  $\frac{1-8x}{5} < 9$   
 $1-8x < 45 \checkmark$   
 $-8x < 44$   
 $8x > -44 \checkmark$   
 $x > -\frac{11}{2} \checkmark$  (3)

b)  $a^3 b^2 = \left[\left(\frac{1}{2}\right)^3\right]^3 \left[\left(\frac{4}{8}\right)^2\right]^2 \checkmark$   
 $= \frac{1}{2^9} \cdot \frac{2^{-8}}{5^{-4}} = \frac{1}{2^9} \cdot \frac{5^4}{2^8} \checkmark$   
 $= 2^{-17} 5^4 \checkmark$  (2)

c)  $1(\sqrt{2}-1) + 2(\sqrt{2}+1) \checkmark$   
 $= 3\sqrt{2}+1 \checkmark$  (2)

2. a)  $= \frac{5}{5-\sqrt{2}} \times \frac{5+\sqrt{2}}{5+\sqrt{2}} \checkmark$   
 $= \frac{25+5\sqrt{2}}{23} \checkmark$  (2)

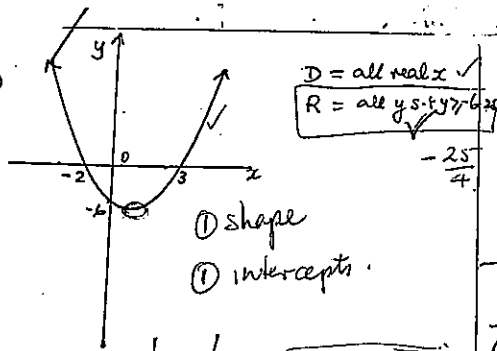
b) (i)  $T = 2\pi \sqrt{\frac{80}{9.8}} \checkmark = 17.95195 \dots$   
 $= 17.95$  (2dp) (2)

(ii)  $T^2 = 4\pi^2 \times \frac{l}{g} \checkmark$   
 $g = \frac{4\pi^2 l}{T^2} \checkmark$  (2)

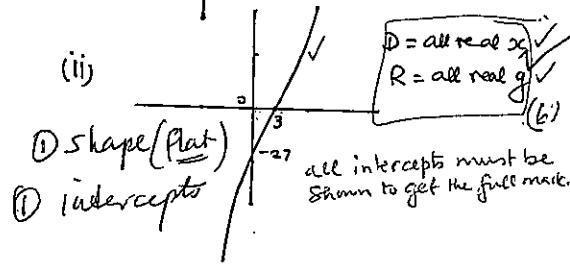
c)  $\frac{3x}{x+1} = \frac{2}{3} \Rightarrow 9x = 2x+2$   
 $7x = 2 \therefore x = \frac{2}{7} \checkmark$  (3)

d)  $x = \frac{47}{330} \checkmark$  (1) (8)

5) a) (i)



(ii)



b)  $|2x+1| \leq 7$   
 $2x+1 \leq 7 \quad - (2x+1) \leq 7$   
 $x \leq 3 \checkmark \quad 2x+1 \geq -7$   
 $-4 \leq x \leq 3$  needed  $2x \geq -8$   
 $x \geq -4$  to get  $x \geq -4$  together (3)

6) a)  $= \frac{3}{x+1} + \frac{2}{(x-1)(x+1)} - \frac{4}{(x-1)}$

$= \frac{3(x-1) + 2 - 4(x+1)}{(x-1)(x+1)} \checkmark$   
 $= \frac{-x-5}{x^2-1} \checkmark$  (3)

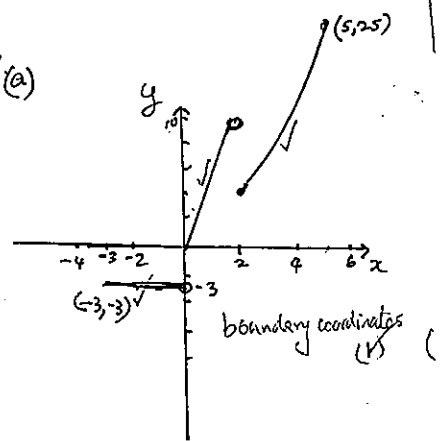
b)  $|5x-1| = 3x+3$   
 $5x-1 = 3x+3 \quad - (5x-1) = 3x+3$   
 $2x = 4 \quad -8x = +2$   
 $x = 2 \checkmark \quad x = -\frac{1}{4} \checkmark$

check: LHS = 9  
 RHS = 9  
 $\checkmark$

LHS =  $\frac{9}{4}$   
 RHS =  $\frac{9}{4} \checkmark$  (3)

(c) Base =  $\sqrt{7 \cdot 6^2 - 5 \cdot 2^2}$   
 $= 5.542 \checkmark$   
 Required area =  $\frac{1}{2} \times 5.542 \times 5.2 - \frac{1 \cdot 3 \cdot 4}{2} \checkmark$   
 $= 5.57$  (2 dp) (1)

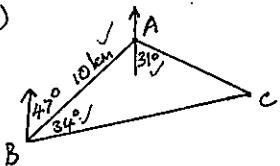
7(a)



b)  $\tan A = \frac{1}{2}$   
 $A = \tan^{-1}(\frac{1}{2}) \checkmark$   
 $A = 26^\circ 34', 206^\circ 34'$  (2)

(c)  $x^2 + \frac{6x}{9} - \frac{4}{9} = 0$   
 $x^2 + \frac{6x}{9} + \left(\frac{3}{9}\right)^2 = \frac{4}{9} + \left(\frac{3}{9}\right)^2 \checkmark$   
 $\left(x + \frac{3}{9}\right)^2 = \frac{5}{9}$   
 $x = \pm \frac{\sqrt{5}}{3} - \frac{1}{3}$   
 $x = \frac{(\sqrt{3}-1)}{3}$  or  $-\frac{(\sqrt{3}+1)}{3}$  (1)

8) a) (i)



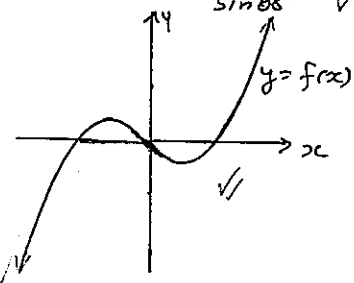
(3)

i)  $\angle BCA = 180 - (34 + 31 + 47) = 68^\circ$

$$\frac{10}{\sin 68^\circ} = \frac{x}{\sin 34^\circ} \checkmark$$

$$x = 10 \times \frac{\sin 34^\circ}{\sin 68^\circ} = 6.03 \text{ km} \quad (2)$$

b)



(2)

(7)

9(a)  $\cos \beta = \sin 43^\circ 50'$

$$\cos (90 - \alpha) = \sin \alpha$$

$$\therefore 90 - \alpha = \beta \checkmark$$

$$\beta = 90 - 43^\circ 50'$$

$$= 46^\circ 10' \checkmark$$

(2)

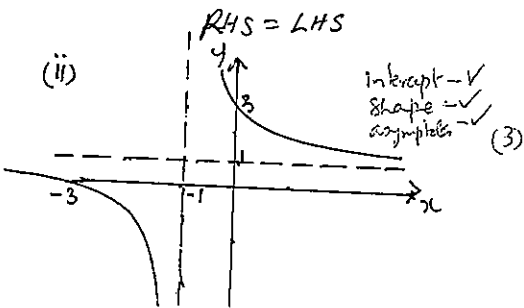
b)  $RHS = 1 + \frac{2}{x+1} = \frac{x+1+2}{x+1} \checkmark$

(1)

$$= \frac{x+3}{x+1} \quad (1)$$

(1)

(ii)



(3)

(iii)  $f(x) = \frac{x^3 - 3x^2 + 1}{x(x-1)}$

$$f\left(\frac{1}{x}\right) = \frac{\left(\frac{1}{x}\right)^3 - 3\left(\frac{1}{x}\right)^2 + 1}{\frac{1}{x}\left(\frac{1}{x} + 1\right)} \checkmark$$

$$= \frac{1 - 3x + x^3}{x^2 x (x-1)}$$

$$= \frac{1 - 3x + x^3}{x(x-1)} \checkmark$$

$$f(1-x) = \frac{(1-x)^3 - 3(1-x)^2 + 1}{(1-x)x}$$

$$= \frac{1 - 3x + 3x^2 - x^3 - 3 + 6x - 3x^2 + 1}{(1-x)x}$$

$$= \frac{-x^3 + 3x - 1}{(1-x)x}$$

$$= -\frac{(x^3 - 3x + 1)}{x(1-x)}$$

$$= \frac{x^3 - 3x + 1}{x(x-1)} \checkmark$$

(3)

Hence  $f\left(\frac{1}{x}\right) = f(1-x)$ .

(9)

