Question 1 (10 marks)

(a) Simplify $2\sqrt{32} + \sqrt{20} - 4\sqrt{18}$ 2

(b) Evaluate correct to 3 significant figures 2

$$\frac{\sqrt[3]{632.15}}{18.27+6.3}$$

(c) Write the following as a single fraction with rational denominator 2

$$\frac{1}{2\sqrt{2}} - \frac{3+\sqrt{2}}{3\sqrt{2}}$$

(d) Express $0.\dot{3}4\dot{2}$ as a fraction in its simplest form. Showing all your working. 2

(e) Given that $F = \frac{9C}{5} + 32$.

(i) Find the value of F when C= 25. 1

(ii) Make C the subject of the formula. 1

Question 2 (Start a new page) (12 marks)

(a) Factorise the following

(i) $x^2 - 10x + 21$ 1

(ii) $2a^2 + 9ab - 5b^2$ 2

(iii) $4a^2 - 4a + 8ab - 8b$ 2

(b) Solve the following equations

(i) $\frac{x+3}{2} - \frac{3x-1}{7} = 2$ 2

(ii) $x + \frac{1}{x} = 3$ 2

(iii) $|3 - 2x| = 5x - 3$ 3

(c) Solve the following inequality and graph the solution on the number line

$|3x - 2| \geq 1$ 2

Question 3 (Start a new page) (12 marks)

(a) By rationalising the denominator find the value of a and b 2

$$\frac{8}{3-\sqrt{5}} = a + b\sqrt{5}$$

(b) Find the domain and range of the following: 4

(i) $y = 9 - x^2$

(ii) $y = \frac{1}{\sqrt{8-4x}}$

(c) $f(x) = \begin{cases} 5 - x^2 & \text{if} \quad x \geq 1 \\ (x-3)^2 & \text{if} \quad x < 1 \end{cases}$ find the value of $f(3) - 2f(-6)$ 2

(d) Determine if $f(x) = \frac{2x}{2x^2-1}$ is an odd function, even function or neither. 2

(e) Solve the equation $2\cos\emptyset = -\sqrt{3}$ where $0 \leq \emptyset \leq 360^{\circ}$ 2

Question 4 (Start a new page) (12 marks)

(a) If $\cos\theta = -0.6$ and $\sin\theta < 0$, find the exact value of $\tan\theta$. 2

(b) Simplify $\frac{2}{x(3-x)} - \frac{1}{x}$ 2

(c) On a number plane shade in the region given by the two inequalities 3

$y > x^2 - 2$ and $y \leq x$. (Indicate where each graph crosses the x and y axes)

(d) Solve for x, if $9^x - 4(3^x) + 3 = 0$ 3

(e) Solve for x and y 2

$$2x + 3y = -1$$
$$-5x + 2y = -7$$

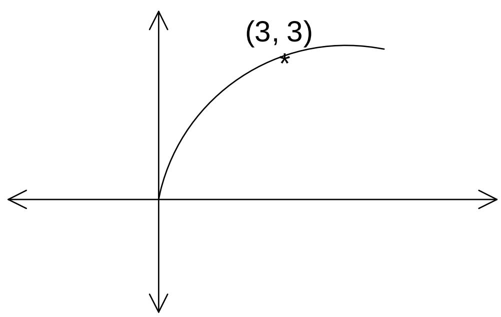
Question 5 (Start a new page) (10 marks)

(a) Solve $\sin^2 x - 2\cos x + 2 = 0$ for $0^\circ \leq x \leq 360^\circ$ 3

(b) Find the exact value of $\dfrac{\sin^2 60^\circ}{\cot 60^\circ + \sec 30^\circ}$ 2

(c) In 1 hour 80 people attend a swimming pool and the money taken at the gate amounts to $139. If adults are charged $2.50 and children are $1.50, how many of each entered the pool that hour? 2

(d) The diagram represents a portion of an even function $y = f(x)$ 2

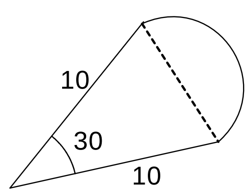


(i) Redraw this diagram and complete the graph .

(ii) Find $f(3) + f(-3)$

Question 6(Start a new page) (10 marks)

(a) A sector is given below. Find the area of the shaded region. 2



(b) Prove $(cosec\beta + \cot\beta)(\mathrm{cosec}\beta - \cot\beta) = 1$ 2

(c) The function $f(x)$ is defined as $f(x) = \begin{cases} 2x & -4 \leq x < 0 \\ 9 - x^2 & 0 \leq x \leq 3 \end{cases}$

(i) Sketch $y = f(x)$ 2

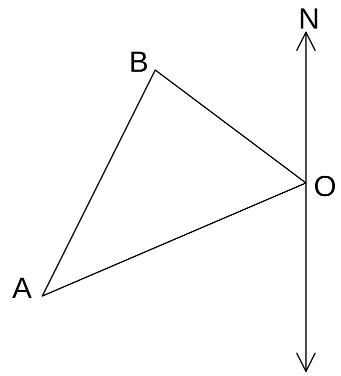
(ii) State the range of $y = f(x)$ 1

Question 6 – Cont

(d) From an observation tower O, a man determines the lighthouse A is on a bearing of 225°T and another lighthouse B is at a bearing of 315°T. Given that A and B are at a distance of 75 and 70 nuatical miles respectively from O, find

(i) the distance between A and B. 2

(ii) The bearing of B from A to the nearest degree. 1



Question 7(Start a new page) (10 marks)

(a) For the parabola $y = x^2 - 2x - 3$

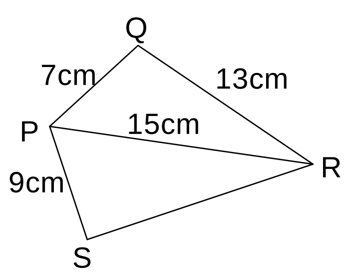
(i) find its x and the y intercepts 1

(ii) find its vertex and hence draw a neat sketch of the curve. 2

(b) Show that $\dfrac{4}{2+\sqrt{5}} - \dfrac{1}{9-4\sqrt{5}}$ is rational. 2

(c) Simplify $\dfrac{1-\dfrac{1}{x+1}}{\dfrac{1}{x+1}}$ 2

(d) In the diagram $\angle SPQ = 147^{\circ}$. Let $\angle QPR = \theta$



(i) Show that $\cos\theta = \frac{1}{2}$ 2

(ii) Find the area of ΔPRS. 1

Question8 (start a new page) (13marks)

(a) Given $x^2 + 12x = p$, use the method of completing square to find an expression for x in terms of p. If $p = 253$, find all possible values of x. 3

(b) If $x + y = 1$ and $x^3 + y^3 = 19$ Find the value of $x^2 + y^2$. 2

(c) (i) Sketch on the same number plane $y = \cos x$ and $y = \sin x$ where $0^\circ \le x \le 90^\circ$ 2

(ii) Find the point of intersection of $y = \cos x$ and $y = \sin x$ for $0^\circ \le x \le 90^\circ$ 1

(iii) Hence or otherwise solve the inequality $sinx > cosx$ for $0° \le x \le 90°$ 1

(d) Simplify $\dfrac{5^{n+3} - 5^n}{5^n}$ 2

(e) Simplify $\dfrac{6x^3 + 48}{24 - 6x^2}$ 2

Question9 (Start a new page) (13 marks)

(a) If $f(x) = 1 - x^2$ and $g(x) = 2x + 1$

(i) find $f(-2)$ 1

(ii) find the value of x for which $f(x) = g(x)$ 2

(iii) find $f[g(x)]$ 1

(b) ΔABC is right-angled at B.

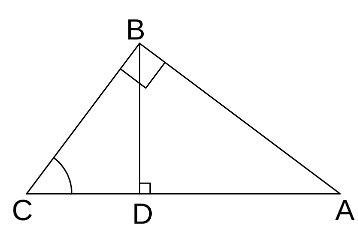
D is a point on AC so that BD is perpendicular to AC, also BC=1unit

DA=CD+1 and $\angle BCA = \theta$

(i) show that $2\cos\theta + 1 = \sec\theta$. 2

(ii) Deduce that $2\cos^2\theta + \cos\theta - 1 = 0$. 2

(iii) Hence find θ. 1



Question 9 – Cont

(c) Find the value of A, B and C if $y^2 + 16y + 94 - 6x$ is expressed in the form 2

$(y + C)^2 - B(x + A)$.

(d) If $\frac{2}{3x-2c} + \frac{3}{2x-3c} = \frac{7}{2c}$ find x in terms of c. 2

- END OF EXAM -

Solution Yr 11 Half yearly 2011

13

1a $2\times4\sqrt{2}+2\sqrt{5}-4\times3\sqrt{2}$ — (1)

$8\sqrt{2}+2\sqrt{5}-12\sqrt{2}$
$2\sqrt{5}-4\sqrt{2}$ } (1)

b) $0.349302\ldots$ — (1)

0.349 — (1)

c) $\frac{\sqrt{2}}{4}-\frac{3\sqrt{2}+2}{6}$ — (1)

$\frac{3\sqrt{2}-6\sqrt{2}-2}{12}$ — (1)

$=\frac{-3\sqrt{2}-4}{12}$ or $-\frac{(3\sqrt{2}+4)}{12}$

d) let $x=0.342342\ldots$

(1) — $1000x=342.342\ldots$

$999x=342$

$x=\frac{342}{999}=\boxed{\frac{38}{111}}$ — (1)

Should be in the simplest form

e) $F=\frac{9c}{5}+32$

(i) $F=9\times\frac{25}{5}+32=77$ — (1)

(ii) $\frac{9c}{5}=F-32$

$9c=5F-160$

$c=\frac{5F-160}{9}$ — (1)

10

Q2 a(i) $(x-7)(x-3)$ — (1)

(ii) $\frac{(2a+10b)(2a-b)}{2}$

$=(a+5b)(2a-b)$ — (1)

(iii) $4a(a-1)+8b(a-1)$

$(a-1)(4a+8b)$ — (1)

$4(a-1)(a+2b)$ — (1)

b) (i) $7x+21-6x+2=28$

$x+23=28$

$x=5$ (2)

(ii) $x^2+1=3x$

$x^2-3x+1=0$

$x=\frac{3\pm\sqrt{9-4}}{2}$

$=\frac{3\pm\sqrt{5}}{2}$ (2)

(iii) $3-2x=5x-3$ | $3-2x=-5x+3$

$7x=6$ | $7x=0$

$x=6/7$ — (1) | $x=0$ — (1)

Check — (1)

$x=0$ | $x=6/7$

LHS $|3-2x|=3$ | LHS $|3-2\times 6/7|=9/7$

RHS $=-3$ | RHS $5\times\frac{6}{7}-3=9/7$

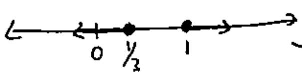
$x=0$ is not a sol. | $\therefore x=6/7$ is a sol.

c) $3x-2\geq1$ | $3x-2\leq-1$

$3x\geq3$ | $3x\leq1$

$x\geq1$ | $x\leq 1/3$

— (1)



— (1)

Q3 a) $\frac{8}{3-\sqrt{5}}\times\frac{3+\sqrt{5}}{3+\sqrt{5}}$

$\frac{24+8\sqrt{5}}{9-5}=\frac{24+8\sqrt{5}}{4}$

$=6+2\sqrt{5}=a+b\sqrt{5}$

$\boxed{a=6}$ (1) $\boxed{b=2}$ (1)

b) (i) D: all real x — (1)

R: $y\leq9$ — (1)

(ii) D: $8-4x>0$

$x<2$ — (1)

R: $y>0$ — (1)

c) $f(3)-2f(-6)=5-3^2-2(-9)^2$

$=-4-162$

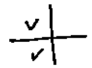
$=-166$ (2)

d) $f(-x)=\frac{-2x}{2x^2-1}=-f(x)$

Hence $f(x)$ is an odd fn — (2)

(with working)

e) $\cos\phi=-\frac{\sqrt{3}}{2}$

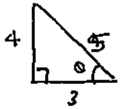


acute $\angle=30°$

$\phi=150°, 210°$ (2)

12

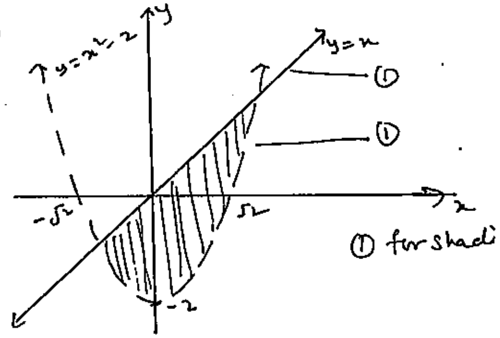
$\cos\theta=\frac{-6}{10}=\frac{-3}{5}$



$\tan\theta=4/3$ or 1.3 (2)

b) $\frac{2-3+x}{x(3-x)}=\frac{x-1}{x(3-x)}$ (2)

c)



d) $3^{2x}-4(3^x)+3=0$

let $3^x=u$ — (1)

$u^2-4u+3=0$

$(u-3)(u-1)=0$

$u=3,\ u=1$ — (1)

$3^x=3$ $\therefore x=1$
$3^x=1$ $x=0$] — (1)

e) $2x+3y=-1\times5$

$-5x+2y=-7\times2$

$10x+15y=-5$

$-10x+4y=-14$

$19y=-19$

$\boxed{y=-1}$

$2x-3=-1$

$2x=2$

$\boxed{x=1}$ (2)

12

Q5 a) $\sin^2 x - 2\cos x + 2 = 0$

$1 - \cos^2 x - 2\cos x + 2 = 0$ — (1)

$\cos^2 x + 2\cos x - 3 = 0$

$(\cos x + 3)(\cos x - 1) = 0$ — (1)

$\cos x \neq -3$

$\cos x = 1$

$x = 0°$ or $360°$] (1)

b) $\dfrac{\frac{3}{4}}{\frac{1}{\sqrt3}+\frac{2}{\sqrt3}}$ — (1) $\quad \dfrac{\sqrt3}{4}$ — (1)

c) $2.5a + 1.5c = 139$

$a + c = 80$

$5a + 3c = 278$

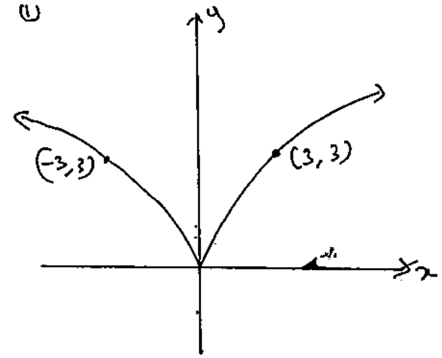
$5a + 240 - 3a = 278$

$2a = 38$

$a = 19$

$c = 61$] (2)

d) (i)



(ii) $f(3) + f(-3) = 3 + 3 = 6$ — (1)

9

Q6 a) A = Ar of Sector − Ar of Δ

$= \frac{1}{2}\pi r^2 \times \left(\frac{60}{360}\right) - \frac{1}{2}\times 10 \times 10 \times \sin 30°$ — (1)

$= \frac{1}{2}\times\frac{\pi}{6}\times 10^2 = 50\times\frac{1}{2}$

$= 50\left(\frac{\pi}{6} - \frac{1}{2}\right) = 25\left(\frac{\pi - 3}{3}\right)$ — (1)

(or any equivalent ans)

b) LHS $\left(\frac{1}{\sin\beta} + \frac{\cos\beta}{\sin\beta}\right)\left(\frac{1}{\sin\beta} - \frac{\cos\beta}{\sin\beta}\right)$

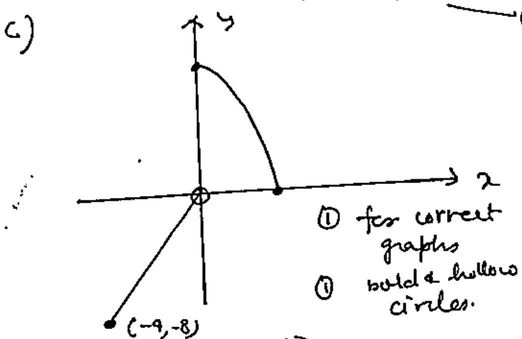
$= \left(\frac{1+\cos\beta}{\sin\beta}\right)\left(\frac{1-\cos\beta}{\sin\beta}\right)$

$= \frac{1-\cos^2\beta}{\sin^2\beta} = \frac{\sin^2\beta}{\sin^2\beta} = 1 =$ RHS — (2)

OR

LHS $\operatorname{cosec}^2\beta - \cot^2\beta = 1 =$ RHS — (2)

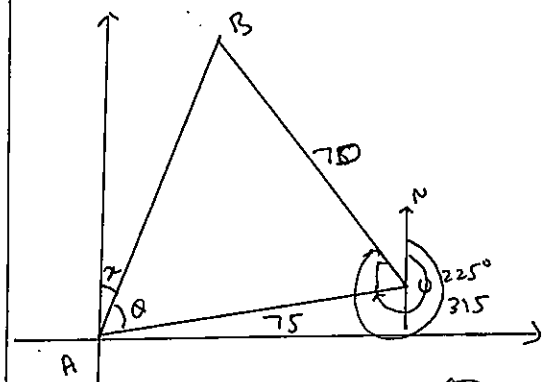
c)



(1) for correct graphs

(1) bold & hollow circles.

(ii) $-8 \le y \le 9$ — (1)



(1)

(i) $\angle AOB = 90°$

$AB^2 = 70^2 + 75^2$

$AB = 102.59$ — (1)

(ii) $\frac{\sin\theta}{70} = \frac{\sin 90}{102.59}$

$\sin\theta = \frac{70}{102.59}$

$\theta = 43°2'$

∴ Bearing B from A

$180 = 135 + 43°2' + x$ (cont. ∠ ∥ lines)

$= 2°$ — (1)

10

Q7 a) (i) y int = −3

x int → $(x-3)(x+1) = 0$

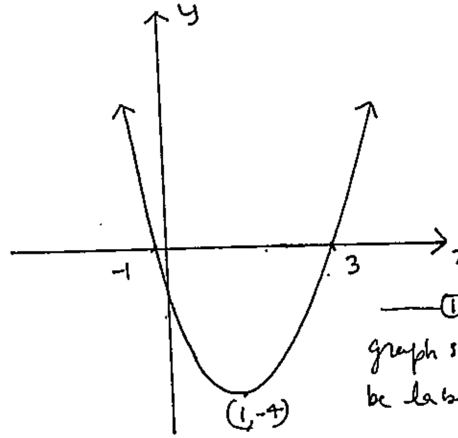
$x = 3, -1$] — (1)

(ii) axis of sym is $x = 1$

∴ y coordinate is

$y = 1 - 2 - 3 = -4$

$V(1,-4)$ — (1)



— (1) graph shd be labelled

b) $\frac{4}{2+\sqrt5}\times\frac{2-\sqrt5}{2-\sqrt5} - \frac{1}{9-4\sqrt5}\times\frac{9+4\sqrt5}{9+4\sqrt5}$ — (1)

$\frac{8-4\sqrt5}{4-5} - \frac{9+4\sqrt5}{81-80}$ — (1)

$-8 + 4\sqrt5 - 9 - 4\sqrt5 = -17$ Hence ratio

c) $\dfrac{\frac{x+1-1}{x+1}}{\frac{1}{x+1}} = x$ — (1) — (1)

d) In ΔPQR

(i) $\cos Q = \frac{15^2 + 7^2 - 13^2}{2\times 15\times 7}$ — (1)

$= \frac{1}{2}$

(ii) $\cos\theta = \frac{1}{2}$

$\therefore \theta = 60°$

$\angle SPQ = 147°$

$\angle SPR = 87°$ — (1)

$A = \frac{1}{2}\times 9\times 15\times\sin 87°$ — (1)

$= 67.4074\ldots$ or 67.4

10

Q8 a) $x^2 + 12x + 36 = p + 36$ — (1)

$(x+6)^2 = p + 36$

$x + 6 = \pm\sqrt{p+36}$

$x = -6 \pm \sqrt{p+36}$ — (1)

for $p = 253$, $x = -6 \pm \sqrt{253+36}$

$= -6 \pm 17$

$x = 11, -23$ (1)

b) $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$

$1^3 = (x^3+y^3) + 3xy(x+y)$

$1 = 19 + 3xy$

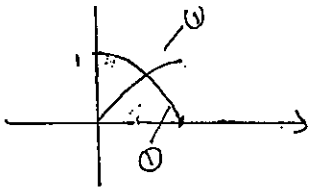
$3xy = -18$ — (1)

$xy = -6$

$(x+y)^2 = x^2 + y^2 + 2xy$

$1 = x^2 + y^2 + 2x - 6$ — (1)

$x^2 + y^2 = 13$



(ii) $x = 45°$, $y = \frac{1}{\sqrt{2}}$ — (1)

(iii) $\sin x > \cos x$
$45° < x \leq 90°$ — (1)

d) $\frac{5^{n+3} - 5^n}{5^n} = \frac{5^n \times 5^3 - 5^n}{5^n}$ — (1)

$\frac{5^n(5^3-1)}{5^n} = 124$ — (1)

e) $\frac{6(x^3+8)}{6(4-x^2)}$ — (1)

$\frac{(x+2)(x^2+4-2x)}{(2-x)(2+x)}$ — (1)

$= \frac{x^2+4-2x}{2-x}$

13

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09 a) (i) $f(-2) = 1-(-2)^2 = 1-2 = -1$ — (1)

(ii) $1-x^2 = 2x+1$

$2x+x^2 = 0$

$x(x+2) = 0$

$x = 0$ (1), $x = -2$ — (1)

(iii) $f(2x+1) = 1-(2x+1)^2$ — (1)

$= 1-(4x^2+1+4x)$

$= 1-4x^2-1-4x$

$= -4(x^2+x)$

b) In Δ $\cos\theta = \frac{CD}{BC}$

$\therefore \cos\theta = CD$

In ΔABC, $\sec\theta = \frac{AC}{BC}$ — (1)

$\sec\theta = AC$

$AC = CD + DA$

But $DA = CD+1$

$\therefore AC = 2CD+1$ — (1)

$\sec\theta = 2\cos\theta+1$

(ii) $\frac{1}{\cos\theta} = 2\cos\theta+1$ — (1)

$1 = 2\cos^2\theta+\cos\theta$ — (1)

$2\cos^2\theta+\cos\theta-1$

(iii) $2\cos^2\theta+\cos\theta-1 = 0$

$(2\cos\theta-1)(\cos\theta+1) = 0$

$2\cos\theta-1 = 0$, $\cos\theta+1 = 0$

$\cos\theta = \frac{1}{2}$ $\quad \cos\theta = -1$

but θ is an acute $\angle$ — (1)

$\therefore \theta = 60$

c) $y^2+16y \cancel{+}64+94-6x$

$(y+8)^2+30-6x$ — (1)

$(y+8)^2-6(x-5)$

$\therefore C = 8$, $B = -6$, $A = -5$ — (1)

d) $\frac{2}{3x-2c}+\frac{3}{2x-3c} = \frac{7}{2c}$

$\frac{4x-6c+9x-6c}{6x^2-9xc-4xc+6c^2} = \frac{7}{2c}$

$\frac{13x-12c}{6x^2-13xc+6c^2} = \frac{7}{2c}$

$26xc-24c^2 = 42x^2-91xc+42c^2$

$42x^2-117xc+66c^2 = 0$ — (1)

$x = \frac{117c \pm \sqrt{(117c)^2-4\times42\times66c^2}}{84}$

$x = \frac{117c \pm \sqrt{2601c^2}}{84}$

$x = 2c, \frac{11}{4}c$ — (1)

13

——— x ——— x ———