BAULKHAM HILLS HIGH SCHOOL

2012

YEAR 11 HALF YEARLY EXAMINATIONS

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
  Black pen preferred
- Board-approved calculators may be used
- Show all necessary working in question 6 – 9
- Marks may be deducted for careless or badly arranged work

Total marks – 65

Section I – Page 2

5 marks

- Attempt Question 1 – 5
- Allow about 10 minutes for this section

Section II – Pages 3 – 6

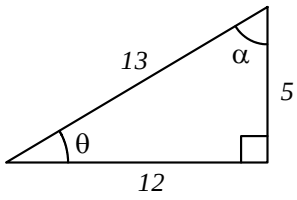
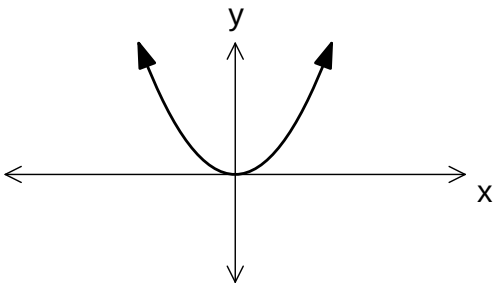
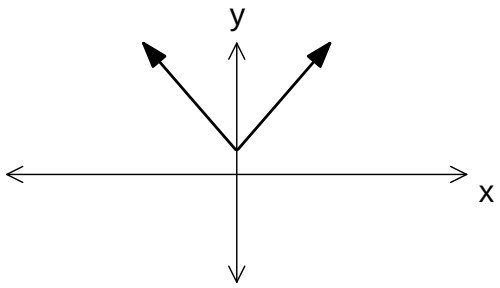
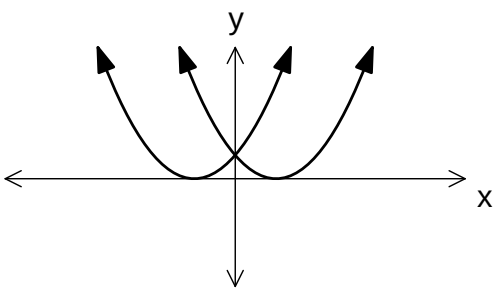
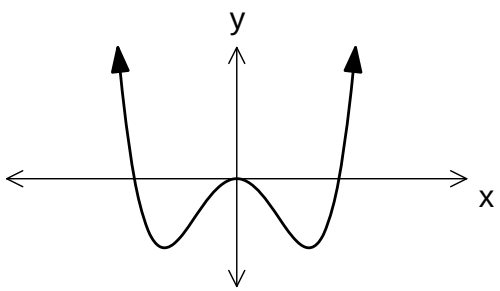
60 marks

- Attempt questions 6 – 9
- Allow about 1 hour and 50 minutes for this section.

Section I

5 marks
Attempt Questions 1 – 5
Allow about 10 minutes for this section.

Use the multiple-choice answer sheet for Questions 1 – 5

Multiple Choice (5 marks) - Answer the following on the answer sheet provided.		**Marks**
1)	Find the value of c if $E = mc^2$ and $E = 7.2 \times 10^{10}$ when $m = 0.8$ (A) 4.5×10^5 (B) 3×10^8 (C) 3×10^5 (D) 9×10^{10}	**1**
2)	Which of the following is a linear equation? (A) $y = x^2 + 7$ (B) $y = 5 - \frac{7}{x}$ (C) $y = 3x - 2$ (D) $y = \sqrt{x} - 5$	**1**
3)	The ratio that has the smallest value is;  (A) $\sin\theta$ (B) $\cos\theta$ (C) $\tan\theta$ (D) $\tan\alpha$	**1**
4)	Express a as the subject of $s = ut + \frac{1}{2}at^2$. (A) $a = \frac{2(s-ut)}{t^2}$ (B) $a = \frac{2s-ut}{t^2}$ (C) $a = \frac{\frac{1}{2}(s-ut)}{t^2}$ (D) $a = \frac{\frac{1}{2}s-ut}{t^2}$	**1**
5)	Identify the curve that does **not** represent an even function. (A)  (B)  (C)  (D)  	**1**
	End of Section 1	

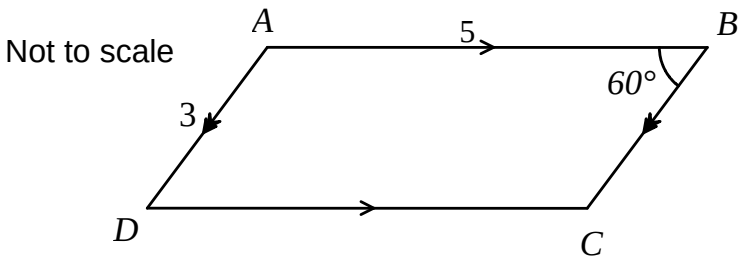
Section II

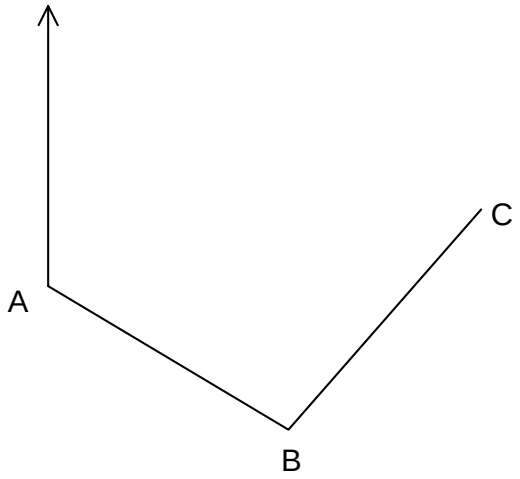
60 marks
Attempt Questions 6 – 9
Allow about 1 hour 50 minutes for this section

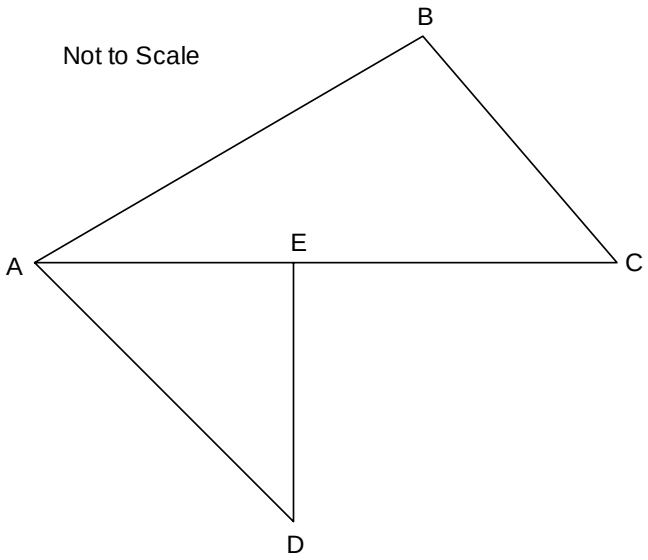
Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

Question 6 (15 marks) - Use a separate answer sheet		**Marks**
a)	Factorise $8x^3 - 125$	**2**
b)	Express $1.9\dot{2}\dot{7}$ as a simplified fraction	**2**
c)	Find the values of a and b $3a + 4b = 5$ $5a - b = 16$	**2**
d)	If $\cos\theta = -\frac{p}{q}$ and $\sin\theta < 0$, find the exact values for $\tan\theta$ and $\operatorname{cosec}\theta$	**3**
e)	Solve for x: $\frac{x-4}{5} \leq 1 + \frac{x}{3}$	**2**
f)	Find A when $\sqrt{A} = \sqrt{52} + \sqrt{117}$	**2**
g)	i) Factorise $x^2 + 3xy - 4y^2$	**1**
	ii) If $x^2 + 3xy - 4y^2 = 0$ and if the ratio $\frac{x}{y}$ has one value equal to 1, find the other value of $\frac{x}{y}$	**1**

Question 7 (15 marks) - Use a separate answer sheet		**Marks**
a)	Find the value of the integers a and b, if $a - b\sqrt{2} = \left(5 - \sqrt{2}\right)^2$	**2**
b)	Solve $3^{6x+3} = 27^{x+2}$	**2**
c)	A function is defined by the following rule: $f(x) = \begin{cases} 1 - x^2 & if\ x < -1 \\ 2 & if\ -1 \leq x \leq 1 \\ x + 1 & if\ x > 1 \end{cases}$ i) Find $f(-2) + f(2)$ ii) Find $f(a^2 + 2)$ iii) Sketch this function	**2** **1** **2**
d)	$ABCD$ is a parallelogram with $AB = 5cm$ and $BC = 3cm$  Find the exact area of the parallelogram $ABCD$	**3**
e)	Solve for θ to the nearest degree $2\sin^2\theta + \sin\theta = 2 \quad for\ 0° \leq \theta \leq 360°$	**3**

Question 8 (15 marks) - Use a separate answer sheet		**Marks**
a)	Given $x^2 + 12x = p$,	
	i) Use the method of completing the square to find an expression for x in terms of p	**1**
	ii) If $p = 253$, find the possible values of x	**2**
b)	i) On the same set of axes, graph $y = \sqrt{25 - x^2}$ *and* $y = 2x - 2$	**2**
	ii) By using the graphs and an algebraic method find the possible solution to the equation $2x - 2 = \sqrt{25 - x^2}$	**3**
	iii) Shade the region described by $y \leq 2x - 2$, $y \leq \sqrt{25 - x^2}$ and $y \geq -5$	**2**
c)	A ship sails $6km$ from A to B on a bearing of $121°T$. It then sails $9km$ to C. The size of $\angle ABC$ is $114°$. Not to Scale  	
	i) Copy the diagram onto your solution sheet and indicate the information given.	
	ii) Find the distance AC (to the nearest km)	**2**
	iii) To the nearest degree, find the true bearing of C from A.	**3**

Question 9 (15 marks) - Use a separate answer sheet		**Marks**
a)	i) Graph $y = 3x^2 - 8x + 5$ (clearly showing the vertex and any intercepts) ii) What is the minimum value of this curve	**3** **1**
b)	Prove that ; $\tan\alpha + \cot\alpha = \operatorname{cosec}\alpha \sec\alpha$	**3**
c)	i) If $a = \frac{15bx}{3b+5x}$ express x in terms of a and b ii) Hence express $\sqrt{\frac{3b-a}{5x-a}}$ in terms of a and b	**1** **2**
d)	If $\angle BAC = 30°, \angle DAE = 45°, AB \perp BC,\ AE \perp DE$ and $EC = 10cm$  Show that i) $2BC - AD = 10 + AE(1 - \sqrt{2})$ ii) $4\sqrt{3}\,AB - 3\sqrt{2}AD = 60$	**3** **2**
	End of Exam	

Yr 11 Half yearly Solns – Mathematics.

2 unit

2012

Start Here →

1. A○ B○ C● D○
2. A○ B○ C● D○
3. A● B○ C○ D○
4. A● B○ C○ D○
5. A○ B○ C● D○

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question 6. /15.

a) $(2x-5)(4x^2+10x+25)$ ✓✓

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b) let $x = 1.92727\ldots$ ①

$1000x = 1927.2727\ldots$ ②

② − ① $990x = 1908$ ✓(working)

$x = \frac{1908}{990} = \frac{106}{55}$

$= 1\frac{51}{55}$ ✓

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c) $3a + 4b = 5$ --①

$5a - b = 16$ --②

② × 4 $20a - 4b = 64$ --③

① + ③ $23a = 69$

$a = 3$ ✓

sub into ①

$9 + 4b = 5$

$b = -1$ ✓

$\therefore$ $a = 3$, $b = -1$

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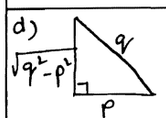
d)



$\tan\theta = \frac{\sqrt{q^2-p^2}}{p}$ ✓

✓

$\operatorname{cosec}\theta = \frac{-q}{\sqrt{q^2-p^2}}$ ✓

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e) $\frac{x-4}{5} \leq 1 + \frac{x}{3}$

$3x - 12 \leq 15 + 5x$ ✓

$-27 \leq 2x$

$x \geq -\frac{27}{2}$ ✓

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f) $\sqrt{A} = 2\sqrt{13} + 3\sqrt{13}$

$= 5\sqrt{13}$ ✓

$= \sqrt{325}$

$A = 325$ ✓

quest 6 cont.

g) i) $(x+4y)(x-y)$ ✓

ii) $(x+4y)(x-y) = 0$

$x = y$ or $x = -4y$

$\frac{x}{y} = 1$ $\frac{x}{y} = -4$ ✓

/15

Question 7. /15

a) $a - b\sqrt{2} = (5 - \sqrt{2})^2$

$= 25 - 10\sqrt{2} + 2$

$= 27 - 10\sqrt{2}$

✓ ✓

$\therefore a = \underline{27}$ $b = \underline{10}$

b) $3^{6x+3} = 27^{x+2}$

$3^{6x+3} = 3^{3x+6}$ ✓

$6x + 3 = 3x + 6$

$3x = 3$

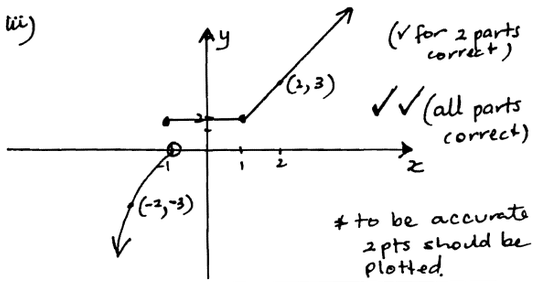
$x = 1$ ✓

c) i) $f(-2) + f(2) = -3 + 3$ ✓

$= 0$ ✓

ii) $f(a^2+2) = a^2 + 3$ ✓

iii)



(✓ for 2 parts correct)

✓✓ (all parts correct)

* to be accurate 2 pts should be plotted.

d) $A = 2 \times \frac{1}{2} \times 3 \times 5 \times \sin 60$ ✓

$= 15 \times \frac{\sqrt{3}}{2}$ ✓

$= \frac{15\sqrt{3}}{2}$ u^2 ✓

quest 7, cont.

e) $2\sin^2\theta + \sin\theta - 2 = 0$

$\therefore \theta = \frac{-1 \pm \sqrt{1+16}}{4}$

$= \frac{-1 \pm \sqrt{17}}{4}$ ✓ some working towards solving.

$= 0.7807..$ or $-1.2807...$

$\therefore \sin\theta \neq -1.2807$ $\therefore$ no soln ✓

$\sin\theta = 0.7807$

$\div 51°, 129°$ ✓ (accept degrees, min)

Question 8 /15

a) $x^2 + 12 = p$

i) $x^2 + x + 36 = p + 36$

$(x)^2 = p + 36$

$x = -6 \pm \sqrt{p+36}$ ✓

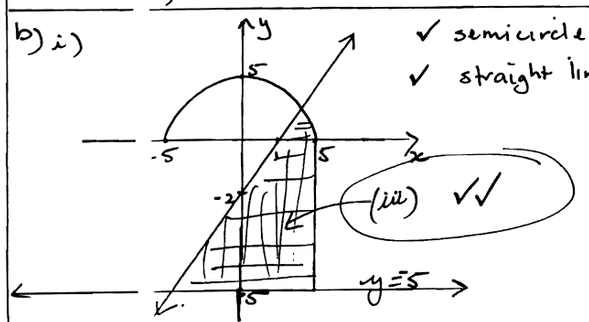
ii) $p = 253$

$x = 6 \pm \sqrt{253+36}$ ✓ working towards soln.

$= 6 \pm 17$

$= 1, -23$ ✓

b) i)



✓ semicircle

✓ straight line

(iii) ✓✓

ii) $2x - = \sqrt{25 - x^2}$

$4x^2 - 8 + 4 = 25 - x^2$ ✓

$5x^2 - 8 - 21 = 0$

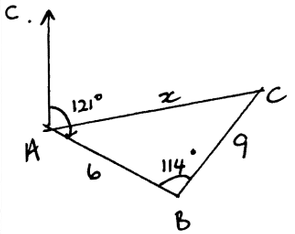
$x = \frac{8 \pm \sqrt{64+420}}{10}$ ✓

$= 3, -1.4$ ✓

$\therefore$ Soln is $\underline{x = 3}$

Quest 8. Cont.

c.



(ii) $x^2 = 6^2 + 9^2 - 2\times 6\times 9\cos 114$ ✓

$= 160.927\ldots$

$x = 12.685\ldots$

$= 13$ km ✓

(iii) $\frac{\sin A}{9} = \frac{\sin 114}{12.685}$ ✓

$\sin A = \frac{9\times\sin 114}{12.685}$

$= 0.685$

$A = 40.4°\ldots$ ✓

Bearing $= 121 - 40.4$

$= 80.6$

$\div 081°$ T ✓

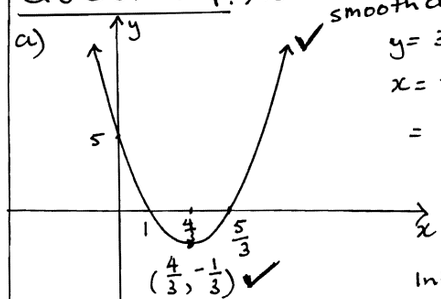
(accept 082 — if used 13km)

when the cosine rule is applied using AC = 13km the result is inaccurate. We accepted this answer though.

Bearing = 084° T

Question 9. /15

a)



✓ smooth curve

✓ intercepts

$y = 3x^2 - 8x + 5$

$x = \frac{-b}{2a}$

$= \frac{8}{6} = \frac{4}{3}$

$y = 3\times(\frac{4}{3})^2 - 8\times\frac{4}{3} + 5$

$x = -\frac{1}{3}$. Vertex $(\frac{4}{3}, -\frac{1}{3})$

Intercepts: $3x^2 - 8x + 5 = 0$

$(3x-5)(x-1) = 0$

$x = 1,\ x = \frac{5}{3}$

$y = 5$.

(ii) min value $-\frac{1}{3}$ ✓

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quest. 9

b. $\tan\alpha + \cot\alpha = \operatorname{cosec}\alpha\sec\alpha$

LHS $= \frac{\sin\alpha}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha}$

$= \frac{\sin^2\alpha + \cos^2\alpha}{\cos\alpha\sin\alpha}$ ✓

$= \frac{1}{\cos\alpha\sin\alpha}$

$= \frac{1}{\sin\alpha}\cdot\frac{1}{\cos\alpha}$ ✓

$= \operatorname{cosec}\alpha\sec\alpha$

$=$ RHS ✓

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c. $a = \frac{15bx}{3b+}$

i) $3ab + 5a = 15bx$

$3a = 15bx - 5ax$

$= x(15b - 5a)$

$x = \frac{3ab}{15b-5a}$ ✓

ii) $\sqrt{\frac{3b-a}{5x-a}} = \sqrt{(3b-a)\div\left(\frac{5\times 3ab}{15b-5a} - a\right)} = \sqrt{(3b-a)\div\frac{3ab-3ab+a^2}{3b-a}}$

$= \sqrt{\frac{(3b-a)^2}{a^2}} = \left|\frac{3b-a}{a}\right|$ ✓ accept without ||

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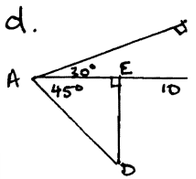
d.



In △AED: $\cos 45 = \frac{AE}{AD}$

$\frac{1}{\sqrt{2}} = \frac{AE}{AD}$

$AD = AE\sqrt{2}$

In △ABC: $\sin 30 = \frac{BC}{AE+10}$ $\sin 60 = \frac{AB}{AE+10}$

$\frac{1}{2} = \frac{BC}{AE+10}$ $\frac{\sqrt{3}}{2} = \frac{AB}{AE+10}$

$2BC = AE + 10$

$AB = \frac{\sqrt{3}}{2}(AE+10)$

i) $2BC - AD = AE + 10 - AE\sqrt{2}$ ✓ ✓✓

$= +AE(1-\sqrt{2})$

ii) $4\sqrt{3}AB - 3\sqrt{2}AD = 4\sqrt{3}\times\frac{\sqrt{3}}{2}(AE+10) - 3\sqrt{2}\times AE\sqrt{2}$ ✓

$= 2\times 3(AE+10) - 3\times 2AE$

✓ $= 6AE + 60 - 6AE = \underline{60}$