BAULKHAM HILLS HIGH SCHOOL

Half -Yearly 2013
YEAR 11 TASK 1

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 2 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-14
- Marks may be deducted for careless or badly arranged work

Total marks – 70
Exam consists of 5 pages.

This paper consists of TWO sections.

<u>Section 1</u> – Page 2 (10 marks)
Questions 1-10
- Attempt Question 1-10
- Answer on answer sheet provided

<u>Section II</u> – Pages 3-5 (60 marks)
- Attempt questions 11-14

Section I - 10 marks

Use the multiple choice answer sheet for question 1-10

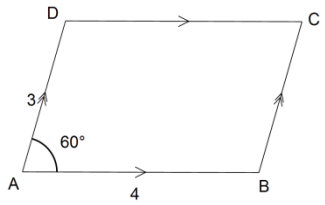
1. $x-4$ is a factor of

(A) x^2-4 (B) x^2+4 (C) $4x^2-16$ (D) x^2-16

2. The solution of $6-\frac{x}{3}>x$ is

(A) $9>x$ (B) $x<\frac{3}{2}$ (C) $x<4.5$ (D) $x>\frac{18}{4}$

3.



The area of the parallelogram $ABCD$ is

(A) $6\times\frac{\sqrt{3}}{2}$ (B) $6\sqrt{3}$

(C) 6 (D) 12

4. Which statement is correct?

(A) $\tan 330°=-\cot 30°$ (B) $\tan 330°=\tan 30°$

(C) $\tan 330°=-\tan 30°$ (D) $\tan 330°=\cot 30°$

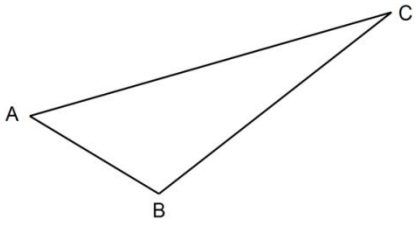
5. The range of $f(x)=\sqrt{9-x^2}$ is

(A) all real y (B) $-3\le y\le 3$ (C) $0\le y\le 3$ (D) $0\ge y\ge -3$

6. The bearing of B from A is 150 ° , C from B is 070°.
What is the bearing of B from C

(A) 030° (B) 250°

(C) 110° (D) 280°



7. $f(x)=(x+4)^2$ is

(A) an even function (B) Neither an even nor odd function

(C) an odd function (D) it's not a function

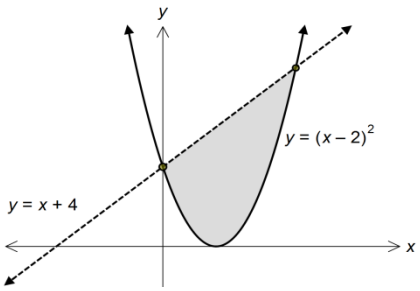
8. If $g(x)=\frac{x}{x^2+1}$ what is $g\left(\frac{1}{x}\right)$ equal to?

(A) $\frac{x^2+1}{x}$ (B) $\frac{x^2}{x^2+1}$ (C) $\frac{x}{1+x^2}$ (D) $\frac{x^2}{x+x^2}$

9. $\left(x-\frac{1}{x}\right)^2=$

(A) $x^2-\frac{1}{x^2}$ (B) $x^2-2+\frac{1}{x^2}$ (C) $x^2+2+\frac{1}{x^2}$ (D) $x^2+\frac{1}{x^2}$

10.



The shaded region is represented simultaneously by the following inequalities

(A) $y\le(x-2)^2$ and $y<x+4$

(B) $y\ge(x-2)^2$ and $y>x+4$

(C) $y\ge(x-2)^2$ and $y<x+4$

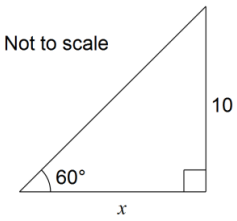
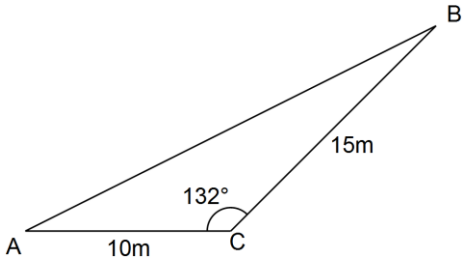
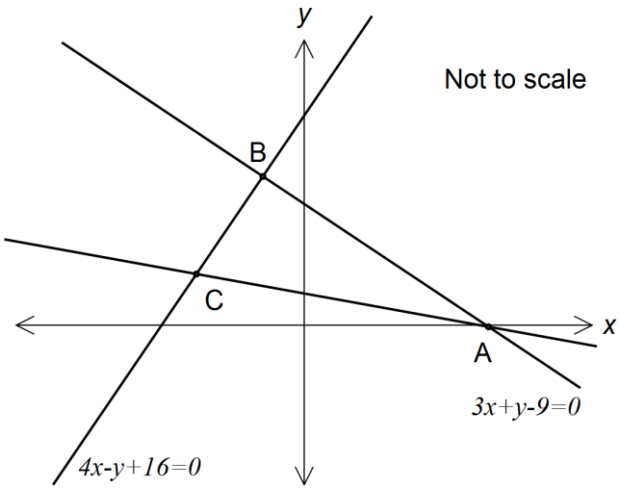
(D) $y\le(x-2)^2$ and $y>x+4$

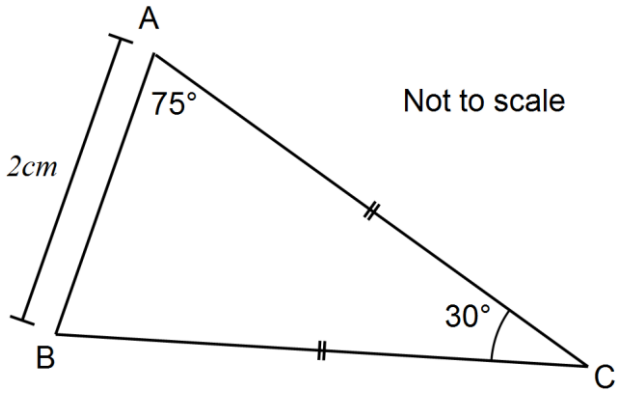
End of Section 1

Section II – Extended Response

Attempt questions 11-14. All necessary working should be shown in every question.

Question 11 (15 marks)		Marks
a)	Simplify $(4x-1)^2-3(x-5)$	2
b)	Evaluate correct to 3 significant figures $$\frac{14.951-11.3}{\sqrt[3]{72}}$$	2
c)	Solve $$a+3b=1$$ $$a+53b=101$$	2
d)	Rationalise the denominator $$\frac{2+\sqrt{3}}{3-2\sqrt{3}}$$	2
e)	Factorise and simplify $$\frac{x^2+x}{x^2-9}\times\frac{x^3-27}{x^2-x-2}$$	3
f)	Consider the function $$f(x)=\begin{cases}-3 & if\ x\le -2\\ x^2 & if\ -2<x<2\\ 2x-3 & if\ x\ge 2\end{cases}$$	
	(i) Evaluate $f(-1)+f(1)$	1
	(ii) Find $f(a^2+2)$	1
	(iii) Sketch the graph of $f(x)$	2
Question 12 (15 marks)		
a)	If $G(x)=4x-x^2$	
	(i) Find the value of $G(-3)$	1
	(ii) Find the values of x for which $G(x)=0$	1
	(iii) Sketch the graph of $G(x)$ showing all intercepts and the vertex.	2
b)	Find all values of θ such that $2\cos^2\theta=1$ for $0°\le\theta\le 360°$	2
c)	(i) Sketch $f(x)=\frac{3}{x-1}-1$ showing all intercepts	3
	(ii) Find the domain and range of $f(x)$	2
d)	If $\sin\theta=-\frac{1}{5}$ and $\tan\theta>0$, find the exact value of $\cos\theta$	2
e)	Given that $(2\sqrt{3}+1)(3-\sqrt{3})=a\sqrt{3}+b$, find a and b	2

Question 13 (15 marks)		Marks
a)	 Find the exact value of x	1
b)	Find the length AB to 2 decimal places. 	2
c)	In the diagram below the lines $3x + y - 9 = 0$ and $4x - y + 16 = 0$ intersect at B. The point A is $(3,0)$ and C is $(-2,8)$ 	
	(i) Show that the line AC has the equation $8x + 5y - 24 = 0$	2
	(ii) Show that the point B has coordinates $(-1,12)$	1
	(iii) Find the equation of a line passing through B and perpendicular to AC. Answer in general form.	2
d)	State whether the function $y = \frac{1}{x^3}$ is even, odd or neither. Justify your answer.	2
e)	Simplify $\frac{2}{3-\sqrt{2}} + \frac{5}{3+\sqrt{2}}$	2
f)	Write $\frac{a^{2x}}{a^{2y}} \times \frac{a^y}{a^2}$ in the form a^k	1
g)	Simplify $\frac{y^{-1}+y}{1+y^2}$	2

Question 14 (15 marks)		Marks
a)	Prove the identity $(\cos\alpha + \cot\alpha)\sec\alpha = 1 + \text{cosec}\alpha$	**2**
b)	Given $y = x^2$ and $y = \lvert 4x - 3 \rvert$	
	(i) Find the points of intersection of the two graphs.	**3**
	(ii) On the same number plane, sketch $y = \lvert 4x - 3 \rvert$ and $y = x^2$ showing the points of intersection.	**2**
	(iii) Hence or otherwise solve: $\lvert 4x - 3 \rvert \geq x^2$	**1**
c)	The diagram shows an isosceles ΔABC with $AC = BC, AB = 2cm, \angle BAC = 75°$ and $\angle ACB = 30°$. 	
	(i) Show that the exact value of $AC = \frac{2}{\sqrt{2-\sqrt{3}}}$	**2**
	(ii) Hence show that the exact value of $\sin 75° = \frac{1}{2\sqrt{2-\sqrt{3}}}$	**2**
	(iii) Find the exact area of triangle ABC	**1**
	(iv) Using part (ii) or otherwise show that the exact value of $\cos 75° = \frac{\sqrt{2-\sqrt{3}}}{2}$	**2**

End of Exam

BAULKHAM HILLS HIGH SCHOOL

YEAR 11 HALF YEARLY ASSESSMENT

2013

Name: Answers

Teacher:

Mathematics
Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: $2+4=$ (A) 2 (B) 6 (C) 8 (D) 9

A ○ B ● C ○ D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ● B ● C ○ D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word *correct* and drawing an arrow as follows.

A ● B ● (correct) C ○ D ○

Start Here →

1.	A ○	B ○	C ○	D ●	6.	A ○	B ●	C ○	D ○
2.	A ○	B ○	C ●	D ○	7.	A ○	B ●	C ○	D ○
3.	A ○	B ●	C ○	D ○	8.	A ○	B ○	C ●	D ○
4.	A ○	B ○	C ●	D ○	9.	A ○	B ●	C ○	D ○
5.	A ○	B ○	C ●	D ○	10.	A ○	B ○	C ●	D ○

Question 11

a) $(4x-1)^2-3(x-5)=16x^2-8x+1-3x+15$
$=16x^2-11x+16$ ✓

b) $0.8776\ldots \overset{1}{=} 0.878$ ✓

c) (1) $a+3b=1$ ⊖ $\therefore a+6=1$
(2) $a+53b=101$ ⊖
(2)−(1) $50b=100$ $\therefore b=2$ $\therefore a=-5$

d) $\frac{2+\sqrt{3}}{3-2\sqrt{3}}\times\frac{3+2\sqrt{3}}{3+2\sqrt{3}}=\frac{6+3\sqrt{3}+4\sqrt{3}+6}{9-12}=\frac{12+7\sqrt{3}}{-3}$

e) $\frac{x^2+x}{x^2-9}\times\frac{x^3-27}{x^2-x-2}=\frac{x(x+1)}{(x-3)(x+3)}\times\frac{(x-3)(x^2+3x+9)}{(x-2)(x+1)}$

$=\frac{x(x^2+3x+9)}{(x+3)(x-2)}$

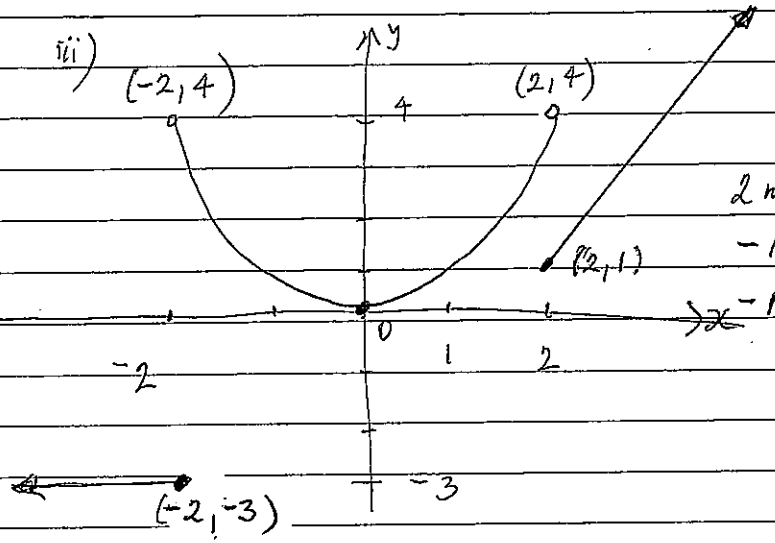
f) $f(x)=\begin{cases}-3 & \text{if } x\le -2\\ x^2 & -2<x<2\\ 2x-3 & x\ge 2\end{cases}$

i) $f(-1)+f(1)=(-1)^2+1^2$
$=2$

ii) $f(a^2+2)=2(a^2+2)-3=2a^2+1$

always ≥ 2

iii)



2 marks
-1 off ← open/close values
-1 off - wrong shape(s)

Question 12

a) $G(x) = 4x - x^2$

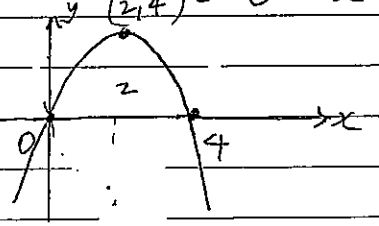
i) $G(-3) = 4(-3) - (-3)^2 = -21$ ✓

ii) $G(x) = 0 = 4x - x^2$

$0 = x(4 - x)$

$x = 0 \quad x = 4$ ✓ for both correct values

iii)



① - shape
① - intercepts and vertex

b) $2\cos^2\theta = 1 \qquad 0° \leq \theta \leq 360°$

$\cos^2\theta = \frac{1}{2}$

$\cos\theta = \pm\frac{1}{\sqrt{2}} \therefore \theta = 45°, 135°, 225°, 315°$

① ①

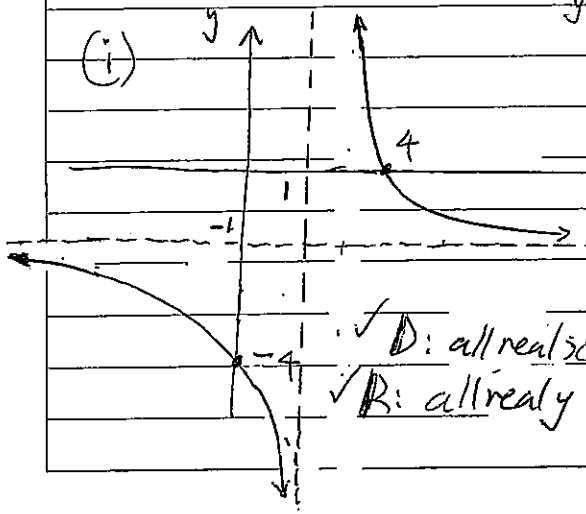
c) $f(x) = \frac{3}{x-1} - 1$

x	0	4
y	-4	0

$0 = \frac{3}{x-1} - 1$

$x - 1 = 3 \therefore x = 4$

(i)

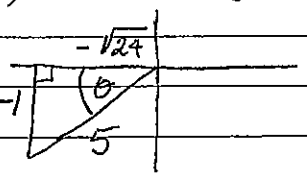


shape – 1
asymptotes – 1
both x & y intercepts

✓ D: all real x except $x \neq 1$

✓ R: all real y except $y = -1$

d) $\sin\theta = \frac{-1}{5}$ and $\tan\theta > 0$



$\therefore \cos\theta = -\frac{\sqrt{24}}{5} = \frac{-2\sqrt{6}}{5}$ ① ①

e) $(2\sqrt{3}+1)(3-\sqrt{3}) = a\sqrt{3}+b$

$6\sqrt{3} - \sqrt{3} + 3 - 6 = a\sqrt{3} + b$

$5\sqrt{3} - 3 = a\sqrt{3} + b$

$\therefore a = 5 \quad b = -3$ ① ①

You may ask for extra writing paper if you need more space to answer question 12

Question 13 **Name:** _____

a)

$\tan 60^\circ = \frac{10}{x}$

$\sqrt{3} = \frac{10}{x} \quad \therefore x = \frac{10}{\sqrt{3}}$ or $\frac{10\sqrt{3}}{3}$ ✓

b) $AB^2 = 10^2 + 15^2 - 2\times 10\times 15\times \cos 132^\circ$ ✓

$= 525.739...$

$AB = 22.929... = \boxed{22.93}$ ✓ (ignore wrong rounding)

c) i) AC: $A(3,0)$ $C(-2,8)$ $m_{AC} = \frac{8-0}{-2-3} = \frac{-8}{5}$

AC: $y - 0 = -\frac{8}{5}(x-3)$ ✓

$y = -\frac{8}{5}(x-3)$

$5y = -8x + 24$

$\therefore \boxed{8x + 5y - 24 = 0}$ ∴ shown

ii) $B(x,y)$ $\begin{cases} 3x+y-9=0 \\ 4x-y+16=0 \end{cases}$

either by substitution ∴ $B(-1,12)$

$3\times -1 + 12 - 9 = 0$

$0 = 0$ ∴ True ✓

B lies on $3x+y-9=0$

$4(-1) - 12 + 16 = 0$ True

∴ B lies on $4x-y+16=0$

OR by solving simultaneously

(1) $3x+y-9=0$

(2) $4x-y+16=0$ (+)

$7x + 7 = 0 \quad \therefore x = -1$ ✓

$\therefore y = -3x - 1 + 9 = 12$

$B(-1,12)$

13c) iii) $B(-1, 12)$

$m_{AC} = \frac{-8}{5}$ $\therefore$ $m_{\perp line} = \frac{5}{8}$ ✓

$\therefore$ equation is $y - 12 = \frac{5}{8}(x - -1)$

$8y - 96 = 5x + 5$

in general form $\therefore 0 = 5x - 8y + 101$ ✓

a) $f(x) = y = \frac{1}{x^3}$

$f(-x) = \frac{1}{-x3} = \frac{-1}{x^3} = -f(x)$ ✓ $\therefore$ odd

(1)

e) $\frac{2}{3-\sqrt{2}} + \frac{5}{3+\sqrt{2}} = \frac{2(3+\sqrt{2}) + 5(3-\sqrt{2})}{(3-\sqrt{2})(3+\sqrt{2})} = \frac{21 - 3\sqrt{2}}{9-2}$

$= \frac{21 - 3\sqrt{2}}{7}$ ✓

f) $\frac{a^{2x}}{a^{2y}} \times \frac{a^{y}}{a^{2}} = a^{2x+y-2y-2} = a^{2x-y-2}$ ✓

g) $\frac{y + y}{1+y^2} = \frac{\frac{1}{y} + \frac{y}{1}}{1+y^2} = \frac{\frac{1+y^2}{y}}{1+y^2} = \frac{1}{y}$ ✓

Question 14 **Name:**_____

a) Prove $(\cos\alpha + \cot\alpha)\sec\alpha = 1 + \operatorname{cosec}\alpha$

$LHS = \left(\cos\alpha + \frac{\cos\alpha}{\sin\alpha}\right) \cdot \frac{1}{\cos\alpha}$ ✓

$= \cos\alpha \cdot \frac{1}{\cos\alpha} + \frac{\cos\alpha}{\sin\alpha} \cdot \frac{1}{\cos\alpha} = 1 + \frac{1}{\sin\alpha} = 1 + \operatorname{cosec}\alpha$

$= RHS$

$\therefore$ proven

b) $y = x^2$ $y = |4x - 3|$

(i) $x^2 = |4x - 3|$

case(1) $x^2 = 4x - 3$ OR case(2) $x^2 = -(4x - 3)$

$x^2 - 4x + 3 = 0$ $x^2 + 4x - 3 = 0$

$(x-3)(x-1) = 0$ ✓ $x = \frac{-4 \pm \sqrt{28}}{2} = \frac{-4 \pm 2\sqrt{7}}{2}$

$x = 3$ $x = 1$ $\therefore x = -2 \pm \sqrt{7}$ ✓

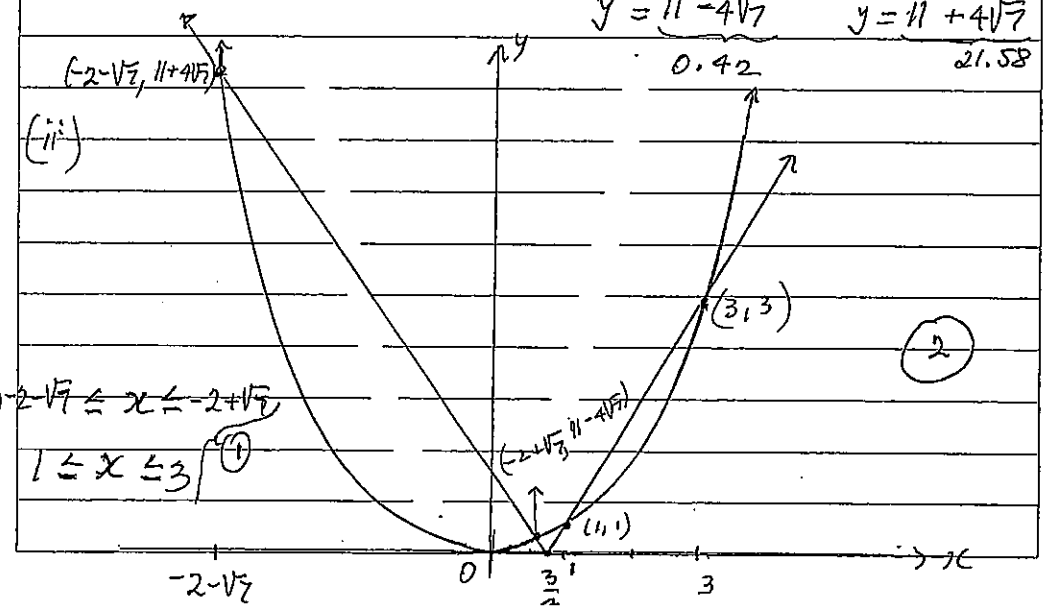
$y = 9$ ✓ $y = 1$ $x = -2 + \sqrt{7}$ $x = -2 - \sqrt{7}$

$y = 11 - 4\sqrt{7}$ $y = 11 + 4\sqrt{7}$

≈ 0.42 ≈ 21.58

(ii) $-2 - \sqrt{7} \leq x \leq -2 + \sqrt{7}$

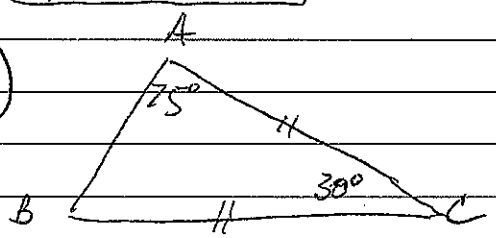
$1 \leq x \leq 3$ (1)



(2)

Question 14

c)



(i) let $AC = BC = a$

$\therefore AB^2 = AC^2 + BC^2 - 2AC \cdot BC \cdot \cos 30^\circ$

$2^2 = a^2 + a^2 - 2a \cdot a \cdot \frac{\sqrt{3}}{2}$ ✓

$4 = a^2(2 - \sqrt{3})$

$a^2 = \frac{4}{2-\sqrt{3}} \therefore a = \oplus \frac{2}{\sqrt{2-\sqrt{3}}} = AC$ ✓ ∴ shown

(only)

ii) $\frac{\sin 75^\circ}{BC} = \frac{\sin 30^\circ}{2}$ ✓

$\sin 75^\circ = \frac{BC \cdot \sin 30^\circ}{2} = \frac{\frac{2}{\sqrt{2-\sqrt{3}}} \times \frac{1}{2}}{2}$

$= \frac{1}{\sqrt{2-\sqrt{3}}} \cdot \frac{1}{2} = \frac{1}{2\sqrt{2-\sqrt{3}}}$ ✓ ∴ shown

iii) $\text{Area} = \frac{1}{2} \times AB \times AC \times \sin 75^\circ$

$= \frac{1}{2} \times 2 \times \frac{2}{\sqrt{2-\sqrt{3}}} \cdot \frac{1}{2\sqrt{2-\sqrt{3}}}$ ✓

$= \frac{1}{2-\sqrt{3}}$

(OR) $\text{Area} = \frac{1}{2} \times AC \times BC \times \sin 30^\circ$ ✓

$= \frac{1}{2} \times \frac{2}{\sqrt{2-\sqrt{3}}} \times \frac{2}{\sqrt{2-\sqrt{3}}} \times \frac{1}{2} = \frac{1}{2-\sqrt{3}}$

iv) $\cos 75^\circ = \frac{AB^2 + AC^2 - BC^2}{2 \times AB \times AC} = \frac{2^2 + 0}{2 \times 2 \times AC}$ ✓

$= \frac{1}{AC}$ ✓ $= \frac{\sqrt{2-\sqrt{3}}}{2}$

(OR) $\cos 75^\circ = \sqrt{1 - \sin^2 75^\circ} = \sqrt{1 - \left(\frac{1}{2\sqrt{2-\sqrt{3}}}\right)^2}$ ✓

$= \sqrt{1 - \frac{1}{4(2-\sqrt{3})}} = \sqrt{1 - \frac{2+\sqrt{3}}{4(4-3)}}$

$= \sqrt{\frac{4-2-\sqrt{3}}{4}} = \frac{\sqrt{2-\sqrt{3}}}{2}$ ✓