BAULKHAM HILLS HIGH SCHOOL

Half -Yearly 2014

YEAR 11 ADVANCED TASK 1

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-15
- Marks may be deducted for careless or badly arranged work

Total marks – 64
Exam consists of 6 pages.

This paper consists of TWO sections.

Section 1 – (10 marks) Pages(2-3)
Questions 1-10

- Attempt Question 1-10
- Answer on answer sheet provided

Section II – (54 marks) Pages(4-6)

- Attempt questions 11-15

Section I - 10 marks

Use the multiple choice answer sheet for question 1-10

1. 0.01208 in scientific notation to 3 significant figures is:

(A) 0.0121

(B) 0.121×10^{-1}

(C) 1.20×10^{-2}

(D) 1.21×10^{-2}

2. Council rates increase by 4% to \$1400. What were rates before the increase, to the nearest dollar?

(A) \$56

(B) \$1344

(C) \$1346

(D) \$1456

3. The solutions to the equation $x^2 - 7x - 2 = 0$ are :

(A) $\frac{7 \pm \sqrt{41}}{2}$

(B) $\frac{7 \pm \sqrt{57}}{2}$

(C) $\frac{-7 \pm \sqrt{41}}{2}$

(D) $\frac{-7 \pm \sqrt{57}}{2}$

4. Which of the following is equivalent to $\frac{1}{2\sqrt{5} - \sqrt{3}}$?

(A) $\frac{2\sqrt{5} - \sqrt{3}}{7}$

(B) $\frac{2\sqrt{5} + \sqrt{3}}{7}$

(C) $\frac{2\sqrt{5} - \sqrt{3}}{17}$

(D) $\frac{2\sqrt{5} + \sqrt{3}}{17}$

5. The solution to the equation $1 + \frac{1 - 3x}{2} \geq 2x$ is :

(A) $x \leq \frac{2}{7}$

(B) $x \geq \frac{2}{7}$

(C) $x \leq \frac{3}{7}$

(D) $x \geq \frac{3}{7}$

6. If $\sin\theta = \frac{3}{7}$ and $\cos\theta < 0$ then $\cot\theta$ is

(A) $\frac{3}{\sqrt{40}}$

(B) $-\frac{3}{\sqrt{40}}$

(C) $\frac{\sqrt{40}}{3}$

(D) $\frac{-\sqrt{40}}{3}$

7. The solution to the equation $|2x + 2| = x - 3$ is:

(A) x= -5 and x=$\frac{1}{3}$

(B) x= -5

(C) x= $\frac{1}{3}$

(D) no solution

8. $\frac{8^{n+1}}{2^{n-2}} =$

(A) 4^{-1}

(B) 4^{3}

(C) 2^{2n+1}

(D) 2^{2n+5}

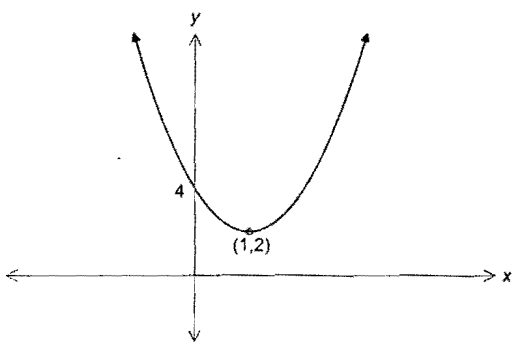
9. $f(x) = \cos x$ is

(A) an even function

(B) an odd function

(C) neither odd nor even

(D) not a function

10. 

The equation for the given parabola is:

(A) $y = (x + 1)^2 + 2$

(B) $y = (x - 1)^2 + 2$

(C) $y = (x - 2)^2 + 1$

(D) $y = 2(x - 1)^2 + 2$

End of Section 1

Section II – Extended Response

Attempt questions 11-15. All necessary working should be shown in every question.

Question 11 (12 marks) **Marks**

a) Express $0.2\dot{3}$ as a fraction in its simplest form. **1**

b) Expand and simplify $\left(3\sqrt{2}-\sqrt{7}\right)^2 =$ **2**

c) If $7\sqrt{2}-\frac{\sqrt{8}}{2}=\sqrt{a}$ find a. **2**

d) Factorise (i) x^2-5x-6 **1**

(ii) $2x^3-16$ **2**

(iii) x^3-x^2-9x+9 **2**

e) Find the exact value of $\sin(240^\circ)$ **2**

End of Question 11

Question 12 (11 marks)

a) Simplify $\frac{1}{x^2-4}+\frac{1}{2x+4}$ **2**

b) Solve (i) $|3x-1|<5$ **2**

(ii) $\frac{x+4}{2}-\frac{x+3}{4}=10$ **2**

c) Simplify $\frac{ab^{-1}-ba^{-1}}{b-a}$ **2**

d) 420 adults and children attended a school musical. There were 120 more children than adults. Form a pair of simultaneous equations and solve them to determine the number of adults and children that attended the musical. **3**

End of Question 12

Question 13 (11 marks) **Marks**

a) Sketch each of the following showing all essential features.

(i) $y = 4 - |x|$ **2**

(ii) $x^2 + (y-2)^2 = 4$ **2**

(iii) State the range for the graphs in part (i) and (ii). **2**

b) State the domain for the function $y = \sqrt{x^2 - 9}$. **2**

c) Sketch the graph of $y = 2 - \frac{1}{x-1}$ **3**

End of Question 13

Question 14 (10marks)

a) Shade the region which satisfies both of the following inequalities :

$y \leq \sqrt{4 - x^2}$ *and* $y \geq x - 2$. **3**

b) Solve the following if $0^\circ < \theta < 360^\circ$

(i) $2 \sin\theta - 1 = 0$ **2**

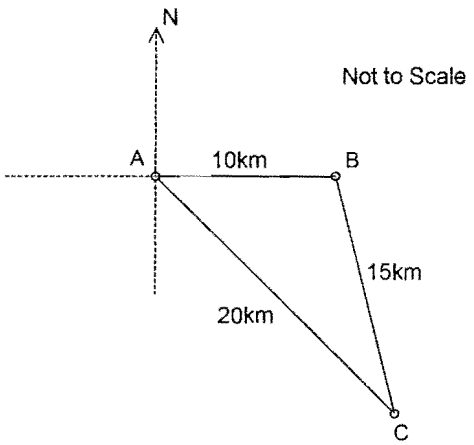
(ii) $\tan^2\theta = 2 \sec\theta + 2$ **3**

c) If $f(x+1) = \frac{2f(x)+3}{3}$ and $f(3) = 4$, find $f(2)$. **2**

End of Question 14

Question 15 (10 marks) **Marks**

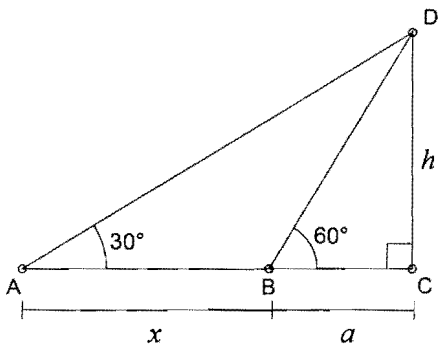
a) Blake's journey is shown in the diagram below. He walks 10 km due east from A to B, 15 km from B to C then 20 km back to A.



(i) Find $\angle ABC$ **2**

(ii) Hence state the bearing of B from C. **1**

b)



Given the above diagram:

(i) Show that $h = \dfrac{x+a}{\sqrt{3}}$ **1**

(ii) Hence or otherwise find an expression for x in terms of a which is independent of h. **2**

c) If k is a solution to $x^2 - x - 1 = 0$. Show that

(i) $k^2 = k + 1$ **1**

(ii) $k^6 = 8k + 5$ **3**

End of Exam

Solutions. Total /64

[10]

1. D 2. C 3. B 4. D 5. C

6. D 7. D 8. D 9. A 10. D

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11. a) Let $x = 0.2333$

[12] $10x = 2.333$

$100x = 23.333$

$\therefore 90x = 21$ (accept $\frac{21}{90}$)

$x = \frac{7}{30}$ (1)

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b) $(3\sqrt{2} - \sqrt{7})^2$

$= 18 - 6\sqrt{14} + 7$

$= 25 - 6\sqrt{14}$ (2) 1 off each error

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c) $7\sqrt{2} - \frac{\sqrt{8}}{2} = 7\sqrt{2} - \frac{2\sqrt{2}}{2}$ (1)

$= 6\sqrt{2}$

$= \sqrt{72}$

$\therefore a = 72$ (1)

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d) (i) $x^2 - 5x - 6$.

$= (x-6)(x+1)$ (1)

(ii) $2x^3 - 16 = 2(x^3 - 8)$ (1)

$= 2(x-2)(x^2+2x+4)$ (1)

(iii) $x^3 - 9x - x^2 + 9$

$= x(x^2-9) - 1(x^2-9)$

$= (x-1)(x^2-9)$ (1)

$= (x-1)(x+3)(x-3)$ (1)

---

e) $\sin 240 = -\sin 60$

$= -\frac{\sqrt{3}}{2}$ (1) (1)

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12. [11]

a) $\frac{1}{x^2-4} + \frac{1}{2x+4}$

$= \frac{1}{(x+2)(x-2)} + \frac{1}{2(x+2)}$ (1)

$= \frac{2 + x - 2}{2(x+2)(x-2)}$

$= \frac{x}{2(x+2)(x-2)}$ (1)

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b) (i) $|3x-1| < 5$ or

(1) $-5 < 3x - 1 < 5$

$-4 < 3x < 6$

(1) $-\frac{4}{3} < x < 2$

(must write as 1 expression)

$3x - 1 < 5$

$x < 2$

$-3x + 1 < 5$ (1)

$-3x < 4$

$x > -\frac{4}{3}$ (1)

$\therefore -\frac{4}{3} < x < 2$

(ii) $\frac{x+4}{2} - \frac{x+3}{4} = 10$

$2x + 8 - x - 3 = 40$ (1)

$x + 5 = 40$

$x = 35$ (1)

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c) $\frac{ab^{-1} - ba^{-1}}{b-a}$

$= \frac{\frac{a}{b} - \frac{b}{a}}{b-a}$ (1)

$= \frac{a^2 - b^2}{ab} \div \frac{(b-a)}{1}$

$= \frac{(a+b)(a-b)}{ab} \times \frac{1}{(b-a)}$

$= \frac{-a-b}{ab}$ (1)

If $\frac{\frac{1}{ab} - \frac{1}{ba}}{b-a}$ the answer is 0 — award (1) only

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d) $a + c = 420$ — (1)

$c = a + 120$ — (2) } (1)

sub (2) into (1)

$a + a + 120 = 420$

$2a = 300$

$a = 150$ (1)

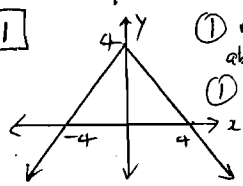
$c = 270$. (1)

adults - 150

children - 270

13a) (i) $y = 4 - |x|$

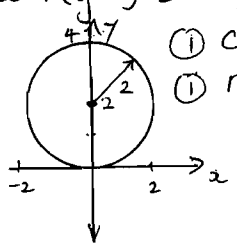
[11]



(1) recognising absolute value

(1) y intercept + 1 other point.

(ii) $x^2 + (y-2)^2 = 4$



(1) Centre

(1) radius

(iii)

Range: $y \leq 4$ (1)

(iv)

$R: 0 \leq y \leq 4$ (1)

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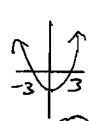
b) $y = \sqrt{x^2 - 9}$



(1) $x^2 - 9 \geq 0$

$x \leq -3$ or $x \geq 3$ (1)

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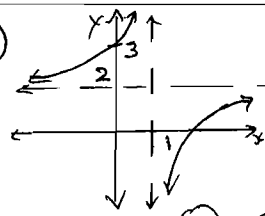
c)



(1) Correct asymptotes

(1) recognising hyperbola

(1) Correct position + 1 other point.

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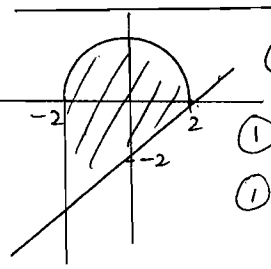
14. a) [10]



(1) semi circle

(1) line

(1) correct region

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b) (i) $2\sin\theta - 1 = 0$ (1) (1)

$\sin\theta = \frac{1}{2} \rightarrow \theta = 30, 150°$

(ii) $\tan^2\theta = 2\sec\theta + 2$

$\sec^2\theta - 1 = 2\sec\theta + 2$ (1)

$\sec^2\theta - 2\sec\theta - 3 = 0$

$(\sec\theta - 3)(\sec\theta + 1) = 0$

$\sec\theta = 3, -1$

$\therefore \cos\theta = \frac{1}{3}$ $\cos\theta = -1$

$\theta = 70°32', 289°28'$ (1) $\theta = 180°$ (1)

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c) $f(x+1) = \frac{2f(x) + 3}{3}$

$f(3) = 4$

$f(3) = f(2+1) = \frac{2f(2) + 3}{3}$

$\therefore 4 = \frac{2f(2) + 3}{3}$ (1)

$2f(2) = 9$

$f(2) = 4.5$ (1)

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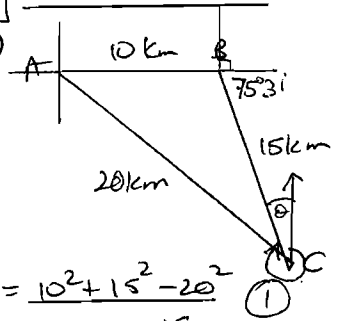
[10]

15a)

(i)



$\cos B = \frac{10^2 + 15^2 - 20^2}{2 \times 10 \times 15}$ (1)

$= 104°29'$ (1)

(ii) Bearing $= 360 - \theta$

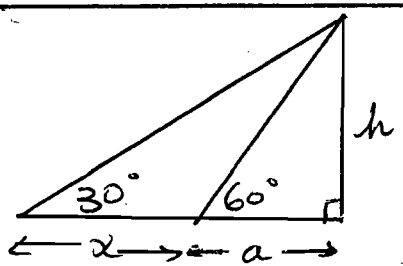
$\theta = 14°29'$

Bearing $= 360 - 14°29'$

$= 345°31'$ (1)

(Accept $345° - 346°$)

b)



(i) $\tan 30° = \frac{h}{x+a}$

$\therefore h = (x+a) \times \tan 30$

$= (x+a) \times \frac{1}{\sqrt{3}}$ (1)

$h = \frac{x+a}{\sqrt{3}}$ — (A)

(ii) $\tan 60° = \frac{h}{a}$

$\therefore h = a \tan 60$

$= a\sqrt{3}$ (1) — (B)

(A) = (B)

$\therefore \frac{x+a}{\sqrt{3}} = a\sqrt{3}$

$x + a = 3a$

$\therefore x = 2a.$ (1)

c) (i) $x^2 - x - 1 = 0$

α is a root $\therefore$

$\alpha^2 - \alpha - 1 = 0$

$\therefore \alpha^2 = \alpha + 1.$ — (1)

(ii) $\alpha^6 = (\alpha^2)^3$

(1) $= (\alpha+1)^3$

$= \alpha^3 + 3\alpha^2 + 3\alpha + 1$

(1) $= \alpha(\alpha^2) + 3(\alpha+1) + 3\alpha + 1$

$= \alpha(\alpha+1) + 3\alpha + 3 + 3\alpha + 1$

$= \alpha^2 + \alpha + 6\alpha + 4$

$= \alpha + 1 + 7\alpha + 4$

(1) $= 8\alpha + 5.$