BAULKHAM HILLS HIGH SCHOOL

2015

YEAR 11

HALF YEARLY EXAMINATIONS

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
  Black pen is preferred
- Board-approved calculators may be used
- Show all necessary working in
  Questions 11 – 14
- Marks may be deducted for careless or
  badly arranged work

Total marks – 58

Section I Pages 2 – 4

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section

Section II Pages 5 – 7

48 marks

- Attempt Questions 11 – 14
- Allow about 1 hour 15 minutes for this section

Section I

10 marks
Attempt Questions 1 – 10
Allow about 15 minutes for this section

Use the multiple-choice answer sheet for Questions 1 – 10

1 $5x^{-\frac{1}{2}} = ?$

(A) $\frac{1}{\sqrt{5x}}$

(B) $\frac{1}{5\sqrt{x}}$

(C) $\sqrt{\frac{5}{x}}$

(D) $\frac{5}{\sqrt{x}}$

2 What is the reciprocal of $x + \frac{1}{x}$?

(A) $\frac{1}{x} + x$

(B) $\frac{x+1}{x}$

(C) $\frac{x}{x+1}$

(D) $\frac{x}{x^2+1}$

3 $a^2 - (b-c)^2 = ?$

(A) $(a-b+c)(a+b-c)$

(B) $(a-b-c)(a+b-c)$

(C) $(a-b+c)(a-b-c)$

(D) $(a-b+c)(a+b+c)$

4 $\frac{2}{\sqrt{2}} + \frac{\sqrt{2}}{2} =$?

(A) 1

(B) 2

(C) $\frac{3\sqrt{2}}{2}$

(D) $\frac{2+\sqrt{2}}{2\sqrt{2}}$

5 Which of the following is a factor of $6x^2 - x - 1$?

(A) $(2x - 1)$

(B) $(2x + 1)$

(C) $(6x - 1)$

(D) $(6x + 1)$

6 The equation of a circle with centre $(-1,2)$ and radius 9 units is ?

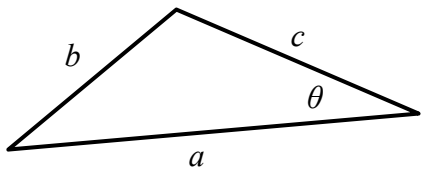
(A) $(x-1)^2 + (y+2)^2 = 9$

(B) $(x+1)^2 + (y-2)^2 = 9$

(C) $(x-1)^2 + (y+1)^2 = 81$

(D) $(x+1)^2 + (y-2)^2 = 81$

7 If $a^2 = b^2 + c^2$ in the triangle given below, then which of the following statements MUST be true?



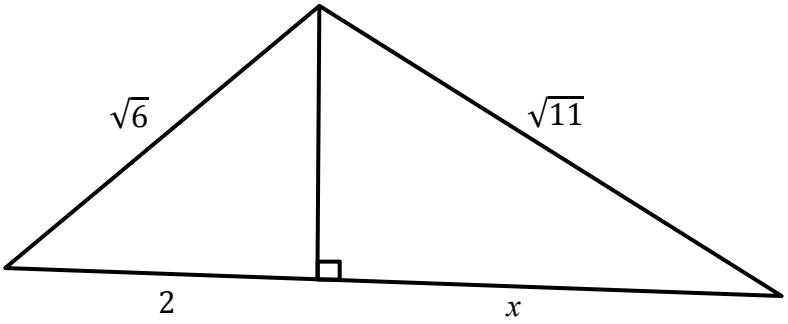
(A) $b = a\sin\theta$

(B) $a = c\cos\theta$

(C) $c = b\tan\theta$

(D) None of these

8 What is the value of x in the diagram below ?



(A) 1

(B) 3

(C) $\sqrt{13}$

(D) $\sqrt{15}$

9 If $P = \dfrac{x+y}{y}$ and $Q = \dfrac{4x}{x+y}$ where x and y are positive integers, which of the following MUST be true?

(A) $P > Q$

(B) $P \geq Q$

(C) $P < Q$

(D) $P \leq Q$

10 The positive integer n is not divisible by 7. The remainder when n^2 is divided by 7 and the remainder when n is divided by 7 are both equal to k.

What is the value of k ?

(A) 6

(B) 4

(C) 2

(D) 1

END OF SECTION I

Section II

48 marks
Attempt Questions 11 – 14
Allow about 1 hour 15 minutes for this section

Answer each question on the appropriate answer sheet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

Marks

Question 11 ***(12 marks)*** Use a *separate* answer sheet

(a) Find the exact value of $\cos\theta$, given that $\tan\theta = 7$ and $\sin\theta < 0$ *2*

(b) Factorise completely.

(i) $x^2 - x - 12$ *2*

(ii) $10x + 2xy - 10y - 2y^2$ *2*

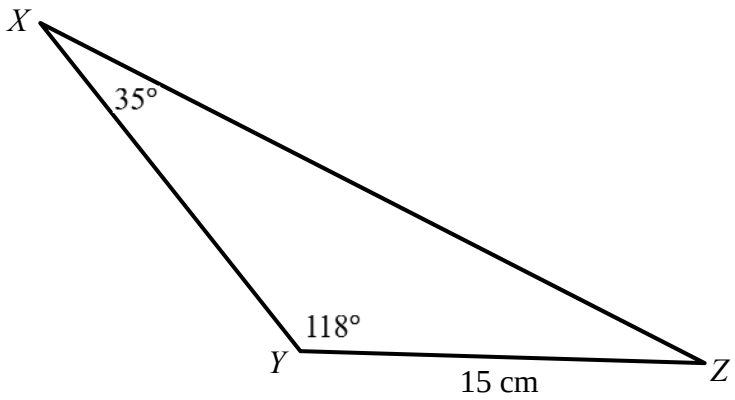
(iii) $8x^3 - 27$ *2*

(c) A function is defined by the rule; *2*

$$f(x) = \begin{cases} 0 & ; x \leq 0 \\ -1 & ; 0 < x < 2 \\ x & ; x \geq 2 \end{cases}$$

Evaluate $f(-2) + f(2) + f(5)$

(d) In ΔXYZ , $\angle X = 35°$, $\angle Y = 118°$ and $YZ = 15$ cm. *2*
Find the length of XZ, correct to one decimal place.



Marks

Question 12 ***(12 marks)*** Use a *separate* answer sheet

(a) For each of the functions (α), (β) and (γ);

(i) draw a neat sketch of the function

(ii) state the domain and range of the function

(α) $y = (x+1)^2$ *2*

(β) $y = |x| + 2$ *2*

(γ) $y = 3 + \dfrac{2}{x-1}$ *2*

(b) Solve the following pair of simultaneous equations *3*

$$7a + 3b = 36$$
$$5a + 2b = 25$$

(c) Show that $\dfrac{4}{2+\sqrt{5}} - \dfrac{1}{9-4\sqrt{5}}$ is rational by expressing it in its simplest form. *3*

Question 13 ***(12 marks)*** Use a *separate* answer sheet

(a) Express $\dfrac{x+1}{x^2-x} - \dfrac{x-1}{x^2+x}$ as a single fraction in its simplest form. *3*

(b) Solve $|x+2| = 2x - 1$ *3*

(c) Prove $(\cot\theta + \operatorname{cosec}\theta)^2 = \dfrac{1+\cos\theta}{1-\cos\theta}$ *3*

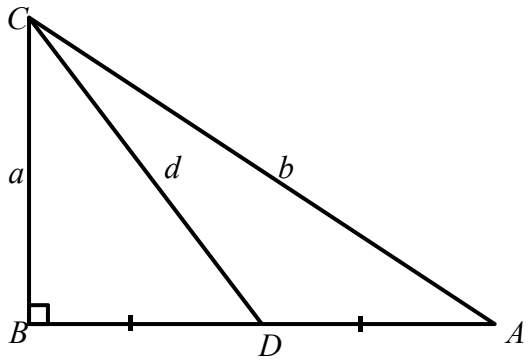
(d) Shade on a number plane the region for which the inequalities $y > \dfrac{1}{x}$ and $y \leq x + 3$ hold simultaneously. *3*

Marks

Question 14 ***(12 marks)*** Use a *separate* answer sheet

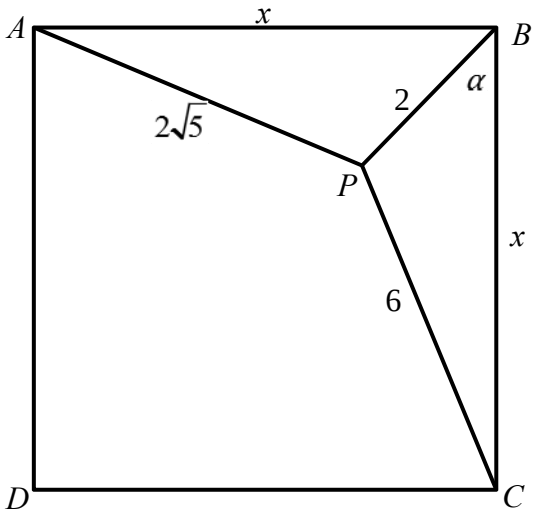
(a) Solve $\sin\theta = -\frac{2}{3}$ for $0° \leq \theta \leq 360°$, giving your answers correct to the nearest degree *2*

(b) In the right-angled triangle *ABC*, the point *D* bisects the base *AB*. *2*



Show that $4d^2 = b^2 + 3a^2$

(c) The diagram shows a square *ABCD* of side *x* cm. $\angle PBC = \alpha$, *PC* = 6 cm, *PB* = 2 cm and *AP* = $2\sqrt{5}$ cm.



(i) Using the cosine rule in $\Delta\ PBC$, show that $\cos\alpha = \frac{x^2 - 32}{4x}$ *2*

(ii) By considering $\Delta\ PBA$, show that $\sin\alpha = \frac{x^2 - 16}{4x}$ *2*

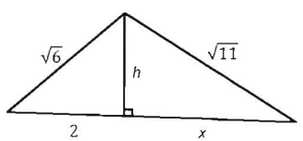
(iii) Hence show that *x* is a solution of $x^4 - 56x^2 + 640 = 0$ *2*

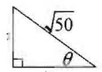
(iv) Hence find the exact value of the side length of the square. *2*

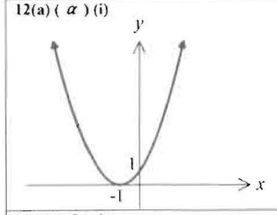
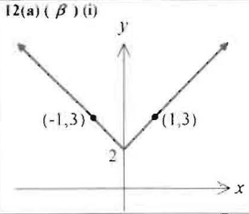
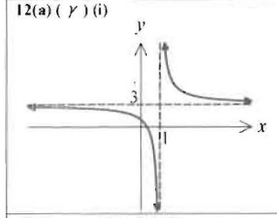
End of paper

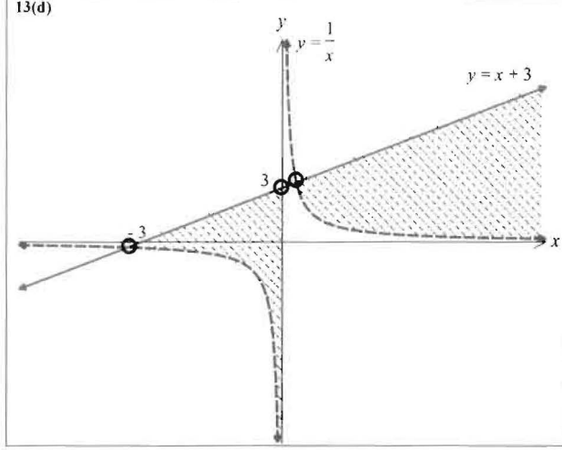
BAULKHAM HILLS HIGH SCHOOL

YEAR 11 MATHEMATICS HALF YEARLY EXAMINATION 2015 SOLUTIONS

Solution	Marks	Comments
SECTION I		
1. **D** - $5x^{\frac{1}{2}} = \dfrac{5}{\sqrt{x}}$	1	
2. **D** - $x + \dfrac{1}{x} = \dfrac{x^2+1}{x}$ $\therefore$ reciprocal is $\dfrac{x}{x^2+1}$	1	
3. **A** - $a^2 - (b-c)^2 = [a-(b-c)][a+(b-c)]$ $= (a-b+c)(a+b-c)$	1	
4. **C** - $\dfrac{2}{\sqrt{2}} + \dfrac{\sqrt{2}}{2} = \dfrac{2\sqrt{2}}{2} + \dfrac{\sqrt{2}}{2}$ $= \dfrac{3\sqrt{2}}{2}$	1	
5. **A** - $6x^2 - x - 1 = 6x^2 - 3x + 2x - 1$ $= 3x(2x-1) + 1(2x-1)$ $= (2x-1)(3x+1)$ $\therefore (2x-1)$ is a factor	1	
6. **D** - $(x+1)^2 + (y-2)^2 = 9^2$ $(x+1)^2 + (y-2)^2 = 81$	1	
7. **A** - If $a^2 = b^2 + c^2$, then a is the hypotenuse of a right angled triangle $\dfrac{b}{a} = \sin\theta$ $b = a\sin\theta$	1	
8. **B** -  $h^2 = (\sqrt{6})^2 - 2^2$ $= 2$ $x^2 = (\sqrt{11})^2 - h^2$ $= 11 - 2$ $= 9$ $x = 3$	1	
9. **B** - $P - Q = \dfrac{x+y}{y} - \dfrac{4x}{x+y}$ $= \dfrac{(x+y)^2 - 4xy}{y(x+y)}$ $= \dfrac{x^2+2xy+y^2-4xy}{y(x+y)}$ $= \dfrac{x^2-2xy+y^2}{y(x+y)}$ $= \dfrac{(x-y)^2}{y(x+y)} \geq 0$ as $x>0$, $y>0$ $\therefore P - Q \geq 0$ $P \geq Q$	1	
10. **D** - $n = 7a + k$, where k a is an integer $n^2 = 7b + k$ $\therefore 7b + k = (7a+k)^2$ $= 49a^2 + 14ak + k^2$ $= 7(7a^2 + 2ak) + k^2$ Thus $b = 7a^2 + 2ak$, which is an integer and $k = k^2$ $k^2 = k$ $k^2 - k = 0$ $k(k-1) = 0$ $k = 0$ or $k = 1$ not a solution	1	

Solution	Marks	Comments
SECTION II		
QUESTION 11		
11(a) $\tan\theta > 0$, $\sin\theta < 0$, $\therefore$ quadrant 3 $\Rightarrow \cos\theta < 0$ $\cos\theta = -\dfrac{1}{\sqrt{50}}$ $= -\dfrac{1}{5\sqrt{2}}$  	2	**2 marks** • Correct solution **1 mark** • Correctly identifies the sign of the cosine ratio • Correctly finds the magnitude of the cosine ratio *Note: surd can be left unsimplified*
11(b) (i) $x^2 - x - 12 = (x-4)(x+3)$	2	**2 marks** • Fully factorised answer **1 mark** • A factorised answer with signs switched
11(b) (ii) $10x + 2xy - 10y - 2y^2 = 2(5x + xy - 5y - y^2)$ $= 2\{x(5+y) - y(5+y)\}$ $= 2(5+y)(x-y)$	2	**2 marks** • Fully factorised answer **1 mark** • Partially factorised answer
11(b) (iii) $4x^3 - 27 = (2x)^3 - 3^3$ $= (2x - 3)(4x^2 + 6x + 9)$	2	**2 marks** • Fully factorised answer **1 mark** • A factorised answer with signs switched • Expressing the second factor as a perfect square
11(c) $f(-2) + f(2) + f(5)$ $= 0 + 2 + 5$ $= 7$	2	**2 marks** • Correct answer **1 mark** • Correctly finds the three function values • Sums an expression with at least two correct function values
11(d) $\dfrac{XZ}{\sin 118^\circ} = \dfrac{15}{\sin 35^\circ}$ $XZ = \dfrac{15\sin 118^\circ}{\sin 35^\circ}$ $= 23.09058227...$ $= 23.1$ cm (correct to 1 decimal place)	2	**2 marks** • Correct answer **1 mark** • Correctly substitutes into the sine rule, or equivalent merit *Note: ignore rounding error*

Solution	Marks	Comments
QUESTION 12		
12(a) (α) (i)  **(ii)** Domain: all real x Range: $y \geq 0$	**2**	**2 marks** • Both of the below dot points **1 mark** • Graph with correct shape and position • Correct domain and range
12(a) (β) (i)  **(ii)** Domain: all real x Range: $y \geq 2$	**2**	**2 marks** • Both of the below dot points **1 mark** • Graph with correct shape and position • Correct domain and range
12(a) (γ) (i)  **(ii)** Domain: all real x except $x = 1$ Range: all real y except $y = 3$	**2**	**2 marks** • Both of the below dot points **1 mark** • Graph with correct shape and position • Correct domain and range
12(b) $7a + 3b = 36$ $5a + 2b = 25$ $\Rightarrow$ $14a + 6b = 72$ ( ) $15a + 6b = 75$ $a = 3$ $\therefore 5(3) + 2b = 25$ $15 + 2b = 25$ $2b = 10$ $b = 5$ $\therefore a = 3, b = 5$	**3**	**3 marks** • Correctly finds a and b **2 marks** • Correctly finds a or b **1 mark** • Rewrites the given equations making the coefficient of a (or b) equivalent • Makes a (or b) the subject of one of the given equations
12(c) $\frac{4}{2+\sqrt{5}} - \frac{1}{9-4\sqrt{5}} = \frac{4}{2+\sqrt{5}} \times \frac{2-\sqrt{5}}{2-\sqrt{5}} - \frac{1}{9-4\sqrt{5}} \times \frac{9+4\sqrt{5}}{9+4\sqrt{5}}$ $= \frac{8-4\sqrt{5}}{4-5} - \frac{9+4\sqrt{5}}{81-80}$ $= -8+4\sqrt{5} - 9 - 4\sqrt{5}$ $= -17$ which is a rational number	**3**	**3 marks** • Correctly simplifies the expression to a rational number **2 marks** • Correctly turns either fraction into an expression with a rational denominator **1 mark** • Attempts to multiply at least one of the fractions by the conjugate of the denominator

Solution	Marks	Comments
QUESTION 13		
13(a) $\frac{x+1}{x^2-x} - \frac{x-1}{x^2+x} = \frac{x+1}{x(x-1)} - \frac{x-1}{x(x+1)}$ $= \frac{(x+1)^2 - (x-1)^2}{x(x+1)(x-1)}$ $= \frac{4x}{x(x+1)(x-1)}$ $= \frac{4}{(x+1)(x-1)}$	**3**	**3 marks** • Correctly solution **2 marks** • Correctly combines into a single fraction **1 mark** • Creates a common denominator
13(b) $\lvert x+2 \rvert = 2x - 1$ $x + 2 = 2x - 1$ or $-x - 2 = 2x - 1$ $x = 3$ $3x = -1$ $x = -\frac{1}{3}$ not a solution $\therefore x = 3$	**3**	**3 marks** • Correctly solution **2 marks** • Correctly identifies two "*possible*" solutions to the equation **1 mark** • Considers two cases in an attempt to solve the equation **0 marks** • Solves without considering the absolute value
13(c) $(\cot\theta + \operatorname{cosec}\theta)^2 = \left(\frac{\cos\theta}{\sin\theta} + \frac{1}{\sin\theta}\right)^2$ $= \left(\frac{\cos\theta + 1}{\sin\theta}\right)^2$ $= \frac{(\cos\theta + 1)^2}{\sin^2\theta}$ $= \frac{(\cos\theta + 1)^2}{1 - \cos^2\theta}$ $= \frac{(\cos\theta + 1)^2}{(1 + \cos\theta)(1 - \cos\theta)}$ $= \frac{1 + \cos\theta}{1 - \cos\theta}$	**3**	**3 marks** • Correctly solution **2 marks** • Makes significant progress with a valid attempt to prove the identity **1 mark** • Uses a trig identity or relationship in an attempt to prove the identity
13(d) 	**3**	**3 marks** • Correctly solution **2 marks** • Boundaries and the points of intersection correctly classified. • Region correctly identified **1 mark** • Boundaries correctly identified: * $y = \frac{1}{x}$ – dotted * $y = x + 3$ solid * $x = 0$ dotted • Identifies the three points of intersection are excluded. • Correctly draws one of the given inequalities

Solution	Marks	Comments
QUESTION 14		
14(a) $\sin\theta = -\frac{2}{3}$ Q3 & Q4 $\sin\alpha = \frac{2}{3}$ $\alpha = 42°$ $\theta = 180° + \alpha\ ,\ 360° - \alpha$ $\theta = 222°\ ,\ 318°$	2	**2 marks** • Correct solution **1 mark** • Correctly finds the size of the acute angle
14(b) In Δ CBD : $BD^2 = d^2 - a^2$ In Δ CBA : $AB = 2BD$ $4BD^2 = b^2 - a^2$ $4(d^2 - a^2) = b^2 - a^2$ $4d^2 - 4a^2 = b^2 - a^2$ $4d^2 = b^2 + 3a^2$	2	**2 marks** • Correct solution **1 mark** • Uses either triangle to find an expression for BD^2
14(c) (i) $\cos\alpha = \frac{x^2 + 2^2 - 6^2}{2 \times x \times 2}$ $= \frac{x^2 + 4 - 36}{4x}$ $= \frac{x^2 - 32}{4x}$	2	**2 marks** • Correct solution **1 mark** • Correctly substitutes into the cosine rule
14(c) (ii) $\cos(90 - \alpha) = \frac{x^2 + 2^2 - (2\sqrt{5})^2}{2 \times x \times 2}$ $\sin\alpha = \frac{x^2 + 4 - 20}{4x}$ $= \frac{x^2 - 16}{4x}$	2	**2 marks** • Correct solution **1 mark** • recognises $\cos(90 - \alpha) = \sin\alpha$
14(c) (iii) $\sin^2\alpha + \cos^2\alpha = 1$ $\left(\frac{x^2 - 16}{4x}\right)^2 + \left(\frac{x^2 - 32}{4x}\right)^2 = 1$ $x^4 - 32x^2 + 256 + x^4 - 64x^2 + 1024 = 16x^2$ $2x^4 - 112x^2 + 1280 = 0$ $x^4 - 56x^2 + 640 = 0$	2	**2 marks** • Correct solution **1 mark** • Attempts to derive given result using an appropriate trig identity
14(c) (iv) $x^4 - 56x^2 + 640 = 0$ $(x^2 - 40)(x^2 - 16) = 0$ $x^2 - 40$ or $x^2 = 16$ $x = \pm 2\sqrt{10}$ $x = \pm 4$ However from Δ PBC , $x > 6$ $\therefore$ the side length of the square is $2\sqrt{10}$ cm	2	**2 marks** • Correct solution **1 mark** • Solves the given "quadratic"