BAULKHAM HILLS HIGH SCHOOL

2016

YEAR 11

HALF YEARLY EXAMINATION

Mathematics

General Instructions

- Reading time – 5 minutes
- Working time – 90 minutes
- Write using black or blue pen
- Black pen is preferred
- Board-approved calculators may be used
- All relevant mathematical reasoning and/or calculations must be shown

Total marks – 58

Section I (Pages 2-3)

10 marks

Attempt Questions 1-10

Allow about 15 minutes for this section

Section II (Pages 4-7)

48 marks

Attempt Questions 11-14

Allow about 1 hour 15 minutes for this section

Section I **Multiple choice.**

10 marks
Allow about 15 minutes for this section.
Use the multiple choice answer sheet for Questions 1-10

1) The solution of $\frac{1-3x}{2} \geq 1\frac{1}{2}$ is

(A) $x \leq -\frac{2}{3}$ (B) $x \geq -\frac{5}{3}$ (C) $x \geq -\frac{2}{3}$ (D) $x \leq -\frac{4}{3}$

2) What is the domain of the function $y = \frac{\sqrt{5-x}}{x-4}$?

(A) $x < 4$ or $x > 4$

(B) $x \leq 5$

(C) $x < 4$ or $4 < x \leq 5$

(D) $x < 4$ or $x < 5$

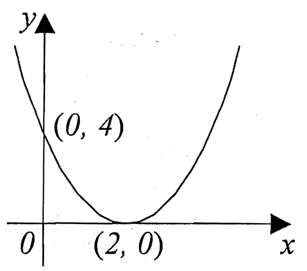
3) If $\tan A = \frac{2rs}{r^2-s^2}$, where A is acute and $r > s > 0$, then $\cos A$ is equal to

(A) $\frac{\sqrt{r^2-s^2}}{2r}$ (B) $\frac{r^2-s^2}{r^2+s^2}$ (C) $\frac{r}{r-s}$ (D) $\frac{rs}{r^2+s^2}$

4) If $(x-3)(2x+1) = 0$ then the possible values of $2x + 1$ are

(A) 0 only (B) 0 and 7 (C) $-\frac{1}{2}$ and 3 (D) 0 and 3

5)



The diagram could be the graph of

(A) $y = (x-2)^2 + 4$

(B) $y = x^2 - 2x$

(C) $y = (x-2)^2$

(D) $y = (x+2)^2$

6) The expression $\frac{k}{3}(k+1)(k+2)+(k+1)(k+2)$ is equal to

(A) $\frac{(k+1)(k+3)(k+4)}{6}$

(B) $\frac{(k+1)(k+2)(k+3)}{3}$

(C) $\frac{k(k+1)(k+2)}{3}$

(D) $\frac{2k}{3}(k+1)(k+2)$

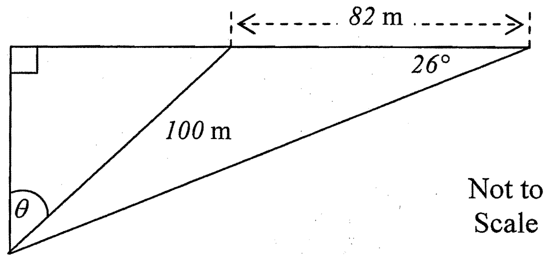
7) If $J = K(G - \frac{1}{T})$, then T equals

(A) $\frac{K}{KG-J}$ (B) $\frac{K}{G-J}$ (C) $\frac{K}{J-KG}$ (D) $\frac{J}{K}-G$

8) If $a^2 = a + 2$, then a^3 equals

(A) $a + 4$ (B) $2a + 8$ (C) $3a + 2$ (D) $27a + 8$

9) What is the value of θ to the nearest degree?



(A) 21° (B) 32° (C) 43° (D) 55°

10) Solve for x, $\sqrt{7x}-\sqrt{3x}=4$

(A) $10-2\sqrt{21}$

(B) $10+2\sqrt{21}$

(C) $\sqrt{7}-\sqrt{3}$

(D) $\sqrt{7}+\sqrt{3}$

END OF SECTION I

SECTION II

48 marks
Attempt questions 11 to 14
Allow about 75 minutes for this section.
Answer each question on the appropriate page of your answer booklet. Each answer sheet must show your name. Extra paper is available.

All necessary working should be shown in every question.

Marks

Question 11 **(12 marks)** Answer on the appropriate page of your answer booklet

(a) Simplify $4x - (2-x)(1+x)$ **2**

(b) Express $\dfrac{2}{\sqrt{5}-1}$ with a rational denominator. **2**

(c) Solve the equation $4 - \dfrac{x}{3} = \dfrac{x}{2}$ **2**

(d) Factorise $3x^2 - 13x + 12$ **2**

(e) Solve $2\sin^2\theta = \sin\theta$ for $0 \le \theta \le 360°$ **2**

(f) Consider the function $f(x) = \begin{cases} x^2 & \text{for } x \ge 0 \\ -x^2 & \text{for } x < 0 \end{cases}$.

Determine, without sketching, whether the function is even, odd or neither. **2**

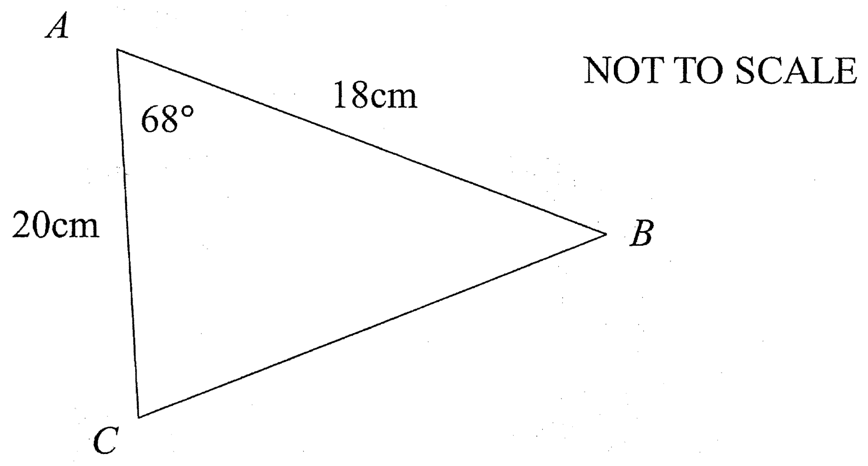
END OF QUESTION 11

Question 12 **(12 marks)** Answer on the appropriate page of your answer booklet

(a) Solve $\cot x = -\frac{\sqrt{3}}{3}$ for $0° \leq x \leq 360°$ **2**

(b) Sketch the region defined by $y \geq x^2 + 2x - 8$ and $y < -2x - 8$ **3**

(c) In the diagram, ABC is a triangle in which $\angle BAC = 68°$, $AC = 20\text{cm}$ and $AB = 18\text{cm}$.



Copy or trace the diagram onto your worksheet.

i) Calculate the area of the triangle ABC, correct to 1 decimal place. **1**

ii) Hence, or otherwise, find the length of the altitude of the triangle, from C to AB giving your answer correct to 1 decimal place. **1**

(d) Town A is 4 km from town P and its bearing from town P is 030°T. Town B is due south of town A and 6km from town P.

(i) Draw a neat sketch that clearly illustrates the above information. **2**

(ii) Find the distance between towns A and B, giving your answer correct to the nearest kilometre. **3**

END OF QUESTION 12

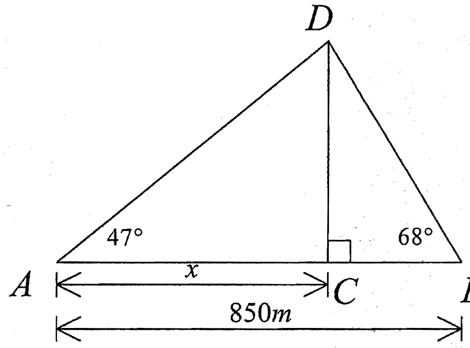
Question 13 **(12 marks)** Answer on the appropriate page of your answer booklet

(a) If a and b are rational numbers and $a + 2b + \sqrt{3a - b} = 2 + 2\sqrt{5}$ find the value of a and the value of b. **3**

(b) (i) Show that $\dfrac{\cos A}{1 - \sin A} - \dfrac{\cos A}{1 + \sin A} = 2\tan A$ **2**

(ii) Hence, or otherwise, solve $\dfrac{\cos A}{1 - \sin A} = 2 + \dfrac{\cos A}{1 + \sin A}$ for $0° \leq A \leq 360°$ **2**

(c) A and B are points directly opposite a hill DC. The angles of elevation of the top of the hill at A and B are 47° and 68° respectively and $AC = x$ m.



(i) Using triangle ADC find an expression for DC in terms of x **1**

(ii) If the distance from A to B is 850m find the height of the hill correct to three significant figures. **2**

(d) Find the value of n if 4^n is equal to one quarter of 2^{88} **2**

END OF QUESTION 13

Question 14 **(12 marks)** Answer on the appropriate page of your answer booklet

(a) Consider the curves $y = \dfrac{7}{x}$ and the line $y = 8 - x$.

(i) Find their points of intersection **2**

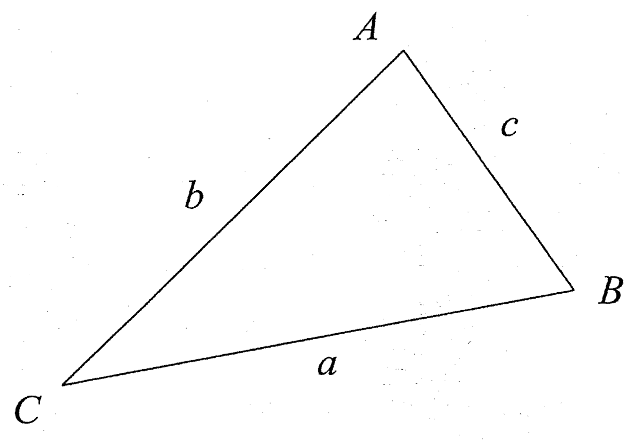
(ii) Sketch both curves on the same number plane. **2**

(iii) Hence solve $\dfrac{7}{x} > 8 - x$ **2**

(iv) What straight line should be drawn to intersect $y = \dfrac{2}{x}$ in order to solve $4x^2 - 3x - 2 = 0$? **1**

(b) Solve $|6 - x| = 2x + 3$ **3**

(c) Consider the following triangle below.



Show that if $\dfrac{1}{a+c} + \dfrac{1}{b+c} = \dfrac{3}{a+b+c}$ then $\angle ACB = 60°$ **2**

END OF EXAMINATION

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1. $\frac{1-3x}{2} \geq \frac{3}{2}$

$1-3x \geq 3$

$-2 \geq 3x$

$x \leq -\frac{2}{3}$

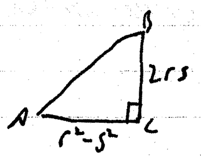
$\therefore$ (A)

2. $5-x \geq 0$ and $x-4 \neq 0$

ie $x \leq 5$ $\quad x \neq 4$

$\therefore$ (C)

3.



$AB^2 = (r^2-s^2)^2 + (2rs)^2$

$= r^4 + 2r^2s^2 + s^4$

$= (r^2+s^2)^2$

$AB = r^2+s^2$

$\cos A = \frac{r^2-s^2}{r^2+s^2}$

$\therefore$ (B)

4. $x-3=0$ or $2x+1=0 \Rightarrow x = -\frac{1}{2}$

$\therefore 2x+1=0$ or $x-3=0 \Rightarrow x=3$

$\therefore 2x+1 = 2\times3+1$

$=7$

$\therefore$ 0 or 7

$\therefore$ B

5. shifted right $y=x^2$ (2 places)

$\therefore$ (C)

6. $\frac{k}{3}(k+1)(k+2) + \frac{3(k+1)(k+2)}{3}$

$= \frac{(k+1)(k+2)(k+3)}{3}$

$\therefore$ (B)

7. $\frac{J}{K} = G - \frac{1}{T}$

$\frac{J}{K} - \frac{GK}{K} = -\frac{1}{T}$

$\frac{1}{T} = \frac{GK-J}{K}$

$T = \frac{K}{GK-J}$

$\therefore$ (A)

8. $a^2 = a+2$

$a^3 = a^2 + 2a$

but $a^2 = a+2$

$\therefore a^3 = a+2+2a$

$= 3a+2$

$\therefore$ (C)

9. $\frac{\sin\alpha}{82} = \frac{\sin 26°}{100}$

$\sin\alpha = \frac{82\sin 26°}{100}$

$\alpha = 21°$ (nearest degree)

$\therefore$ ext $\angle = 47° = (21+26)°$

$\therefore \theta = 180-90-47$

$= 43°$ $\quad$ (C?)

10. $\sqrt{7}\sqrt{x} - \sqrt{3}\sqrt{x} = 4$

$\sqrt{x}(\sqrt{7}-\sqrt{3}) = 4$

$\sqrt{x} = \frac{4}{\sqrt{7}-\sqrt{3}} \cdot \frac{\sqrt{7}+\sqrt{3}}{\sqrt{7}+\sqrt{3}}$

$= \frac{4(\sqrt{7}+\sqrt{3})}{7-3}$

$x = (\sqrt{7}+\sqrt{3})^2$

$= 10 + 2\sqrt{21}$

$\therefore$ (B)

Section II

11 a) $4x - (2+x-x^2)$

$= 3x - 2 + x^2$

(2) correct answer
(1) {expands $(2-x)(1+x)$ / simplifies & expands out negative

b) $\frac{2(\sqrt{5}+1)}{(\sqrt{5}-1)(\sqrt{5}+1)}$

$= \frac{2(\sqrt{5}+1)}{5-1}$

$= \frac{\sqrt{5}+1}{2}$

(2) correct answer
(1) {multiplies by conjugate

c) $4 - \frac{x}{3} = \frac{x}{2}$

$24 - 2x = 3x$

$5x = 24$

$x = \frac{24}{5}$ or $4\frac{4}{5}$ or 4.8

(2) correct answer
(1) attempts to find common denominator or multiplies by 6

d) $3x^2 - 13x + 12 = 3x^2 - 9x - 4x + 12$

$= 3x(x-3) - 4(x-3)$

$= (3x-4)(x-3)$

(2) correct answer
(1) progress towards ie splits middle term / finds one factor

e) $\sin\theta(2\sin\theta - 1) = 0$

$\sin\theta = 0$ or $2\sin\theta = 1$

$\theta = 0°, 180°, 360°$ $\sin\theta = \frac{1}{2}$

$\theta = 30°, 150°$

$\therefore \theta = 0°, 30°, 150°, 180°, 360°$

(2) correct solution
(1) solves $\sin\theta = 0$, $\sin\theta = \frac{1}{2}$ correctly.

f) If $a > 0$, $f(a) = a^2$,
$f(-a) = -(-a)^2$
$= -a^2$
$= -f(a)$

If $a < 0$ $f(a) = -(a)^2$
$= -a^2$
$f(-a) = (-a)^2$
$= a^2$
$= -f(a)$

If $a = 0$, $f(0) = 0$ $\therefore$ odd function

(2) correct solution with working for both cases
(1) attempts to find $f(-a)$ and compares to $f(a)$

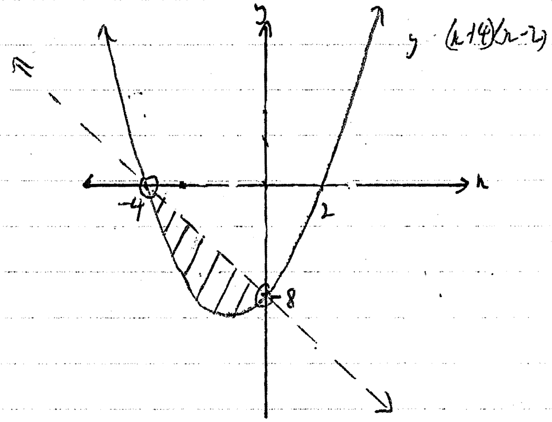
12 a) $\tan x = \frac{-3}{\sqrt{3}}$

$\tan x = -\sqrt{3}$

$x = 120°, 300°$

(2) correct solution
(1) • finds acute θ for tan
• finds one correct ∠
• finds 150°, 330°

b)



(3) correct solution
(2) • both graphs correct
• 1 region correct
(1) 1 graph correct
(ignore open circles)

c) i) $A = \frac{1}{2} \times 20 \times 18 \times \sin 68°$
$= 166.9\ cm^2$

(1) correct answer

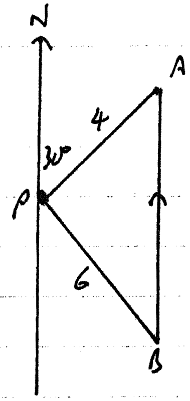
ii) $166.893... = \frac{18h}{2}$

$h = 18.5436...$

$\therefore$ Altitude $= 18.5$ cm 1dp

(1) correct answer

(d) (i)



(2) all information correct
(1) correct without vertices or all measurements

ii) $\angle PAB = 30°$

By cosine rule $6^2 = AB^2 + 4^2 - 2 \times 4\,AB \cos 30°$

$AB^2 - 4\sqrt{3}AB - 20 = 0$

$AB = \frac{4\sqrt{3} \pm \sqrt{48 + 80}}{2}$

$= \frac{4\sqrt{3} \pm \sqrt{128}}{2}$

$= 9.1209...$

$= 9$ km (nearest kilometre)

(3) correct solution
(2) • uses cosine rule with 30° angle to form quadratic
• correctly solves incorrect quadratic
(1) uses cosine rule

13 a) $a + 2b + \sqrt{3a - b} = 2 + \sqrt{20}$

$a + 2b = 2$ (1)

$3a - b = 20$ (2)

(1)×3 $3a + 6b = 6$ (1A)

(1A) − (2) $7b = -14$

$b = -2$

sub in (1) $a - 4 = 2$

$a = 6$

$\therefore a = 6, b = -2$

(3) correct answer
(2) • finds a or b from correct simultaneous equations
• correctly finds a and b from incorrect equations
(1) correctly establishes simultaneous equations
(1) solves for a or b only from incorrect simultaneous equations

13 (b) (i) LHS $= \frac{\cos A}{1-\sin A} \times \frac{1+\sin A}{1+\sin A} - \frac{\cos A(1-\sin A)}{(1+\sin A)(1-\sin A)}$ $(A \neq 90°)$ ② correct solution

$= \frac{\cos A + \cos A \sin A - \cos A + \cos A \sin A}{1-\sin^2 A}$ ① expresses with common denominator

$= \frac{2\cos A \sin A}{\cos^2 A}$

$= \frac{2\sin A}{\cos A}$

$= 2\tan A$

$=$ RHS

(ii) $\frac{\cos A}{1-\sin A} = 2 + \frac{\cos A}{1+\sin A}$ ② correct solution ① uses (i) to find $2\tan A = 2$

$\frac{\cos A}{1-\sin A} - \frac{\cos A}{1+\sin A} = 2$

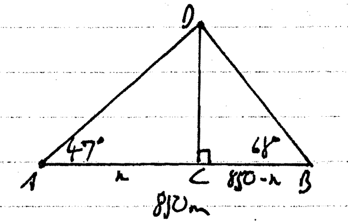
$2\tan A = 2$

$\tan A = 1$

$A = 45°, 180 + 45°$

$\therefore A = 45°, 225°$

c)



② correct solution
① forms equation by equating DC in 2 triangles
OR ① finds AD or BD
① finds DC from incorrect AD or BD

① correct solution

(i) In ΔADC

$\tan 47° = \frac{DC}{x}$

$DC = x\tan 47°$

(ii) In ΔCDB

$\tan 68° = \frac{DC}{850-x}$

$DC = (850-x)\tan 68°$

$\therefore x\tan 47° = (850-x)\tan 68°$

$x(\tan 47° + \tan 68°) = 850\tan 68°$

$x = \frac{850\tan 68°}{\tan 47° + \tan 68°}$

$x = 593.051...$

$x = 593$

$\therefore 47° = \frac{DC}{x}$

$DC = x\tan 47°$

$= 635.9698...$

$= 636\text{m}$ (3 sig fig)

13 d) $4^n = \frac{2^{88}}{4}$ ② correct solution ① uses indices rules to simplify expression

$2^{2n} = \frac{2^{88}}{2^2}$

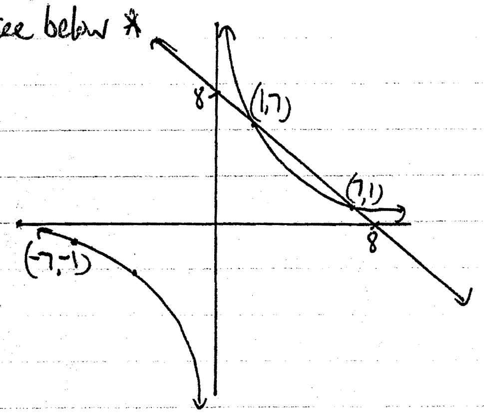
$2^{2n} = 2^{86}$

$2n = 86$

$n = 43$

14 (a) (i) see below *

14 a) (ii)



② correct solution
① one correct graph

* i) $\frac{7}{x} = 8 - x$ ② correct solutions ① forms quadratic ; finds 1 point of intersection

$7 = 8x - x^2$

$x^2 - 8x + 7 = 0$

$(x-7)(x-1) = 0$

$x = 1, 7$

$\therefore$ Pts of intersection $(1,7)$ and $(7,1)$

(iii) $0 < x < 1$ or $x > 7$ ① correct solution

(iv) $4x^2 - 3x - 2 = 0$ ② correct solution ① divides by x

$4x - 3 - \frac{2}{x} = 0$ $(\div x)$

$4x - 3 = \frac{2}{x}$

$\therefore y = 4x - 3$ is the required equation

b) $|6-x| = 2x+3$

$6-x = 2x+3$ or $6-x = -2x-3$

$3 = 3x$ $\quad$ $x = -9$

$x = 1$ $\quad$ no soln

NB $2x+3 \geq 0$

$2x \geq -3$

$x \geq -\frac{3}{2}$

$\therefore x = 1$ is the only solution

(3) correct solution (with justification)

(2) $x=1$ with no justification/explanation for other solution

(2) $x=1$ and $x=-9$

(1) $x=1$ no justification

c) $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$

$(a+b+c)(b+c) + (a+c)(a+b+c) = 3(a+c)(b+c)$

$ab + ac + b^2 + bc + bc + c^2 + a^2 + ab + ac + ac + bc + c^2 = 3(ab + ac + bc + c^2)$

$2ab + 3ac + b^2 + 3bc + 2c^2 + a^2 = 3ab + 3ac + 3bc + 3c^2$

$a^2 + b^2 - ab = c^2$

Since $\cos 60° = \frac{1}{2}$

$c^2 = a^2 + b^2 - 2ab\cos C$ which gives the cosine rule for a triangle ABC with angle $C = 60°$.

(2) correct solution

(1) simplifies to $c^2 = a^2+b^2-ab$ or equivalent.