BAULKHAM HILLS HIGH SCHOOL

Half -Yearly 2017
YEAR 11 ADVANCED TASK 1

Mathematics

General Instructions
- Reading time – 5 minutes
- Working time – 1.5 hours
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Questions 11-15
- Marks may be deducted for careless or badly arranged work

Total marks – 76
Exam consists of 9 pages.

This paper consists of TWO sections.

<u>Section 1</u> – (10 marks) Pages(2-4)
Questions 1-10
- Attempt Question 1-10
- Answer on answer sheet provided

<u>Section II</u> – (66 marks) Pages(5-9)
- Attempt questions 11-15

Section I - 10 marks

Use the multiple choice answer sheet for question 1-10

1. The factorisation of $x^3 - 8$ is

(A) $(x-2)(x^2-2x+4)$

(B) $(x-2)(x^2+2x+4)$

(C) $(x-2)(x^2-x+4)$

(D) $(x-2)(x^2+x+4)$

2. The solutions to the equation $x^2 - 5x + 2 = 0$ are :

(A) $\frac{5\pm\sqrt{17}}{2}$

(B) $\frac{-5\pm\sqrt{17}}{2}$

(C) $\frac{5\pm\sqrt{33}}{2}$

(D) $\frac{-5\pm\sqrt{33}}{2}$

3. Which of the following is equivalent to $\frac{1}{2\sqrt{5}-\sqrt{3}}$?

(A) $\frac{2\sqrt{5}-\sqrt{3}}{7}$

(B) $\frac{2\sqrt{5}+\sqrt{3}}{7}$

(C) $\frac{2\sqrt{5}-\sqrt{3}}{17}$

(D) $\frac{2\sqrt{5}+\sqrt{3}}{17}$

4. $\frac{8^{n+1}}{2^{n-2}} =$

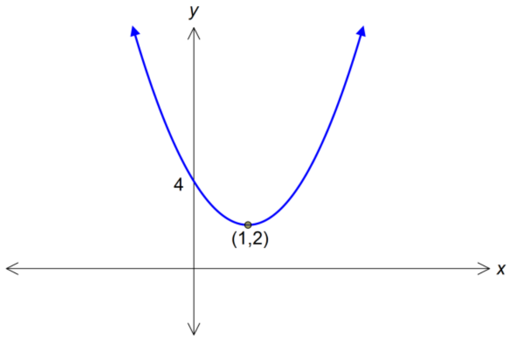
(A) 4^{-1}

(B) 4^{3}

(C) 2^{2n+1}

(D) 2^{2n+5}

5.



The equation for the given parabola is:

(A) $y = (x + 1)^2 + 2$
(B) $y = (x - 1)^2 + 2$
(C) $y = (x - 2)^2 + 1$
(D) $y = 2(x - 1)^2 + 2$

6. How many solutions of the equation $(sinx - 1)(tanx + 2) = 0$ lie between 0° and 360°?

(A) 1

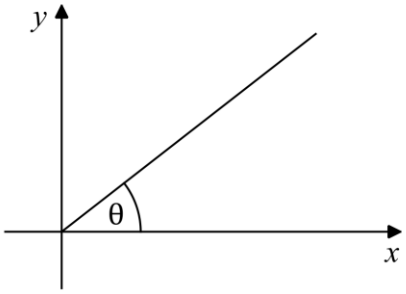
(B) 2

(C) 3

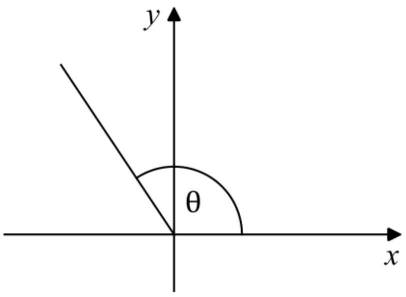
(D) 4

7. For the angle θ, $\sin\theta = \frac{7}{25}$ and $\cos\theta = -\frac{24}{25}$. Which diagram best shows the angle θ?

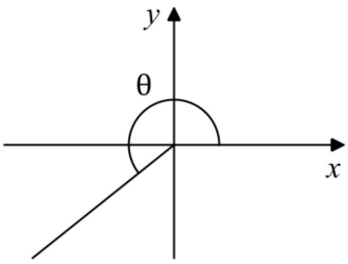
(A)



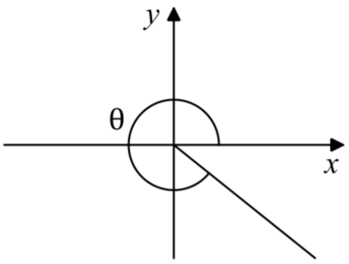
(B)



(C)

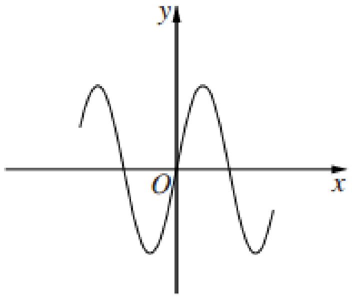


(D)

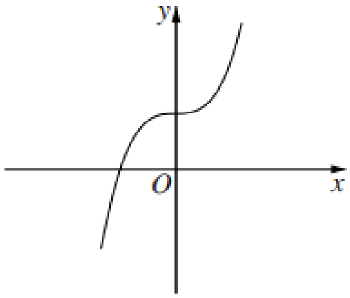


8. Which diagram shows the graph of an odd function?

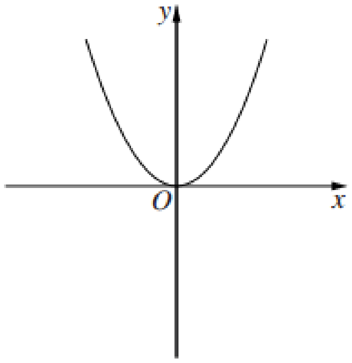
(A)



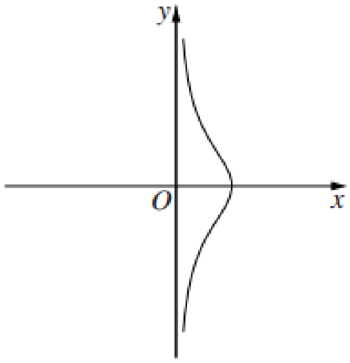
(B)



(C)



(D)



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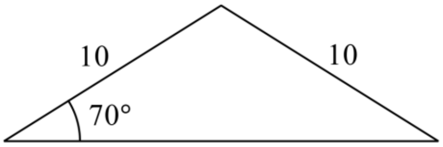


NOT DRAWN TO SCALE

The area for the given triangle, correct to two decimal places, is:

(A) 38.30

(B) 46.98

(C) 32.14

(D) 43.30

10. How many solutions does the equation $|\cos(2x)| = 1$ have for $0° \leq x \leq 360°$?

(A) 1

(B) 3

(C) 4

(D) 5

Section II – Extended Response

Attempt questions 11-15. All necessary working should be shown in every question.

Question 11 (13 marks) Use the Question 11 section of the writing booklet.		**Marks**
a)	Solve (i) $6-\frac{2x+1}{4}=3x$ (ii) $\lvert 3x - 1\rvert = 6$	**2** **2**
b)	Simplify (i) $(x-2)(x+2)-(3-x)$ (ii) $\frac{1}{x}-\frac{1}{x-1}$	**2** **2**
c)	If $(2\sqrt{3}-2)^2 = a - \sqrt{b}$, find the values of a and b	**2**
d)	Council rates increased by 8% to $1296. What were the rates prior to the increase?	**1**
e)	Solve simultaneously $4x+6y=11$ $17x-5y=1$	**2**
	End of Question 11	

Question 12 (13 marks) Use the Question 12 section of the writing booklet.

a) If $f(x) = x^2 - 4x$, find:

(i) $f(-2)$ **1**

(ii) $f(2\sqrt{3})$ **1**

(iii) the exact value of x, in simplest form, if $f(x) = 2$. **2**

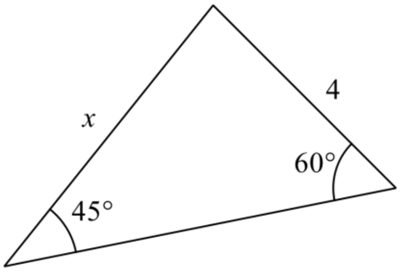
b) Decide if the function $f(x) = (2x^2 - 5x)^2$ is even, odd or neither. Justify your answer **2**

c) Sketch the region in the Cartesian plane for which inequalities **3**

$$y \leq 2x - 2$$

$$x^2 + y^2 \leq 4 \text{ hold simultaneously}$$

d) Given the triangle below, find the exact value of x. **2**



e) Find the exact value of $\tan\theta$ if $\cos\theta = \frac{\sqrt{2}}{5}$ and $270^\circ \leq \theta \leq 360^\circ$. **2**

End of Question 12

Question 13	(13 marks) Use the Question 13 section of the writing booklet.	**Marks**
a)	Consider the curve $y = \frac{2}{x-1} - 2$	
	(i) State the domain and range	**2**
	(ii) Find the intercepts	**2**
	(iii) Sketch the curve showing all important features	**2**
b)	Solve for $0° \leq \theta \leq 360°$	
	(i) $\sqrt{2}\sin\theta = 1$	**2**
	(ii) $2\sin^2\theta - \cos\theta = 1$	**3**
c)	Show that $\left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right)^2 = \sec^2\theta - 2\tan\theta$	**2**
	End of Question 13	

Question 14 (13marks) Use the Question 14 section of the writing booklet.

a) Solve | 2x + 5 | = 3x + 9 **3**

b) Factorise $4x^3 - 12x^2 - x + 3$ **2**

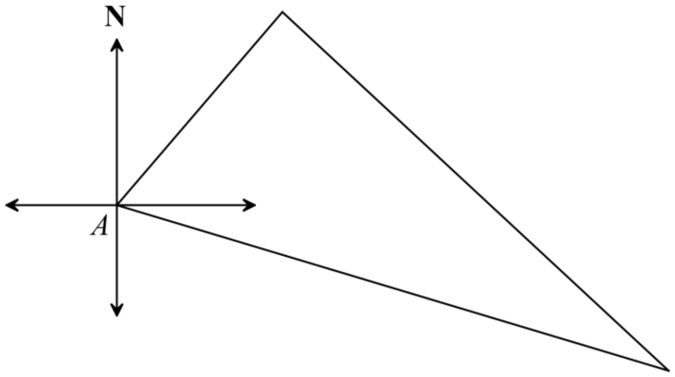
c) Given

$$f(x) = \begin{cases} x + 2 & for \quad x \le -2 \\ \sqrt{4 - x^2} & for \quad -2 < x < 2 \\ 2 - x & for \quad x \ge 2 \end{cases}$$

(i) Sketch the function **2**

(ii) Hence or otherwise find the range of $f(x)$. **1**

d) A hiker left camp A and walked 15 km on a bearing of $N32^{\circ}E$ to B. He then turned and walked for 25 km to the point C, then 35 km back to A.

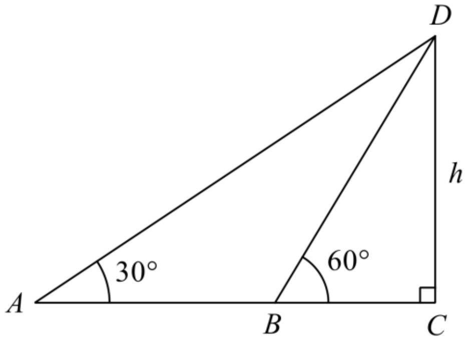


i) Redraw the diagram into your booklet showing the given information. **1**

ii) Find the size of $\angle ABC$. **2**

iii) Hence or otherwise find the bearing of B from C. **2**

End of Question 14

	Question 15 (14 marks) Use the Question 15 section of the writing booklet.	**Marks**
a)	Solve $\left(\frac{15}{x}+x\right)^2 - 11\left(\frac{15}{x}+x\right) + 24 = 0$	**3**
b)	Find an expression for the exact length of AB in terms of *h*.  	**2**
c)	i) Prove that $\tan A \sin A + \cos A = \sec A$ ii) Hence or otherwise solve $\tan A \sin A + \cos A = \operatorname{cosec} A$ for $0^\circ \leq A \leq 360^\circ$	**2** **2**
d)	If $f(x) = 2 - x^2$ and $g(x) = 2x - 1$ i) Find $f\left(g(5)\right)$. ii) Show that $f\left(g(x)\right) = -4x^2 + 4x + 1$. iii) Find the value(s) of x for which $f\left(g(x)\right) = g\left(f(x)\right)$.	 **1** **1** **3**
	End of Exam	

Multiple choice   Advanced 2017
Half Yearly

① $x^3 - 8 = (x-2)(x^2+2x+4)$ Ⓑ

② $x^2 - 5x + 2 = 0$

$\therefore x = \frac{-(-5) \pm \sqrt{(-5)^2 - 4(1)(2)}}{2(1)} = \frac{5 \pm \sqrt{17}}{2}$ Ⓐ

③ $\frac{1}{2\sqrt{5}-\sqrt{3}} \times \frac{2\sqrt{5}+\sqrt{3}}{2\sqrt{5}+\sqrt{3}} = \frac{2\sqrt{5}+\sqrt{3}}{20-3} = \frac{2\sqrt{5}+\sqrt{3}}{17}$ Ⓓ

④ $\frac{8^{n+1}}{2^{n-2}} = \frac{(2^3)^{n+1}}{2^{n-2}} = \frac{2^{3n+3}}{2^{n-2}} = 2^{3n+3-n+2} = 2^{2n+5}$ Ⓓ

⑤ $y = a(x-m)^2 + n$ vertex $(1,2)$

$\therefore y = a(x-1)^2 + 2$ sub. $(0,4)$ $\therefore a = 2$ $\therefore$ Ⓓ

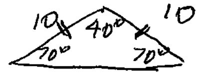
⑥ $\underbrace{\sin x - 1 = 0}_{\underbrace{\sin x = 1}_{1 \text{ solns.}}}$ or $\underbrace{\tan x + 2 = 0}_{\underbrace{\tan x = -2}_{2 \text{ solns.}}}$ Ⓒ

⑦ $\underbrace{\sin\theta = \frac{7}{25}}_{\text{I or II}}$ & $\underbrace{\cos\theta = -\frac{24}{25}}_{\text{II or III quadrant}}$ $\therefore$ Ⓑ

second quadrant

⑧ Symmetrical around $x \times y$-axis $\therefore$ Ⓐ

⑨ $A = \frac{1}{2} ab \cdot \sin C$



$\therefore A = \frac{1}{2} \times 10 \times 10 \times \sin 40^\circ = 32.13938\ldots$ $\therefore$ Ⓒ

⑩ $|\cos(2x)| = 1$ $0 \le x \le 360^\circ$

$\pm \cos(\underbrace{2x}_{\alpha}) = 1$ $0 \le \underbrace{2x}_{\alpha} \le 720^\circ$ $\therefore$ Ⓓ

$\cos\alpha = \pm 1$

$\alpha = 0^\circ, 180^\circ, 360^\circ, 540^\circ, 720^\circ$ $\therefore$ (5 solns)

BAULKHAM HILLS HIGH SCHOOL

MATHEMATICS

Section I – Multiple Choice

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely.

Sample: 2 + 4 =   (A) 2   (B) 6   (C) 8   (D) 9

A ○   B ●   C ○   D ○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A ●   B ☒   C ○   D ○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word ***correct*** and drawing an arrow as follows.

A ☒   B ☒ ← *correct*   C ○   D ○

Start Here →

1.	A ○	B ●	C ○	D ○	**6.**	A ○	B ○	C ●	D ○
2.	A ●	B ○	C ○	D ○	**7.**	A ○	B ●	C ○	D ○
3.	A ○	B ○	C ○	D ●	**8.**	A ●	B ○	C ○	D ○
4.	A ○	B ○	C ○	D ●	**9.**	A ○	B ○	C ●	D ○
5.	A ○	B ○	C ○	D ●	**10.**	A ○	B ○	C ○	D ●

Question 11 (13 marks)

	Marks
a) Solve (i) $6 - \frac{2x+1}{4} = 3x$	
	2 - correct solns.
$24 - (2x+1) = 12x$	1 - simplifying
$24 - 2x - 1 = 12x$	
$23 = 14x$	
$\frac{23}{14} = x$	
ii) $\lvert 3x - 1 \rvert = 6$	2 - correct solns
$3x - 1 = \pm 6$	1 - one correct answer
$3x-1=6$ $\quad$ $3x-1=-6$	
$3x=7$ $\quad$ $3x=-5$	
$x = 7/3$ $\quad$ $x = -5/3$	
$\therefore$ solns. $x = \frac{7}{3}, \frac{-5}{3}$	
b) Simplify	
i) $(x-2)(x+2) - (3-x)$	2 - correct answer
$= x^2 - 4 - 3 + x$	1 - expanding one
$= x^2 + x - 7$	of brackets correctly

	Marks
b) ii) $\frac{1}{x} - \frac{1}{x-1}$	2 - correct answer
$= \frac{x-1-x}{x(x-1)}$	1 - correct denominator
$= \frac{-1}{x(x-1)}$ or $\frac{-1}{x^2-x}$	
c) $(2\sqrt{3} - 2)^2 = a - \sqrt{b}$	2 - correct values for a and b
$4 \times 3 - 8\sqrt{3} + 4 = a - \sqrt{b}$	
$16 - 8\sqrt{3} = a - \sqrt{b}$	1 - correct value
$16 - \sqrt{192} = a - \sqrt{b}$	for a or b
$\therefore a = 16, \quad b = 192$	
d) 108% of $x = 1296$	1 - correct answer
$x = \$1200$	
$\therefore$ The rates were \$1200	

	Marks
e) (1) $4x + 6y = 11 \quad / \times 5$	2 - correct
(2) $\underline{17x - 5y = 1} \quad / \times 6$	values for x&y
(1) $20x + 30y = 55$	1 - correct value
(2) $\underline{102x - 30y = 6}$	for x or y
$122x = 61$	
$x = \frac{61}{122}$	1 - correct elimination
	of x or y
$x = \frac{1}{2}$	
	1 — making x or y
sub. $x = \frac{1}{2}$ into (1)	the subject and
$\therefore 4(\frac{1}{2}) + 6y = 11$	subst. into the
$2 + 6y = 11$	other equation
$6y = 9$	
$y = \frac{9}{6}$ or $\frac{3}{2}$	
$\therefore$ solutions are $\therefore$ $x = \frac{1}{2}, y = \frac{3}{2}$	

You may ask for extra writing paper if you need more space to answer question 11

Question 12

	Marks
a) $f(x) = x^2 - 4x$	
i) $f(-2) = (-2)^2 - 4(-2)$	1 - correct answer
$= 4 + 8 = 12$	
ii) $f(2\sqrt{3}) = (2\sqrt{3})^2 - 4(2\sqrt{3})$	1 - correct answer
$= 4 \times 3 - 8\sqrt{3}$	exact or decimal
$= 12 - 8\sqrt{3}$	
or $-1.8564...$	
iii) $f(x) = 2$	
$x^2 - 4x = 2$	2 - correct soln s,
$x^2 - 4x - 2 = 0$	in surd form simplified
$x = \frac{+4 \pm \sqrt{(-4)^2 - 4(1)(-2)}}{2}$	
$x = \frac{4 \pm \sqrt{24}}{2}$	
$x = \frac{4 \pm 2\sqrt{6}}{2}$	1 - correct soln s in surd form unsimplified
$x = 2 \pm \sqrt{6}$	

Marks

b) $f(x) = (2x^2 - 5x)^2$

2 - correct working

$f(-x) = (2(-x)^2 - 5(-x))^2$

$= (2x^2 + 5x)^2 \neq f(x)$

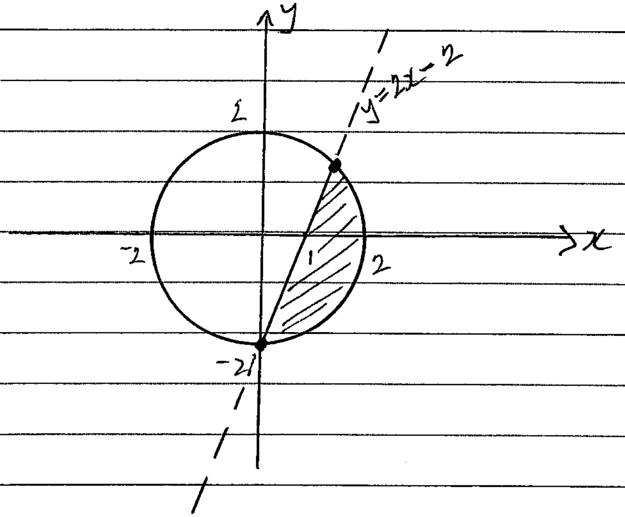
1 - gets the expression for $f(-x)$

$\therefore$ not even

also $(2x^2 + 5x)^2 \neq -f(x)$

$\therefore$ neither

c)



3 - correct region

2 - one correct region and boundary

2 - two correct boundaries

1 - one of the boundaries correct

Marks

d) $\dfrac{x}{\sin 60^\circ} = \dfrac{4}{\sin 45^\circ}$

2 - correct answer

$\dfrac{x}{\frac{\sqrt{3}}{2}} = \dfrac{4}{\frac{1}{\sqrt{2}}}$

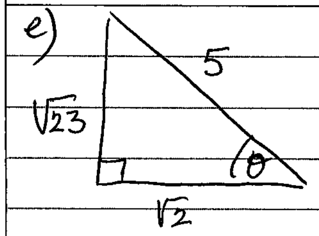
1 - both exact values correct

$x = \dfrac{4 \times \frac{\sqrt{3}}{2}}{\frac{1}{\sqrt{2}}} = \dfrac{4\sqrt{3}}{2} \times \sqrt{2}$

1 - uses sine rule correctly

$x = 2\sqrt{6}$

e)



$270^\circ \leq \theta \leq 360^\circ$

2 - correct answer

$\therefore \theta$ in IV quadrant

1 - correct ratio for $\tan\theta$

$\therefore \tan\theta < 0$

$\tan\theta = -\dfrac{\sqrt{23}}{\sqrt{2}}$ or $\tan\theta = -\sqrt{\dfrac{23}{2}}$

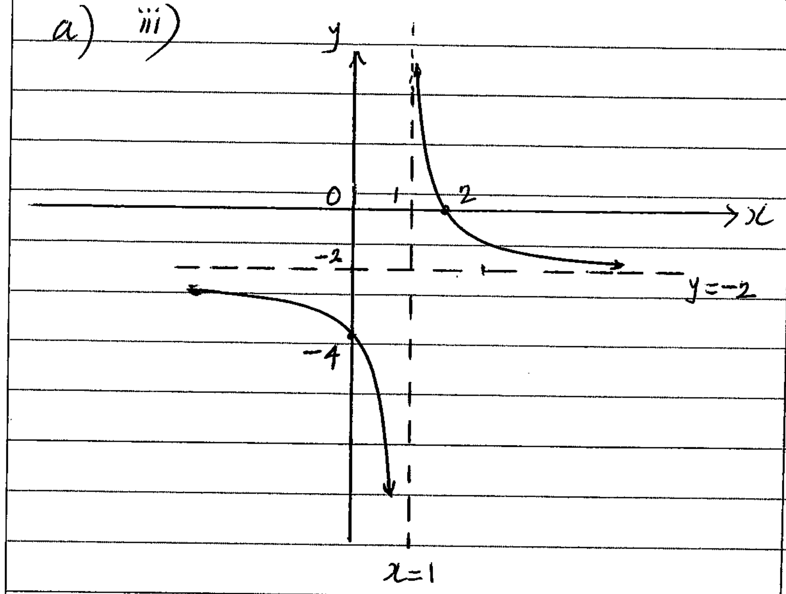
1 - correct sign for $\tan\theta$

OR $\tan\theta = -\dfrac{\sqrt{46}}{2}$

You may ask for extra writing paper if you need more space to answer question 12

Question 13

	Marks
a) $y = \dfrac{2}{x-1} - 2$	
i) Domain= all real x, where $x \neq 1$ Range= all real y except $y = -2$	2 - correct solns. 1 - correct Domain 1 - correct Range
ii) x-intercept $\therefore y = 0$ $\therefore 0 = \frac{2}{x-1} - 2$ $2 = \frac{2}{x-1}$ $2(x-1) = 2$ $x - 1 = 1$ $x = 2$ y-intercept $\therefore x = 0$ $\therefore y = \frac{2}{0-1} - 2$ $\therefore y = -4$	2 - correct x & y intercepts 1 - correct x or y-intercept

	Marks
a) iii) 	2 - correct shape with asymptot and intercepts 1 - correct shape 1 - intercepts and asymptotes
b) i) $\sqrt{2}\sin\theta = 1$ $\sin\theta = \frac{1}{\sqrt{2}}$ $\therefore \theta = 45^\circ, 135^\circ$	2 - correct answer. 1 - correct acute angle
ii) $2\sin^2\theta - \cos\theta = 1$ $2(1-\cos^2\theta) - \cos\theta - 1 = 0$ $-2\cos^2\theta - \cos\theta + 1 = 0$ $2\cos^2\theta + \cos\theta - 1 = 0$ $(2\cos\theta - 1)(\cos\theta + 1) = 0$ $\therefore \cos\theta = \frac{1}{2}$ OR $\cos\theta = -1$ $\therefore \theta = 60^\circ, 300^\circ, 180^\circ$	3 - correct answer. 2 - uses correct identity & create quad. trig. equatic with one trig. ratio 2 - gets two sets of solns. from incorrect quad. eqn. 1 - uses correct trig. identity

c) Show $\left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right)^2 = \sec^2\theta - 2\tan\theta$

$LHS = \left(\frac{\cos\theta - \sin\theta}{\cos\theta}\right)^2$

$= \frac{(\cos\theta - \sin\theta)^2}{\cos^2\theta}$

$= \frac{\cos^2\theta - 2\cos\theta.\sin\theta + \sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta + \sin^2\theta - 2\cos\theta.\sin\theta}{\cos^2\theta}$

$= \frac{1 - 2\cos\theta.\sin\theta}{\cos^2\theta}$

$= \frac{1}{\cos^2\theta} - \frac{2\cos\theta.\sin\theta}{\cos^2\theta}$

$= \sec^2\theta - \frac{2\sin\theta}{\cos\theta}$

$= \sec^2\theta - 2\tan\theta = RHS$

$\therefore$ proven

Marks

2 - correct proof

1 - recognizes one of the trig. identities and correctly expands bracket

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Question 14

a) Solve $|2x+5| = 3x+9$

$\pm(2x+5) = 3x+9$

$2x+5 = 3x+9$

$-4 = x$ ①

$-2x-5 = 3x+9$

$-14 = 5x$

① $x = -14/5$

① By checking $x \neq -4$ (not a solution)

$\therefore$ the only solution is $x = \frac{-14}{5}$

b) Factorise

$4x^3 - 12x^2 - x + 3$

$= 4x^2(x-3) - (x-3)$

$= (x-3)(4x^2-1)$

$= (x-3)(2x-1)(2x+1)$

c) $f(x) = x+2$

x	-3	-2
$x+2$	-1	0

$f(x) = \sqrt{4-x^2}$ semicircle

$f(x) = 2-x$

x	2	3
$2-x$	0	-1

Marks

3 - correct solution

2 - finds both x-values correctly

2 - finds one x-value correctly and checks

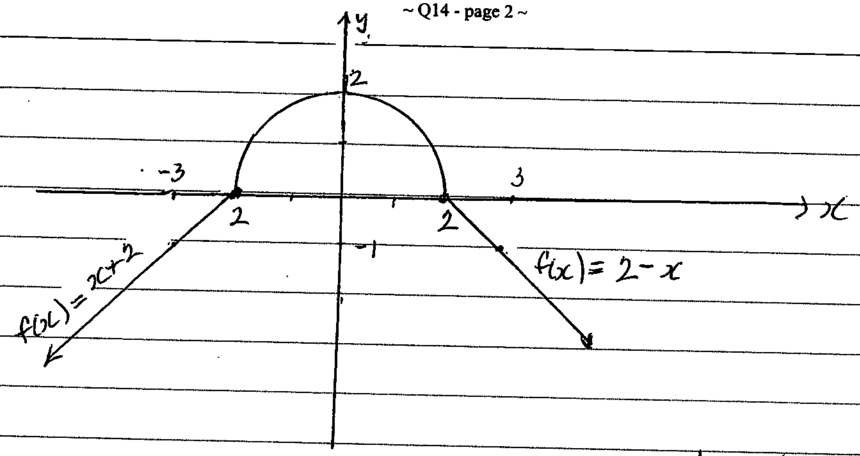
1 - finds one of the x-values correctly

2 - correct factors

1 - gets 2 of 3 correct factors

2 - all three sections correct

1 - two of the three sections of the peacemeal function correct



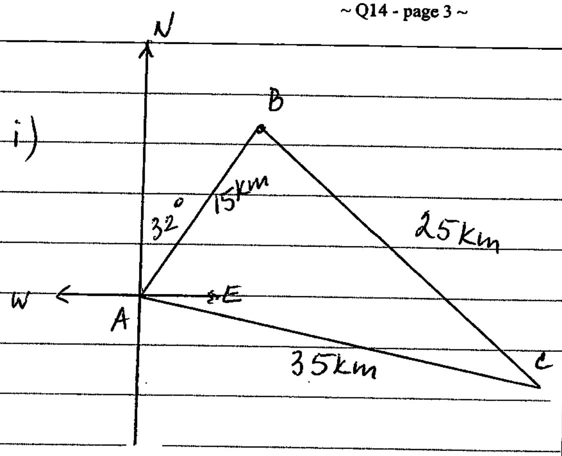
Marks

ii) Range - all real y where $y \le 2$.

1-correct answer

Marks

d) i)



1- correct diagram

ii) by cosine rule

$$\cos(\angle ABC) = \frac{15^2 + 25^2 - 35^2}{2 \times 15 \times 25}$$

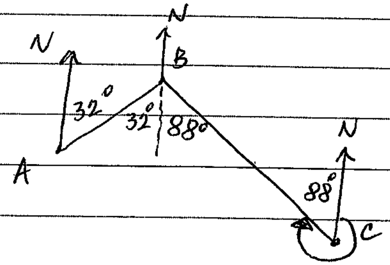
$$\cos(\angle ABC) = -\frac{1}{2}$$

$$\therefore \angle ABC = 120°$$

2- correct answer

1- uses cosine rule correctly

iii)



bearing of B from C

$360° - 88° = 272°T$

or N 88° W

2 - correct answer

1 - recognises one correct pair of angles on parrallel lines

Question 15

Marks

a) Solve $\left(\frac{15}{x}+x\right)^2-11\left(\frac{15}{x}+x\right)+24=0$

let $\frac{15}{x}+x=m$

$\therefore m^2-11m+24=0$

$(m-8)(m-3)=0$

$m=8$ or $m=3$ ①

$\therefore \frac{15}{x}+x=8$, $\frac{15}{x}+x=3$

$15+x^2=8x$, $15+x^2=3x$

$\therefore x^2-8x+15=0$, $x^2-3x+15=0$

$(x-5)(x-3)=0$, $x=\frac{3\pm\sqrt{-51}}{2}$

$x=5, x=3$ ① , $\therefore$ no solution ①

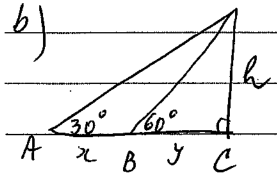
$\therefore$ solutions. $x=5, x=3$

3- correct answers

2- uses substitution to lead to quadr. equation and solves it correctly then significant progress towards solution

1- uses substitution correctly

b)



let $\boxed{AB=x}$

$BC=y$

(1) $\tan 30°=\frac{h}{x+y}$ (2) $\tan 60°=\frac{h}{y}$

$\therefore \frac{1}{\sqrt{3}}=\frac{h}{x+y}$, $\sqrt{3}=\frac{h}{y}$

$\therefore y=\frac{h}{\sqrt{3}}$

subst. $y=\frac{h}{\sqrt{3}}$ into (1)

$\frac{1}{\sqrt{3}}=\frac{h}{x+\frac{h}{\sqrt{3}}}$

2- correct expression

1- uses correctly exact values of tan30° & tan60° to find AB & BC

15b) cont.

$x+\frac{h}{\sqrt{3}}=\sqrt{3}h$

where $x=AB$

$\therefore \boxed{x=\sqrt{3}h-\frac{h}{\sqrt{3}}}$

OR $x=h\left(\sqrt{3}-\frac{1}{\sqrt{3}}\right)$ OR $h\left(\frac{3-1}{\sqrt{3}}\right)$

OR $\boxed{x=\frac{2}{\sqrt{3}}h}$ ✓

Marks

c) i) Prove $\tan A\sin A+\cos A=\sec A$

Proof: LHS $=\tan A\sin A+\cos A$

$=\frac{\sin A}{\cos A}.\sin A+\cos A$

$=\frac{\sin^2 A}{\cos A}+\cos A$ ①

$=\frac{\sin^2 A+\cos^2 A}{\cos A}$

$=\frac{1}{\cos A}=\sec A=\text{RHS}$

$\therefore$ proven

2- correct proof

1- significant progress toward solution

1- uses correctly both identities tan A & $\sin^2 A+\cos^2$

	Marks
c) ii) Now Solve	
$\tan A \sin A + \cos A = \operatorname{cosec} A$	2-correct solns.
for $0 \leq A \leq 360°$	
from part (i) $\tan A \sin A + \cos A = \sec A$	1 - uses part(i) correctly
∴ Solve $\sec A = \operatorname{cosec} A$	1 - progress towards answers
$\frac{1}{\cos A} = \frac{1}{\sin A}$	
$\sin A = \cos A$	1 - gets only one of the solns.
$\therefore \tan A = 1$	
$\therefore A = 45°, 225°$	
d) $f(x) = 2-x^2, \; g(x) = 2x-1$	
	1-correct answer
i) Find $f(g(5)) = f(2(5)-1)$	
$= f(9) = 2 - 9^2 = -79$.	
ii) $f(g(x)) = f(2x-1)$	
$= 2-(2x-1)^2$	1-correct solns.
$= 2-(4x^2-4x+1)$	
$= 2 - 4x^2 + 4x - 1$	
$= -4x^2 + 4x + 1$	

	Marks
15d) cont.	
iii) Find $g(f(x)) = g(2-x^2)$	3-correct answer
$= 2(2-x^2) - 1$	
$= 4 - 2x^2 - 1$	2- finds $g(f(x))$ and equates
$\therefore g(f(x)) = 3 - 2x^2$ ①	$f(g(x))$ & $g(f(x))$ correctly
Now solve $f(g(x)) = g(f(x))$	1- finds $g(f(x))$
$\therefore -4x^2+4x+1 = 3-2x^2$ ①	
$\therefore 0 = 2x^2 - 4x + 2$	
$0 = x^2 - 2x + 1$	
$0 = (x-1)^2$	
$\therefore x=1$ ①	