Name: _____

Teacher: _____

Class: _____

FORT STREET HIGH SCHOOL

2009

PRELIMINARY SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 1

Mathematics

TIME ALLOWED: 1 HOUR

Outcomes Assessed	Questions	Marks
Performs routine arithmetic manipulation involving surds and simple rational expressions	1	
Chooses and applies appropriate algebraic techniques to simplify algebraic expressions	2	
Solves problems involving equations	3	
Chooses and applies appropriate geometric techniques and provides appropriate reasoning to support conclusions in proofs	4	

Question	1	2	3	4	Total	%
Marks	/16	/16	/16	/16	/64	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each new question is to be started on a new booklet

Question One: (16 marks) **Start a new booklet**

a) Find $|-2.5|-|8|$ **[1]**

b) Simplify $1\frac{3}{5}\div 2\frac{2}{3}$ **[1]**

c) Find $\frac{1}{2.82-1.97}$ (correct to 3 significant figures) **[2]**

d) Calculate $6.31^3\times\sqrt{4.301}\div 9.38$ (correct to 2 decimal places) **[2]**

e) Simplify $\sqrt{24}+3\sqrt{48}-2\sqrt{54}+4\sqrt{75}$ **[2]**

f) Calculate $\sqrt{\frac{305^2+4071}{6\div 8.6^3}}$ (give you answer in scientific notation to 4 significant figures) **[2]**

g) Expand $\left(2\sqrt{2}-1\right)\left(\sqrt{2}+3\right)$ **[2]**

h) Evaluate $\frac{a^3b^4}{c^2}$ if $a=\left(\frac{2}{3}\right)^2 \quad b=\left(\frac{3}{4}\right)^3 \quad c=\left(\frac{3}{8}\right)^4$, leaving your answer as an improper fraction **[2]**

i) Rationalize the denominator: $\frac{2\sqrt{3}+3}{1-\sqrt{3}}$ **[2]**

Question Two: (16 marks) **Start a new booklet**

a) Expand $(x-2)(2x+5)$ **[1]**

b) Factorize $x^2+14x-32$ **[1]**

c) Simplify $6x^2+5x-2x^2+4x-3$ **[2]**

d) Simplify $\frac{a+2}{3}-\frac{a}{4}$ **[2]**

e) Fully factorize x^4-4x^2 **[2]**

f) Simplify $(2x+1)^2-(x-2)^2$ **[2]**

g) Given $S=\frac{a(1-r^n)}{1-r}$, find S when $a=2, \quad r=\frac{1}{3}, \quad n=12$ **[2]**

h) Simplify $\frac{p^2-5p+4}{p^2-1}$ **[2]**

i) Factorise $27x^3+125$ **[2]**

Question Three: (16 marks) **Start a new booklet**

a) Solve $-2(3x+1)=8$ **[1]**

b) Draw a neat number-line to show the solution to $\frac{x+5}{3}+3>4$ **[2]**

c) Solve $x^2-3x-10=0$ **[2]**

d) Use the quadratic formula to find the roots of $x^2-6x+7=0$ in surd form. **[2]**

e) Solve $|x-3|=2x-1$ **[3]**

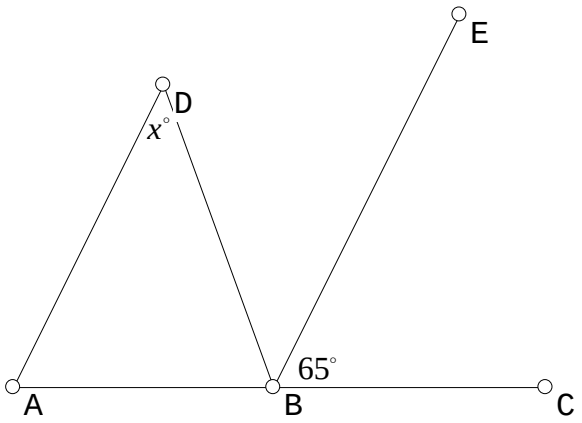
f) Solve $y=x^2-4x-1$ and $y=2x+3$ simultaneously. **[4]**

g) A regular polygon has an interior angle of 156°. Find how many sides it has. **[2]**

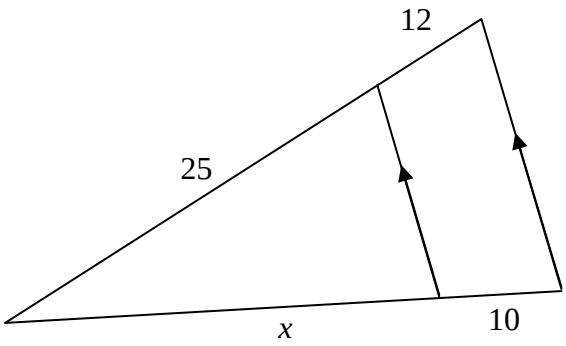
Question four: (16 marks) **Start a new booklet**

a) Find the value of x in the diagrams below

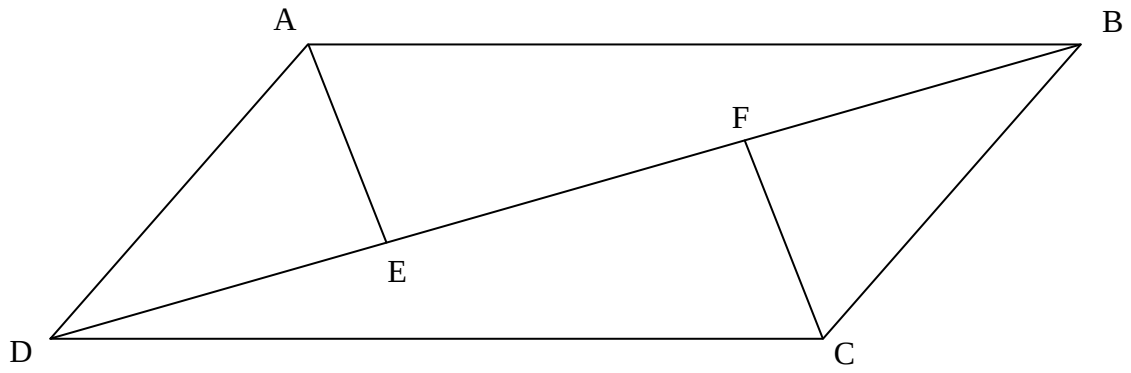
i. $AD=DB$ and $AD \parallel BE$. Give reasons. **[3]**



ii. **[2]**



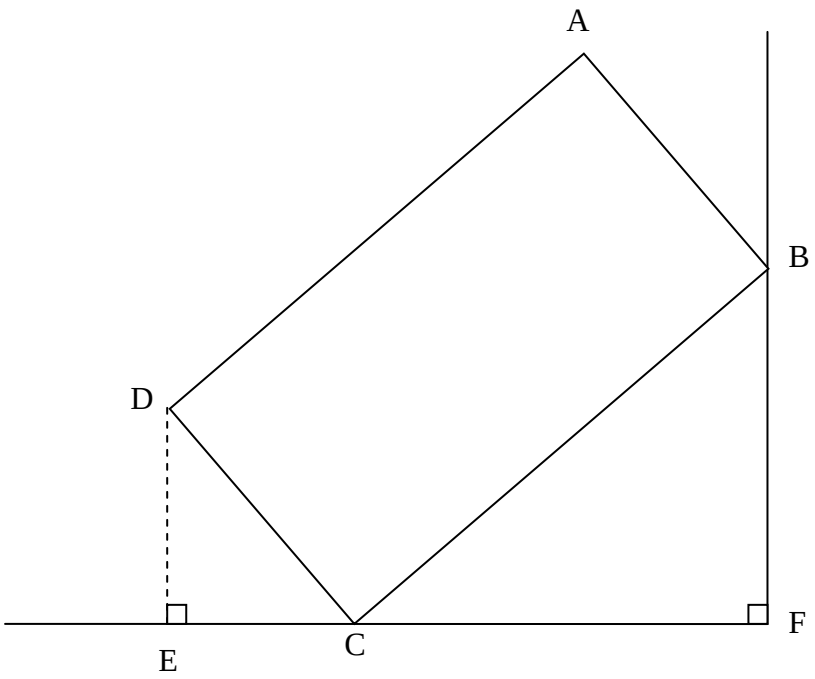
b) $ABCD$ is a parallelogram, with AE and CF drawn perpendicular to the diagonal BD.



i. Prove $\Delta AEB \equiv \Delta DFC$ **[4]**

ii. Hence prove $AE = FC$ **[2]**

c) $ABCD$ is a rectangular block 10cm by 6cm leaning against a vertical wall. Point D is 5cm above the ground.



i. Prove ΔCED and ΔFCB are similar. **[3]**

ii. Find, in simplest surd form, the height of point B above the ground. **[2]**

Question One: (16 marks) **Start a new booklet**

		Marking	Comments
a)	$\lvert -2.5\rvert - \lvert 8\rvert$ **[1]**		
	$= 2.5 - 8$		- for questions marked §, most students lost marks because they did not read carefully and so did not meet the requirements of the question!
	$= -5.5$	❶ answer	
b)	$1\frac{3}{5} \div 2\frac{2}{3}$ **[1]**		
	$= \frac{8}{5} \times \frac{3}{8}$		
	$= \frac{3}{5}$	❶ answer	
c)	$\frac{1}{2.82 - 1.97}$ **[2]**		
	$= \frac{1}{0.85}$		
	$= 1.176470588$	❶ answer	§
	$= 1.18$	❶ 3 sig figs correct	
d)	$6.31^3 \times \sqrt{4.301} \div 9.38$ **[2]**		
	$= 55.54819617$	❶ answer	§
	$= 55.55$	❶ 2 dp correct	
e)	$\sqrt{24} + 3\sqrt{48} - 2\sqrt{54} + 4\sqrt{75}$ **[2]**		- students had trouble reducing $\sqrt{48}$ and $\sqrt{75}$ to multiples of $\sqrt{3}$
	$= \sqrt{4 \times 6} + 3\sqrt{16 \times 3} - 2\sqrt{9 \times 6} + 4\sqrt{25 \times 3}$	❶ surds reduced correctly	
	$= 2\sqrt{6} + 3 \times 4\sqrt{3} - 2 \times 3\sqrt{6} + 4 \times 5\sqrt{3}$	❶ combined like surds correctly	
	$= 32\sqrt{3} - 4\sqrt{6}$		
f)	$\sqrt{\frac{305^2 + 4071}{6 \div 8.6^3}}$ **[2]**		
	$= 3208.283377$	❶ answer	§
	$= 3.208 \times 10^3$	❶ sci. not. correct	
g)	$\left(2\sqrt{2} - 1\right)\left(\sqrt{2} + 3\right)$ **[2]**		- as with numerator in i) below, many had problems expanding with surds.
	$= \sqrt{2}\left(2\sqrt{2} - 1\right) + 3\left(2\sqrt{2} - 1\right)$	❶ correct expansion	
	$= 4 - \sqrt{2} + 6\sqrt{2} - 3$		
	$= 1 + 5\sqrt{2}$	❶ collected like terms correctly	

h) **[2]**

$a=\left(\frac{2}{3}\right)^2 \quad b=\left(\frac{3}{4}\right)^3 \quad c=\left(\frac{3}{8}\right)^4$

$\therefore \frac{a^3b^4}{c^2}$

$=\frac{\left(\left(\frac{2}{3}\right)^2\right)^3\times\left(\left(\frac{3}{4}\right)^3\right)^4}{\left(\left(\frac{3}{8}\right)^4\right)^2}$

$=\frac{\left(\frac{2}{3}\right)^6\times\left(\frac{3}{4}\right)^{12}}{\left(\frac{3}{8}\right)^8}$

$=\frac{2^6}{3^6}\times\frac{3^{12}}{2^{24}}\times\frac{2^{24}}{3^8}$

$=\frac{2^6}{3^2}$

$=\frac{64}{9}$

Marking

❶ powers resolved correctly

❶ answer

Comments

- many had problems leaving or converting to powers of 2 and 3, so had problems resolving as a fraction

i) **[2]**

$\frac{2\sqrt{3}+3}{1-\sqrt{3}}$

$=\frac{2\sqrt{3}+3}{1-\sqrt{3}}\times\frac{1+\sqrt{3}}{1+\sqrt{3}}$

$=\frac{\left(2\sqrt{3}+3\right)+\sqrt{3}\left(2\sqrt{3}+3\right)}{1^2-\left(\sqrt{3}\right)^2}$

$=\frac{2\sqrt{3}+3+6+3\sqrt{3}}{1-3}$

$=-\frac{\left(9+5\sqrt{3}\right)}{2}$

Marking

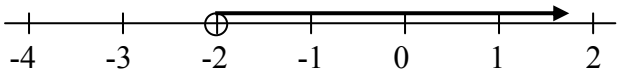
❶ correct conjugate

❶ answer

Comments

- those losing marks on this question generally did not have the conjugate correct.

	Marking	Comments
<u>Question Two:</u> (16 marks) **Start a new booklet**		
a) $(x-2)(2x+5)$ **[1]** $=x(2x+5)-2(2x+5)$ $=2x^2+5x-4x-10$ $=2x^2+x-10$	❶ answer	Note: a), b) and c) are EXPRESSIONS - too many treated them as equations and answered as $x=\ldots$
b) $x^2+14x-32$ **[1]** $=(x+16)(x-2)$	❶ answer	
c) $6x^2+5x-2x^2+4x-3$ **[2]** $=4x^2+9x-3$	❶ x^2 term correct ❶ x term correct	Note: this is not a factorising question!
d) Simplify $\frac{a+2}{3}-\frac{a}{4}$ **[2]** $=\frac{4(a+2)-3a}{12}$ $=\frac{4a+8-3a}{12}$ $=\frac{a+8}{12}$	❶ correct cross multiplication ❶ answer	
e) x^4-4x^2 **[2]** $=x^2(x^2-4)$ $=x^2(x+2)(x-2)$	❶ x^2 factored ❶ difference of squares correct	- many missed the x^2 factor.
f) $(2x+1)^2-(x-2)^2$ **[2]** $=4x^2+4x+1-(x^2-4x+4)$ $=3x^2+8x-3$	❶ expansions ❶ answer	- many made errors subtracting the expansion; brackets often missed
g) $a=2,\quad r=\frac{1}{3},\quad n=12$ **[2]** $S=\frac{a(1-r^n)}{1-r}$ $=\frac{2\left(1-\left(\frac{1}{3}\right)^{12}\right)}{1-\frac{1}{3}}$ $=2\left(1-\frac{1}{3^{12}}\right)\times\frac{3}{2}$ $=3\left(\frac{3^{12}-1}{3^{12}}\right)$ $=2.999994355$	❶ substitutions & powers correct ❶ answer	Note: the question does not ask for a number of decimal places; in these cases, quote all from your calculator; an answer of 3 is incorrect (no mark deducted if the answer show was subsequently rounded to 3)

		Marking	Comments
h) $\frac{p^2-5p+4}{p^2-1}$	[2]		
$=\frac{(p-4)(p-1)}{(p+1)(p-1)}$		❶ factors correct	
$=\frac{(p-4)}{(p+1)}$		❶ answer	
i) $27x^3+125$	[2]	❶ cube root bracket	- this pattern poorly known
$=(3x+5)(9x^2-15x+25)$		❶ squares bracket	
Question Three: (16 marks) **Start a new booklet**			
a) $-2(3x+1)=8$	[1]		- simplest form! $\frac{-10}{6}$ accepted (this time)…
$-6x-2=8$			
$-10=6x$			
$x=\frac{-5}{3}$		❶ answer with working	
b) $\frac{x+5}{3}+3>4$	[2]		Note: a) and c) are EQUATIONS – they DO need an answer of $x=...$
$\frac{x+5}{3}>1$			
$x+5>3$			
$x>-2$		❶ resolved inequality	
		❶ number line correct	
c) $x^2-3x-10=0$	[2]		- too many stopped at the factored form, and did not give the x values.
$(x-5)(x+2)=0$		❶ factors correct	
$x=5,-2$		❶ answers	

	Marking	Comments
d) **[2]**		
$x^2 - 6x + 7 = 0$		
$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$		
$= \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times 7}}{2 \times 1}$	❶ correct substitution in quadratic formula	- resolving $\sqrt{8}$ to $2\sqrt{2}$ an issue for some, leading to errors giving the simplest form of the surd
$= \dfrac{6 \pm \sqrt{36 - 28}}{2}$		
$= \dfrac{6 \pm \sqrt{8}}{2}$		
$= \dfrac{6 \pm 2\sqrt{2}}{2}$		
$= 3 \pm \sqrt{2}$	❶ answers	
e) $\lvert x - 3 \rvert = 2x - 1$ **[3]**		- several students couldn't get two answers for the two cases
$x - 3 = 2x - 1$ or $-(x - 3) = 2x - 1$		
$-2 = x$ $\qquad$ $4 = 3x$	❶ both cases shown	
$x = -2$ $\qquad$ $x = \dfrac{4}{3}$	❶ possible solutions correct	
Check:		- most common error was not checking the solutions. You MUST do this step every time there is an absolute value = a non-absolute value.
$LHS = \lvert -2 - 3 \rvert$ $\qquad$ $LHS = \left\lvert \frac{4}{3} - 3 \right\rvert$		
$= \lvert -5 \rvert$ $\qquad$ $= \left\lvert -\frac{5}{3} \right\rvert$		
$= 5$ $\qquad$ $= \dfrac{5}{3}$		
$RHS = 2 \times -2 - 1$		
$= -5$ $\qquad$ $RHS = 2 \times \dfrac{4}{3} - 1$		
$\neq LHS$ $\qquad$ $= \dfrac{5}{3}$	❶ checking correct	
$= LHS$		
$\therefore x = \dfrac{4}{3}$ is the only solution.		

f) $y = x^2 - 4x - 1$ and $y = 2x + 3$. **[4]**

$2x + 3 = x^2 - 4x - 1$

$0 = x^2 - 6x - 4$

$x = \dfrac{6 \pm \sqrt{(-6)^2 - 4 \times 1 \times -4}}{2 \times 1}$

$= \dfrac{6 \pm \sqrt{52}}{2}$

$= \dfrac{6 \pm 2\sqrt{13}}{2}$

$= 3 \pm \sqrt{13}$

$x = 3 + \sqrt{13}$ $\quad x = 3 - \sqrt{13}$

$y = 2\left(3 + \sqrt{13}\right) + 3$ $\quad y = 2\left(3 - \sqrt{13}\right) + 3$

$= 6 + 2\sqrt{13} + 3$ $\quad = 6 - 2\sqrt{13} + 3$

$= 9 + 2\sqrt{13}$ $\quad = 9 - 2\sqrt{13}$

$\therefore$ Points of intersection are

$\left(2 + \sqrt{13}, 9 + 2\sqrt{13}\right)$ $\quad \left(2 - \sqrt{13}, 9 - 2\sqrt{13}\right)$

g) Regular polygon with interior angle of 156°: **[2]**

$\theta = \dfrac{180(n-2)}{n}$

$156 = \dfrac{180(n-2)}{n}$

$156n = 180n - 360$

$360 = 24n$

$n = 15$

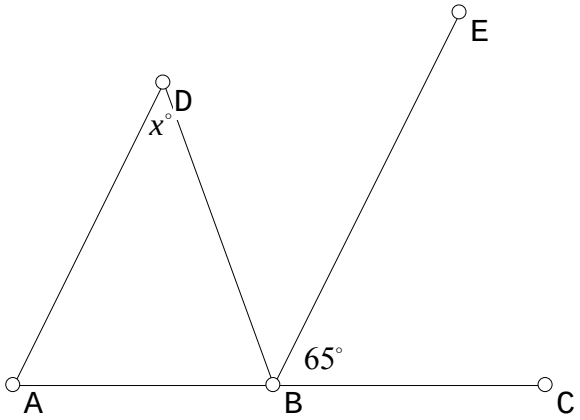
$\therefore$ the polygon has 15 sides.

Marking	Comments
❶ quadratic correctly formed	- many did not put as quadratic =0, so were unable to proceed
❶ quadratic formula substitution correct	
❶ x values correct	- solving simultaneously means finding the points of intersection – many did not find the y-values.
❶ y values correct	
❶ correct substitution in polygon formula	- some students did not use a valid formula and were
❶ answer	

Question Four: (16 marks) **Start a new booklet**

a) Find the value of x in the diagrams below

i. $AD = DB$ and $AD \parallel BE$. Give reasons. **[3]**



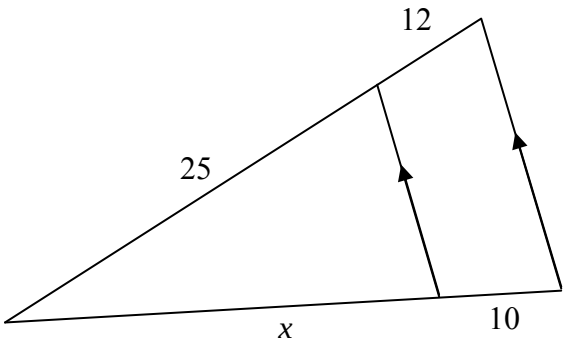
$\angle DAB = 65^\circ \quad (cor\angle' s AD \parallel BE)$

$\angle ABD = 65^\circ \quad (base\angle' s Isos\triangle)$

$x + 2 \times 65 = 180 \quad (\triangle Sum)$

$x = 50$

ii. **[2]**



From similar triangles:

$\frac{x}{25} = \frac{10}{12}$ OR $\frac{x}{25} = \frac{x+10}{25+12}$

$12x = 250$ $\quad 37x = 25x + 250$

$x = \frac{125}{6}$ $\quad 12x = 250$

$x = 20\frac{5}{6}$ $\quad x = \frac{125}{6}$

$x = 20\frac{5}{6}$

Marking

❶ value with reason

❶ value with reason

❶ answer with reason

❶ correct ratio setup

❶ answer

Comments

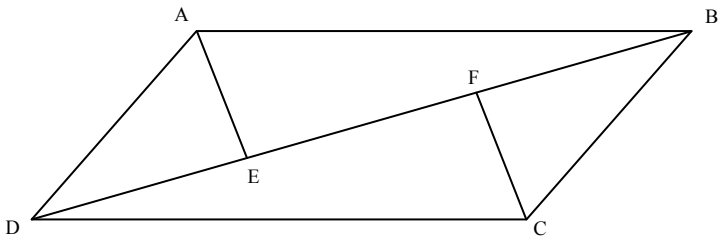
For both parts of a)

- generally not well set out; it was often difficult to follow reasoning.

- references to parallel line properties MUST mention the parallel lines in question (no marks deducted – this time!)

- mostly well done; method depended on how students saw the ratios – parallel lines OR similar triangles.

b) $ABCD$ is a parallelogram, with AE and CF drawn perpendicular to the diagonal BD.



i. Prove $\Delta AEB \equiv \Delta DFC$ **[4]**

In Δ's AEB, DFC

$i) \angle AEB = \angle CFD (= 90^\circ)$ $\left(given\ AE \perp BD,\ CF \perp BD\right)$

$ii) \angle EBA = \angle FDC$ $\left(given\ BC \parallel AD,\ ABCD\ \parallel gram\right)$

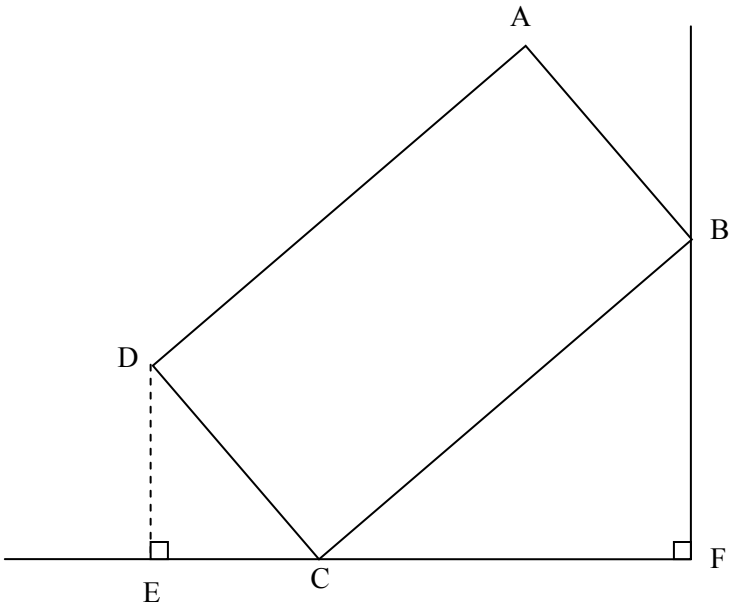
$iii) AD = BC$ $\left(op.\ sides\ of\ \parallel gram\ ABCD\right)$

$\therefore \Delta AEB \equiv \Delta DFC$ (AAS)

ii. Hence prove $AE = FC$ **[2]**

$\therefore AE = FC$ $(cor.\ \angle's\ cong.\ \triangle's\ above)$

c) $ABCD$ is a rectangular block 10cm by 6cm leaning against a vertical wall. Point D is 5cm above the ground.



Marking

❶ set up proof

❷ three congruency components; subtract ❶ per error

❶ conclusion with correct reason

❶ corresponding as reason

❶ reference to congruent triangles/proof above

Comments

NOTE: GEOMETRY PROOFS GENERALLY!

These are very formal in structure – while content of a proof may vary, there is a right way in terms of structure (and no other)!

- many omitted the set-up step.

- several students claimed "BD common" and a RHS proof – clearly not using the correct triangles!

i. Prove ΔCED and ΔFCB are similar. **[3]**

$\angle ECD + \angle DCB + \angle BCF = 180^\circ \quad (st.\ line)$

$but\ \angle DCB = 90^\circ \quad (ABCD\ a\ \text{rectangle})$

$hence\ \angle ECD + \angle BCF = 90^\circ$

$so\ \angle ECD = 90^\circ - \angle BCF$

$also\ \angle BCF + \angle CBF + \angle BFC = 180^\circ \quad (\triangle sum)$

$and\ \angle BFC = 90^\circ \quad (given\ BF \perp EB)$

$so\ \angle BCF + \angle CBF = 90^\circ$

$hence\ \angle CBF = 90^\circ - \angle BCF$

$but\ \angle ECD = 90^\circ - \angle BCF \quad (shown\ above)$

$\therefore \angle ECD = \angle CBF$

$\therefore In\ \triangle's\ DCE, CBF$

$i) \angle ECD = \angle CBF \quad (shown\ above)$

$ii) \angle CED = \angle BFC = 90^\circ \quad (DE \perp EF, BF \perp EF)$

$hence\ \triangle DCE \,|||\, \triangle CBF \quad (equal\ \angle's)$

Marking

❷ argument connecting angles between the two triangles (any method); subtract ❶ per error

❶ similarity proof correct

Comments

- connecting arguments and reasoning very poorly communicated, if at all.

- many erroneously tried to say $\angle DCE$ and $\angle BCF$ were equal!

ii. Height of point B. **[2]**

$$hence$$

$$\frac{BF}{EC} = \frac{BC}{CD}$$

$$\frac{BF}{\sqrt{6^2 - 5^2}} = \frac{10}{6}$$

$$BF = \frac{10\sqrt{11}}{6}$$

$$= \frac{5\sqrt{11}}{3}$$

❶ correct ratio setup

❶ answer

- even if you can't do part (i), assume it is shown and complete part (ii) anyway – many did not do this (this is standard for any HSC-style question).

NOTE: timing – make sure you give some time to every question on the paper – some students clearly "ran out of time")