

Name:

Teacher:



Fort Street High School

2011 Assessment task 1

Mathematics

Time allowed: 1 HOUR

PLUS 5 minutes reading time

Outcomes Assessed	Questions	Marks
Demonstrates the ability to manipulate and simplify numeric and algebraic expressions and solves problems involving equations	1,2	
Solves problems involving absolute values, inequalities, indices and logs	3,4	

Question	1	2	3	4	Total	%
Marks	/14	/16	/10	/16	/56	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question.
Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- Each Question is to be started in a new booklet.

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- a) A customer is given a 7.5% discount on the price of a new car. If they paid \$32 000, what was the price of the car before the discount? Give your answer correct to the nearest dollar. **2**
- b) Simplify $2\sqrt{8} - \sqrt{18}$ **2**
- c) Simplify $\frac{(\sqrt{2} a^3 b^2)^2 \times 16a^7 b}{(2a^2 b)^5}$ **2**
- d) Find the value of $2\pi\sqrt{\frac{l}{g}}$ correct to two decimal places if $l = 32.8$ and $g = 9.8$. **2**
- e) Express $0.\dot{6}2$ as a fraction in its simplest form. **3**
- f) Find rational numbers a and b such that $\frac{1}{3-\sqrt{5}} = a + b\sqrt{5}$ **3**

- a) i) Factorise $m^2 - 7m + 10$ 1
- ii) Factorise $125m^3 - 8$ 1
- iii) Hence, simplify $\frac{125m^3 - 8}{m^2 - 7m + 10} \times \frac{m - 5}{25m^2 - 4}$ 2
- b) Use the *completing the square method* to find the exact value for x in $x^2 - 8x + 1 = 0$ 2
- c) i) Factorise fully $6x^2 + 9x - 6$ 2
- ii) Hence or otherwise solve $4 - 8x = (3x + 2)(2x - 1)$ 2
- d) Factorise fully $x^6 - 7x^3 - 8$ 3
- e) Solve for u , v and w .
- | | | | | | | | |
|------|---|-----|---|------|---|------|---|
| $3u$ | + | v | - | $4w$ | = | 23 | |
| u | - | v | + | $2w$ | = | -9 | |
| $4u$ | - | v | + | $3w$ | = | -6 | 3 |

-
- a) Evaluate $|-5|-|2|$ 1
- b) Solve $|2-3x|=5$ 2
- c) Solve $5x^2+2x-7=0$ 3
- d) Solve $|5x-9|>21$ 2
- e) i) Express $\frac{1}{32}$ as a number with a negative index 1
- ii) Hence solve $2^{4-3x}=\frac{1}{32}$ 1

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- a) Evaluate $\log_5 4$ correct to 3 significant figures. 2
- b) Simplify $\frac{8^{x-1} \times 2^4}{4^{2x}} \div \frac{64^{\frac{1}{3}}}{2^{3x}}$ 3
- c) Simplify $\log_2 96 - \log_2 6$ 2
- d) Given $\log_7 2 = 0.36$ and $\log_7 5 = 0.83$, find $\log_7 350$ 2
- e) How many positive integer powers of 6 are less than 2.5×10^9 ? 2
- f) Solve $2^{5x+1} = 3.6$, correct to 2 decimal places. 2
- g) If $\log_{10} \frac{1}{x} + \log_{10} \frac{1}{x^2} + \log_{10} \frac{1}{x^3} = -12$, find the value of x . 3



Fort Street High School

2011 Assessment task 1

Mathematics

Solutions

Question 1. (14 marks)

a) A customer is given a 7.5% discount on the price of a new car. If they paid \$32 000, what was the price of the car before the discount?

Give your answer correct to the nearest dollar.

Solution

$$\begin{aligned}92.5\% &= 32000 \\1\% &= \frac{32000}{92.5} \\&= 345.94559... \\100\% &= 34594.59...\end{aligned}$$

∴ The price of the car before the discount is \$34595 correct to nearest dollar.

Marking Criteria

2 For correct response or

1 For answer with incorrect rounding or an arithmetic error.

b) Simplify $2\sqrt{8} - \sqrt{18}$

Solution

$$\begin{aligned}2\sqrt{8} - \sqrt{18} &= 2\sqrt{4 \times 2} - \sqrt{9 \times 2} \\&= 4\sqrt{2} - 3\sqrt{2} \\&= \sqrt{2}\end{aligned}$$

Marking Criteria

2 For correct response or

1 For answer with one error made in the solution.

c) Simplify $\frac{(\sqrt{2} a^3 b^2)^2 \times 16a^7 b}{(2a^2 b)^5}$

Solution

$$\begin{aligned}\frac{(\sqrt{2} a^3 b^2)^2 \times 16a^7 b}{(2a^2 b)^5} &= \frac{2a^6 b^4 \times 16a^7 b}{32a^{10} b^5} \\&= \frac{32a^{13} b^5}{32a^{10} b^5} \\&= a^3\end{aligned}$$

Marking Criteria

2 For correct response or

1 For answer with one error made in the solution.

d) Find the value of $2\pi\sqrt{\frac{l}{g}}$ correct to two decimal places if $l = 32.8$ and $g = 9.8$.

Solution

$$\begin{aligned}2\pi\sqrt{\frac{l}{g}} &= 2\pi\sqrt{\frac{32.8}{9.8}} \\&= 11.49\end{aligned}$$

Marking Criteria

2 For correct response or

1 For answer not corrected to 2 d.p.

e) Express $0.\dot{6}\dot{2}$ as a fraction in its simplest form.

Solution

$$x = 0.622222\dots$$

$$10x = 6.22222\dots$$

$$100x = 62.2222\dots$$

$$90x = 56$$

$$x = \frac{56}{90}$$

$$= \frac{28}{45}$$

Marking Criteria

3 For correct response or

2 For not simplifying answer or

1 For some evidence of correct procedure.

f) Find rational numbers a and b such that

$$\frac{1}{3-\sqrt{5}} = a + b\sqrt{5}$$

Solution

$$\frac{1}{3-\sqrt{5}} = \frac{1}{3-\sqrt{5}} \times \frac{3+\sqrt{5}}{3+\sqrt{5}}$$

$$= \frac{3+\sqrt{5}}{4}$$

$$= \frac{3}{4} + \frac{\sqrt{5}}{4}$$

$$\therefore a = \frac{3}{4} \text{ and } b = \frac{1}{4}$$

Marking Criteria

3 For correct response or

2 For writing $\frac{3}{4} + \frac{\sqrt{5}}{4}$ but not stating a and b

1 For rationalising to $\frac{3+\sqrt{5}}{4}$

Question 2. (16 marks)

a) i) Factorise $m^2 - 7m + 10$

Solution

$$m^2 - 7m + 10 = (m-5)(m-2)$$

Marking Criteria

1 For correct response

ii) Factorise $125m^3 - 8$

Solution

$$125m^3 - 8 = (5m-2)(25m^2 + 10m + 4)$$

Marking Criteria

1 For correct response

iii) Hence, simplify $\frac{125m^3 - 8}{m^2 - 7m + 10} \times \frac{m-5}{25m^2 - 4}$

Solution

$$= \frac{125m^3 - 8}{m^2 - 7m + 10} \times \frac{m-5}{25m^2 - 4}$$

$$= \frac{(5m-2)(25m^2 + 10m + 4)}{(m-5)(m-2)} \times \frac{m-5}{(5m-2)(5m+2)}$$

$$= \frac{25m^2 + 10m + 4}{(m-2)(5m+2)}$$

Marking Criteria

2 For correct response or

1 Not cancelling all factors correctly

b) Use the *completing the square method* to find the exact value for x in $x^2 - 8x + 1 = 0$

Solution

$$x^2 - 8x + 1 = 0$$

$$x^2 - 8x = -1$$

$$x^2 - 8x + 16 = 15$$

$$(x-4)^2 = 15$$

$$x-4 = \pm\sqrt{15}$$

$$x = 4 \pm \sqrt{15}$$

Marking Criteria

2 For correct response or

1 One arithmetic error

c) i) Factorise fully $6x^2 + 9x - 6$

Solution

$$6x^2 + 9x - 6 = 3(2x^2 + 3x - 2)$$

$$= 3(2x^2 + 4x - x - 2)$$

$$= 3(2x(x+2) - 1(x+2))$$

$$= 3(2x-1)(x+2)$$

Marking Criteria

2 For correct response

1 One for factored for without the HCF

ii) Hence or otherwise solve $4 - 8x = (3x + 2)(2x - 1)$

Solution

$$4 - 8x = (3x + 2)(2x - 1)$$

$$4 - 8x = 6x^2 + x - 2$$

$$6x^2 + 9x - 6 = 0$$

$$3(2x-1)(x+2) = 0 \quad \text{from part i}$$

$$2x-1=0 \quad \text{or} \quad x+2=0$$

$$x = \frac{1}{2} \quad x = -2$$

Marking Criteria

2 For correct response or

1 One mark for attaining the line

$$6x^2 + 9x - 6 = 0$$

d) Factorise fully $x^6 - 7x^3 - 8$

Solution

$$x^6 - 7x^3 - 8 = (x^3)^2 - 7(x^3) - 8$$

Let $x^3 = m$

$$= m^2 - 7m - 8$$

$$= (m-8)(m+1)$$

but $x^3 = m$

$$= (x^3 - 8)(x^3 + 1)$$

$$= (x-2)(x^2 + 2x + 4)(x+1)(x^2 - x + 1)$$

Marking Criteria

3 For correct response

2 For finding $(x^3 - 8)(x^3 + 1)$

1 For recognising it could be reduced to a quadratic.

e) Solve for u , v and w .

$$3u + v - 4w = 23 \quad \text{---1.}$$

$$u - v + 2w = -9 \quad \text{---2.}$$

$$4u - v + 3w = -6 \quad \text{---3.}$$

Solution

Eqn. 1. + Eqn 2.

$$4u - 2w = 14 \quad \text{---4.}$$

Eqn. 1. + Eqn 3.

$$7u - w = 17 \quad \text{---5.}$$

Eqn. 5. \times 2

$$14u - 2w = 34 \quad \text{---5A.}$$

Eqn. 5A. - Eqn 4.

$$10u = 20$$

$$u = 2$$

Sub. $u = 2$ into Eqn. 4.

$$4(2) - 2w = 14$$

$$w = -3$$

Sub. $u = 2$ and $w = -3$ into eqn. 2

$$(2) - v + 2(-3) = -9$$

$$v = 5$$

Marking Criteria

3 For correct response

2 For one arithmetic error

1 For evidence of correct procedure with errors.

Question 3. (10 marks)

a) Evaluate $|-5| - |2|$

Solution

$$\begin{aligned} |-5| - |2| &= 5 - 2 \\ &= 3 \end{aligned}$$

Marking Criteria

1 For correct response

b) Solve $|2 - 3x| = 5$

Solution

$$\begin{aligned} 2 - 3x &= +5 & 2 - 3x &= -5 \\ -3x &= 3 & \text{or} & -3x = -7 \\ x &= -1 & & x = \frac{7}{3} \end{aligned}$$

Marking Criteria

2 For correct response.

1 One correct solution.

c) Solve $5x^2 + 2x - 7 = 0$

Solution

Using the quadratic formula

$$x = \frac{-2 \pm \sqrt{4 - 4.5.(-7)}}{2.5}$$

$$= \frac{-2 \pm \sqrt{144}}{10}$$

$$= 1 \quad \text{or} \quad \frac{-7}{5}$$

Marking Criteria

3 For correct response

2 For one correct solution

1 For evidence of correct procedure with errors.

d) Solve $|5x - 9| > 21$

Solution

$$5x - 9 > 21 \quad 5x - 9 < -21$$

$$5x > 30 \quad \text{or} \quad 5x < -12$$

$$x > 6 \quad x < \frac{-12}{5}$$

Marking Criteria

2 For correct response.

1 One correct solution.

e) i) Express $\frac{1}{32}$ as a number with a negative index.

Solution

$$\frac{1}{32} = \frac{1}{2^5}$$

$$= 2^{-5}$$

Marking Criteria

1 For correct response

ii) Hence solve $2^{4-3x} = \frac{1}{32}$

$$2^{4-3x} = \frac{1}{32}$$

$$2^{4-3x} = 2^{-5}$$

$$4 - 3x = -5$$

$$x = 3$$

Marking Criteria

1 For correct response

Question 4. (16 marks)

a) Evaluate $\log_5 4$ correct to 3 significant figures.

Solution

$$\log_5 4 = \frac{\log 4}{\log 5}$$

$$= 0.861 \quad \text{correct to 3 sig. figs.}$$

Marking Criteria

2 For correct response.

1 For finding the change of base.

b) Simplify $\frac{8^{x-1} \times 2^4}{4^{2x}} \div \frac{64^{\frac{1}{3}}}{2^{3x}}$

Solution

$$\frac{8^{x-1} \times 2^4}{4^{2x}} \div \frac{64^{\frac{1}{3}}}{2^{3x}} = \frac{(2^3)^{x-1} \times 2^4}{(2^2)^{2x}} \div \frac{\sqrt[3]{64}}{2^{3x}}$$

$$= \frac{2^{3x-3} \times 2^4}{2^{4x}} \times \frac{2^{3x}}{4}$$

$$= \frac{2^{3x+1}}{2^{4x}} \times \frac{2^{3x}}{2^2}$$

$$= \frac{2^{6x+1}}{2^{4x+2}}$$

$$= 2^{2x-1}$$

Marking Criteria

3 For correct response

2 For one arithmetic error

1 For evidence of correct procedure with errors.

c) Simplify $\log_2 96 - \log_2 6$

Solution

$$\begin{aligned}\log_2 96 - \log_2 6 &= \log_2 \frac{96}{6} \\ &= \log_2 16 \\ &= \log_2 2^4 \\ &= 4\log_2 2 \\ &= 4\end{aligned}$$

Marking Criteria

2 For correct response.

1 For finding $\log_2 16$.

d) Given $\log_7 2 = 0.36$ and $\log_7 5 = 0.83$,
find $\log_7 350$

Solution

$$\begin{aligned}\log_7 350 &= \log_7 (7 \times 5 \times 5 \times 2) \\ &= \log_7 7 + \log_7 5^2 + \log_7 2 \\ &= 1 + 2\log_7 5 + \log_7 2 \\ &= 1 + 2(0.83) + 0.36 \\ &= 3.02\end{aligned}$$

Marking Criteria

2 For correct response.

1 For partial solution.

e) How many positive integer powers of 6 are less than 2.5×10^9 ?

Solution

$$\begin{aligned}6^n &< 2.5 \times 10^9 \\ \log 6^n &< \log(2.5 \times 10^9) \\ n \log 6 &< \log(2.5 \times 10^9) \\ n &< \frac{\log(2.5 \times 10^9)}{\log 6}\end{aligned}$$

$$n < 12.07... \quad \text{from calculator}$$

$$\therefore n = 12$$

Marking Criteria

2 For correct response.

1 For finding $n < 12.07...$

f) Solve $2^{5x+1} = 3.6$, correct to 2 d.p.

Solution

$$\begin{aligned}2^{5x+1} &= 3.6 \\ \log(2^{5x+1}) &= \log 3.6 \\ (5x+1)\log 2 &= \log 3.6\end{aligned}$$

$$5x+1 = \frac{\log 3.6}{\log 2}$$

$$x = \frac{\frac{\log 3.6}{\log 2} - 1}{5}$$

$$= 0.1695...$$

$$= 0.17$$

g) If $\log_{10} \frac{1}{x} + \log_{10} \frac{1}{x^2} + \log_{10} \frac{1}{x^3} = -12$, find the value of x .

Solution

$$\log_{10} \frac{1}{x} + \log_{10} \frac{1}{x^2} + \log_{10} \frac{1}{x^3} = -12$$

$$\log_{10} \left(\frac{1}{x} \times \frac{1}{x^2} \times \frac{1}{x^3} \right) = -12$$

$$\log_{10} \frac{1}{x^6} = -12$$

$$\log_{10} x^{-6} = -12$$

$$10^{-12} = x^{-6}$$

$$x^{-6} = 10^{2 \times -6}$$

$$x^{-6} = (10^2)^{-6}$$

$$\therefore x = 10^2 = 100$$

Marking Criteria

3 For correct response

2 For finding $10^{-12} = x^{-6}$

1 For simplifying $\log_{10} \frac{1}{x^6} = -12$