Name: _____

Teacher: _____

FORT STREET HIGH SCHOOL

2012

PRELIMINARY HIGH SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 1

Mathematics

TIME ALLOWED: 1 HOUR
PLUS 5 MINUTES READING TIME

Outcomes Assessed	Questions	Marks
Demonstrates the ability to manipulate and simplify numeric and algebraic expressions	1	
Solves problems involving equations and inequalities	2	
Uses appropriate techniques to solve problems involving plane geometry	3	
Solves problems involving indices and logs	4	

Question	1	2	3	4	Total	%
Marks	/14	/14	/13	/14	/55	

Directions to candidates:

- Attempt all questions
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used
- **Each Question is to be started in a new booklet.**

Question 1 (Basic arithmetic and algebra) 14 marks

a) Mr Fraser bought an antique chair valued at $980. Each year its value appreciated by 12%. Calculate the value of the chair after 5 years. Answer correct to the nearest 5 cents. 2

b) Calculate correct to four significant figures: 2

$$\frac{\left(\frac{2}{5}\right)^4 x \left(\frac{3}{4}\right)^5}{\left(\frac{6}{7}\right)^2 + \left(\frac{2}{3}\right)^3}$$

c) Expand and simplify: 2

$$\frac{12x}{x+1} \div \frac{6x}{x^2+2x+1}$$

d) Factorise fully: 2

$$y^3 - 64$$

e) Express the following recurring decimal as a simplest fraction. 3

$$0.\dot{2}9\dot{7}$$

f) Express as a single fraction with rational denominator. Express your answer in simplest form. 3

$$\frac{2}{6-3\sqrt{3}} - \frac{1}{3+2\sqrt{3}}$$

Question 2 (Equations and Inequalities) 14 marks

a) Solve for a: 2

$$\frac{a}{4}-\frac{a+2}{3}=9$$

b) Solve for x: 2

$$\frac{x-3}{7}-\frac{3}{4}\geq 9$$

c) Solve by completing the square. Answer in exact form. 2

$$x^2-10x+20=0$$

d) Solve for x: 2

$$x^2-11x+18>0$$

e) Solve for x: 3

$$|x-2|=7-2x$$

f) Solve simultaneously: 3

$$a+b=5$$

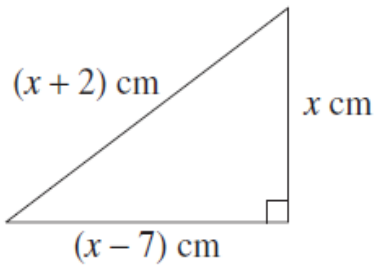
$$2a+b+c=4$$

$$a-b-c=5$$

Question 3 (Geometry) 13 marks

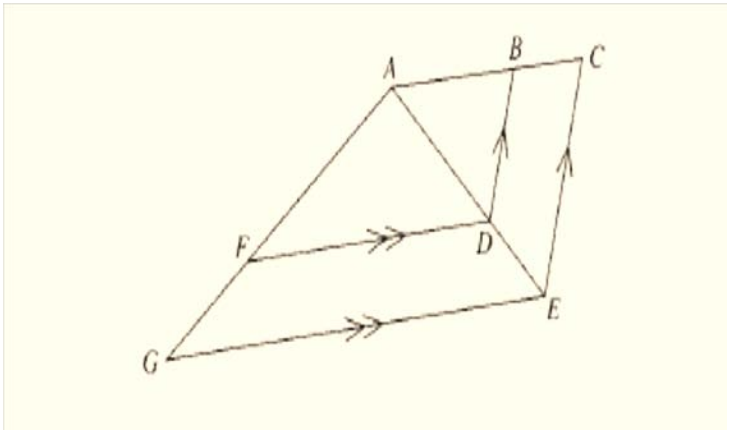
a) The interior angles in a regular polygon are 140^o. Calculate the number of sides of the polygon. 2

b) Solve for x: 2



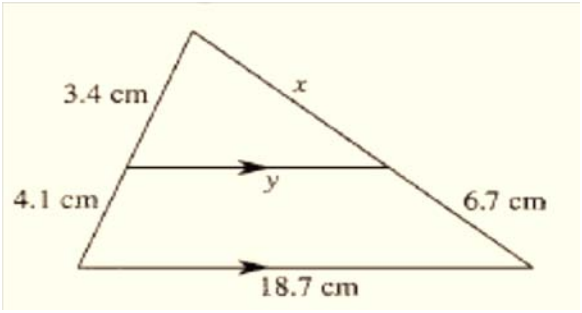
c) Prove that $\frac{AF}{AG} = \frac{AB}{AC}$

2



d) Find the value of the pronumerals. 3

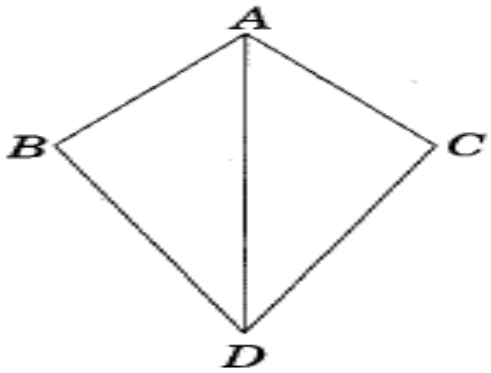
(Answer to 1 decimal place)



e) Given AB = AC and AD bisects $\angle BAC$. Prove that 3

i) $\triangle ABD \equiv \triangle ACD$, and

ii) Hence give a reason why DB=DC 1



Question 4 (Logarithms) 14 marks

a) Evaluate: 1

$\log_6 6\sqrt{6}$

b) Solve for x: $\log_3 x = 5$ 1

c) Given $\log_7 2 = 0.36$ and $\log_7 5 = 0.83$, find $\log_7 70$. 2

d) Solve $\log_9 200$ to 1 decimal place. 2

e) Solve: $3^{x+2} = 81$ 2

f) Evaluate:

i) $\log_5 500 - \log_5 4$ 3

ii) $2\log_3 6 + \log_3 18 - 3\log_3 2$ 3

END

MATHEMATICS

2012 Assessment 1 : Preliminary HSC Solutions

Question 1 (14)

a. $A = P(1+r)^n$

$A_5 = \$980(1.12)^5$ ✓ (2)

$= \$1727.10$ ✓

b. $= 0.00589239\ldots$ ✓ (2)

$= 0.005892$ ✓

c. $\frac{12x}{x+1} \times \frac{x^2+2x+1}{6x}$

$= \frac{12x^2}{x+1} \times \frac{(x+1)(x+1)}{6x}$ ✓ factorised (2)

$= 2(x+1)$ ✓

d. $y^3 - 64 = y^3 - 4^3$

$= (y-4)(y^2+4y+16)$ ✓ ✓ (2)

e) let $x = 0.297297297\ldots$ — ①

$1000x = 297.297\ldots$ — ②

② − ①

$999x = 297$

$x = \frac{297}{999}$ ✓

$= \frac{11}{37}$ ✓

(3)

f. $\frac{2(6+3\sqrt{3})}{(6-3\sqrt{3})(6+3\sqrt{3})} - \frac{(3-2\sqrt{3})}{(3+2\sqrt{3})(3-2\sqrt{3})}$

$= \frac{12+6\sqrt{3}}{36-27} - \frac{(3-2\sqrt{3})}{9-12}$ ✓

$= \frac{12+6\sqrt{3}}{9} - \frac{(3-2\sqrt{3})3}{-3\times 3}$

(3)

$= \frac{12+6\sqrt{3}+9-6\sqrt{3}}{9}$ ✓

$= \frac{21}{9}$

$= \frac{7}{3}$ (or $2\frac{1}{3}$)

✓

* $\frac{0}{3}$ not × by conjugate

(2)

Question 2

a. $\left[\frac{a}{4} - \frac{(a+2)}{3} = 9\right] 12$

$3a - 4(a+2) = 108$ ✓

$-a - 8 = 108$

$a = -116$ ✓ (2)

b. $\left[\frac{x-3}{7} - \frac{3}{4} \geq 9\right] 28$

$4(x-3) - 21 \geq 252$

$4x - 12 - 21 \geq 252$ ✓

$4x \geq 285$ (2)

$x \geq \frac{285}{4}$ ✓ $\left(71\frac{1}{4}\right)$

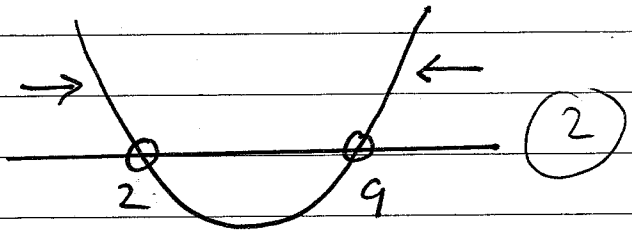
c. $x^2 - 10x + (-5)^2 = -20 + 25$

$(x-5)^2 = 5$ ✓ (2)

$x - 5 = \pm\sqrt{5}$

$x = 5 \pm \sqrt{5}$ ✓

d. $x^2 - 11x + 18 > 0$



(2)

$(x-9)(x-2) > 0$

$x < 2$ or $x > 9$

✓ ✓ (3)

e. $|x-2| = 7-2x$

$x-2 = \pm(7-2x)$

case 1:

$x-2 = 7-2x$

$3x = 9$

$x = 3$ ✓

case 2: $x-2 = 2x-7$

$x = +5$ ✓

Check: (3)

When $x = 3$: LHS $= 1$, RHS $= 1$, $\therefore$ LHS $=$ RHS

$x = +5$: LHS $= 3$, RHS $= -3$, LHS $\neq$ RHS ✓

$\therefore$ $x = 3$ is the only Solution.

f. $a+b = 5$ —— (1)

$2a+b+c = 4$ —— (2)

$a-b-c = 5$ —— (3)

(2) + (3)

$3a = 9$

$a = 3$ ✓

Sub $a = 3$ into (1) (3)

$b = 2$ ✓

Sub $a = 3$, $b = 2$ into 5

$3-2-c = 5$

$c = -4$ ✓ (4)

Question 3

a. Interior angle $= 140°$

$\therefore$ Exterior angle $= 40°$

Sum of exterior angles $= 360°$

$\therefore$ number of sides $= \frac{360°}{40°}$ ✓

$= 9$ ✓ (2)

b. $(x+2)^2 = x^2 + (x-7)^2$

$x^2 + 4x + 4 = x^2 + x^2 - 14x + 49$

$x^2 - 18x + 45 = 0$ ✓ (2)

$(x-3)(x-15) = 0$

$x = 3$ or 15 ✓

c) $\frac{AF}{AG} = \frac{AD}{AE}$ ✓ ——— (1) ($\triangle AGE$)

Ratio properties of a triangle

Also $\frac{AD}{AE} = \frac{AB}{AC}$ ——— (2) ($\triangle ACE$) ✓

Sub (1) into (2) (2)

$\therefore \frac{AF}{AG} = \frac{AB}{AC}$

(5)

d.

$$\frac{x}{3.4} = \frac{6.7}{4.1}$$

$x = 5.6$ cm ✓

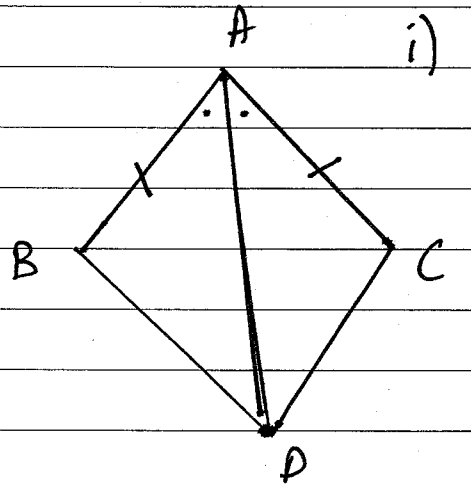
$$\frac{x}{y} = \frac{(x + 6.7)}{18.7}$$

$$y = \frac{5.6\,(18.7)}{12.3}$$

✓ (3)

$= 8.5$ cm ✓

e.



i) In $\triangle$ ABD and ACD

AB = AC (given) ✓

$\angle BAD = \angle CAD$ (given) ✓

AD is common ✓ (4)

$\therefore \triangle ABD \equiv \triangle ACD$ (SAS) ✓

ii) $\therefore$ DB = Dc (corresponding sides of congruent triangles) ✓

(6)

Question 4

a) $\log_6 6\sqrt{6} = \log_6 6^{\frac{3}{2}}$

$= \frac{3}{2}\log_6 6$

$= \frac{3}{2}$ ✓ (with working shown) (1)

b) $\log_3 x = 5$

$x = 3^5$

$= 243$ ✓ (1)

c. $\log_7 70 = \log_7 (7 \times 5 \times 2)$

$= \log_7 7 + \log_7 5 + \log_7 2$ ✓

$= 1 + 0.83 + 0.36$ (2)

$= 2.19$ ✓

d. $\log_9 200 = x$

$9^x = 200$

$x = \frac{\ln 200}{\ln 9}$ ✓ (2)

$= 2.4$ ✓

(7)

e. $3^{x+2} = 81$ or $x+2 = \frac{\ln 81}{\ln 3}$

$3^{x+2} = 3^4$ ✓

$x+2 = 4$

$x = 2$ ✓

(2)

$x+2 = 4$

$x = 2$

f.

i) $\log_5 500 - \log_5 4$

$= \log_5 \left(\frac{500}{4}\right)$ ✓

$= \log_5 125$ ✓

$= \log_5 5^3$

$= 3 \log_5 5$

$= 3$ ✓

or

$= \frac{\ln 125}{\ln 5}$

$= 3$

(3)

ii) $\log_3 6^2 + \log_3 18 - \log_3 2^3$

$= \log_3 \left(\frac{36 \times 18}{8}\right)$ ✓

$= \log_3 81$ ✓

$= \log_3 3^4$

$= 4 \log_3 3$

$= 4$ ✓

(3)

END

(8)