Year 11   May, 2003
Task 2 (Half Yearly)   Time: 80 min

Instructions: 1. Attempt all questions.
                    2. Write your answers on your own paper.
                    3. All necessary working must be shown.
                    4. Marks will be deducted for careless or badly arranged work.

QUESTION ONE : (**23** MARKS)

a)   Write $1.3\overset{\bullet}{2}7\overset{\bullet}{4}$ as a rational number   *1*

b)   Evaluate $\sqrt[3]{\dfrac{8.3 \times 4.1}{0.2 + 5.4 \div 1.3}}$ correct to 3 significant figures   **2**

c)   If $\sqrt{800} = x\sqrt{2}$ find the value of x   **2**

d)   Simplify

       i) $ab + 2b - 3ab + 8b$ ii) $8 - 4(2y + 1) + y$ **5**

e)   Simplify $\dfrac{\left(x^2\right)^{-3} \times \left(y^3\right)^2}{x^{-1} \times y^4}$ **2**

f)   Expand and simplify

       i) $(2y - 3)(y + 5)$ ii) $(2x + 3)^2$ iii) $(5a - b)(5a + b)$

                                                                                              9

       iv) $\left(\sqrt{6} + 3\sqrt{2}\right)\left(\sqrt{6} - 3\sqrt{2}\right)$ v) $\left(\sqrt{5} + \sqrt{2}\right)^2$

g)   Express $\dfrac{2\sqrt{3}}{\sqrt{5}}$ with a rational denominator   **2**

QUESTION TWO : (51 MARKS)

a) Factorise fully

i) $4x^2y - 2xy$
    ii) $xm + 5x + 7m + 35$
    iii) $m^2 - 9m + 18$

iv) $4x^2 + 4x - 3$
    v) $3x^2 - 12y^2$
    vi) $8 + 27y^3$ 13

b) Simplify

i) $\dfrac{x^2}{5x^2 - 2x}$
    ii) $\dfrac{t^2 + t - 2}{t^2 + 5t + 6}$
    iii) $\dfrac{2xy + 2x - 6 - 6y}{4x^2 - 16x + 12}$

iv) $\dfrac{3}{b+2} \times \dfrac{b^2 + 2b}{6a - 3}$
    v) $\dfrac{3ab^2}{5xy} \div \dfrac{12ab - 6a}{x^2y + 2xy^2}$
    vi) $\dfrac{p-3}{6} + \dfrac{p+2}{2}$

vii) $\dfrac{2}{x^2 - 4} - \dfrac{3}{x+2}$ 22

c) Simplify

i) $\sqrt{12} - \sqrt{27}$
    ii) $2\sqrt{8} - \sqrt{18}$
    iii) $4\sqrt{48} + 3\sqrt{147} + 5\sqrt{12}$ 11

d) Find a and b if $\dfrac{\sqrt{3} - 4}{2 + 3\sqrt{3}} = a + b\sqrt{3}$ 5

QUESTION THREE : (**38** MARKS)

a)   Solve the following

   i) $2(3x + 7) = 6 - (x - 1)$ ii) $3y - 4 > 5y + 12$

   iii) $-1 \le 2x + 3 \le 5$ iv) $\dfrac{a-2}{3} < 2 + \dfrac{3a}{4}$

   v) $|2x + 1| = 3x - 2$ vi) $|2t - 3| \ge 5$ 19

b)   Solve giving answers in simplified surd form where necessary

   i) $3a^2 - 14a + 8 = 0$ ii) $4x^2 + 12x + 1 = 0$ 8

c)   If $x = 2$ is one solution to $x^2 - ax - 2 = 0$, find the other solution.   3

d)   Solve the simultaneous equations

   i) $3a + 2b = 5$ ii) $y = x^2 + 4x$ 8
        $2a - b = -6$ $2x - y - 1 = 0$

QUESTION FOUR : (15 MARKS)

a)   Find correct to 3 decimal places

   i) $\sin 38^0 25'$ ii) $\tan 125^0 36'$ 2

b)   Find θ in degrees and minutes if θ is acute

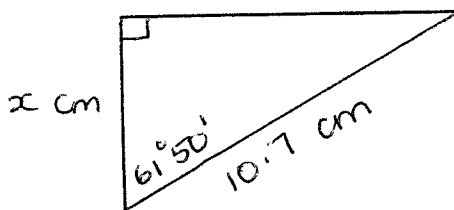   i) $\cos \theta = 0.827$ ii) $\sin \theta = \dfrac{10}{14}$ 2

c)

   i)   Find x to one decimal place   ii)   Find θ in degrees and minutes
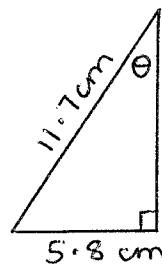
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d)   The angle of depression from the top of an 80 metre building to a car below is $61^0 29'$.
       How far is the car from the building to one decimal place ?
       (Draw a diagram.)   3

e)   A girl rides a motorbike through her property, starting at her house. If she rides south for
       1.3 km, then rides west for 2.4 km , what is her bearing from the house , to the nearest
       degree ?
       (Draw a diagram.)   4

QUESTION FIVE : (33 MARKS)

a)   Find the exact values of

   i) $\sin 60^0 + \cos 60^0$ ii) $\cos^2 45^0$ iii) $2\cosec 30^0$

   iv) $\dfrac{\tan 45^0 + \tan 60^0}{1 - \tan 45^0 \tan 60^0}$ v) $\sin 225^0$ vi) $\cos 390^0$

   vii) $\tan(-135^0)$ viii) $\sin 240^0$ 18

b)   If $\cos x = \dfrac{-3}{8}$ and $\sin x < 0$, find the exact value of $\tan x$ 3

c)   Find the value of θ to the nearest minute for $0^0 \le \theta \le 360^0$ for which

   i) $\sin \theta = 0.463$ ii) $\cos \theta = -0.237$ iii) $2 \sin \theta = \sqrt{3}$
                                                                      12

   iv) $\tan 2\theta = \sqrt{3}$ v) $\cos^2 \theta = \dfrac{1}{2}$

QUESTION SIX : (38 MARKS)

a) Simplify

    i) $\sin\theta\cot\theta$ ii) $3 + 3\tan^2\theta$ iii) $\cot\theta - \cot\theta\cos^2\theta$ 8

b) Prove that

    i) $\dfrac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$

                                                                      8

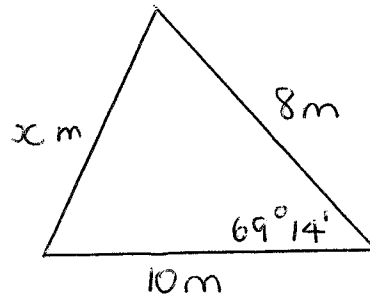   ii) $\dfrac{\cos\theta(\sin\theta + \cos\theta)}{(1 + \sin\theta)(1 - \sin\theta)} = 1 + \tan\theta$
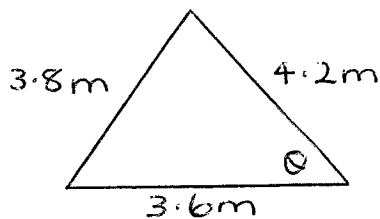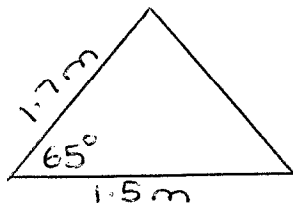
c) Find x to one decimal place

    i)   ii)

                                                                                 6
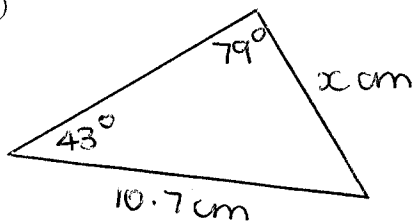
d) Find θ to the nearest minute.

                                                                                    3

e) Find the area of the sail below, correct to one decimal place.

                                                                                 2

f) Sam drove from his home for 200km on a bearing of 040^0, then drives on a bearing of 157^0 for 345km.

    i)   Draw a diagram showing this information.   2

    ii)   How far from home is Sam to the nearest km ?   3

    iii)   What is the bearing of Sam from his home to the nearest degree ?   3

g) Sketch the graph of $y = \cos x$ for $0^0 \le x \le 360^0$ 3

Q1 a) $1.3\overline{274} = 1\frac{3274-3}{9990} = 1\frac{3271}{9990}$ ①

b) 1.98 ②

c) $\sqrt{800}$
$= \sqrt{2 \times 400}$
$= 20\sqrt{2}$
$\therefore x = 20$ ②

d) i) $-2ab + 10b$ ② ii) $8 - 8y - 4 + y$
$= 4 - 7y$ ③

e) $\frac{x^{-6} \times y^6}{x^{-1} \times y^4}$
$= x^{-5} y^2$ ②

f) i) $2y^2 + 10y - 3y - 15$
$= 2y^2 + 7y - 15$ ②

ii) $4x^2 + 12x + 9$ ②

iii) $25a^2 - b^2$ ①

iv) $6 - 9 \times 2$
$= 6 - 18$
$= -12$ ②

v) $5 + 2\sqrt{10} + 2$
$= 7 + 2\sqrt{10}$ ②

g) $\frac{2\sqrt{3}}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{2\sqrt{15}}{5}$ ②

Q2 a) i) $2xy(2x-1)$ ②

ii) $x(m+5) + 7(m+5)$
$= (m+5)(x+7)$ ②

iii) $(m-6)(m-3)$ ②

iv) $4x^2 + 4x - 3$
$= 4x^2 + 6x - 2x - 3$
$= 2x(2x+3) - 1(2x+3)$
$= (2x+3)(2x-1)$ ③

v) $3(x^2 - 4y^2)$
$= 3(x+2y)(x-2y)$ ②

vi) $2^3 + (3y)^3$
$= (2+3y)(4 - 6y + 9y^2)$ ②

b) i) $\frac{x^2}{x(5x-2)}$
$= \frac{x}{5x-2}$ ②

ii) $\frac{(t+2)(t-1)}{(t+3)(t+2)}$
$= \frac{t-1}{t+3}$ ③

iii) $\frac{2x(y+1) - 6(y+1)}{4(x^2 - 4x + 3)}$
$= \frac{(2x-6)(y+1)}{4(x-3)(x-1)}$
$= \frac{2(x-3)(y+1)}{4(x-3)(x-1)}$
$= \frac{y+1}{2(x-1)}$ ⑤

Q2 b) iv) $\frac{3}{b+2} \times \frac{b(b+2)}{3(2a-1)}$
$= \frac{b}{2a-1}$ ③

v) $\frac{3ab^2}{5xy} \times \frac{xy(x+2y)}{6a(2b-1)}$
$= \frac{b^2(x+2y)}{10(2b-1)}$ ③

vi) $\frac{p-3}{6} + \frac{3p+6}{6}$
$= \frac{4p+3}{6}$ ②

vii) $\frac{2}{(x+2)(x-2)} - \frac{3(x-2)}{(x+2)(x-2)}$
$= \frac{2 - 3x + 6}{(x+2)(x-2)}$
$= \frac{8 - 3x}{(x+2)(x-2)}$ ④

c) i) $\sqrt{4 \times 3} - \sqrt{9 \times 3}$
$= 2\sqrt{3} - 3\sqrt{3}$
$= -\sqrt{3}$ ③

ii) $2\sqrt{4 \times 2} - \sqrt{9 \times 2}$
$= 4\sqrt{2} - 3\sqrt{2}$
$= \sqrt{2}$ ③

iii) $4\sqrt{16 \times 3} + 3\sqrt{49 \times 3} + 5\sqrt{4 \times 3}$
$= 16\sqrt{3} + 21\sqrt{3} + 10\sqrt{3}$
$= 47\sqrt{3}$ ⑤

d) $\frac{\sqrt{3} - 4}{2 + 3\sqrt{3}} \times \frac{2 - 3\sqrt{3}}{2 - 3\sqrt{3}}$
$= \frac{2\sqrt{3} - 9 - 8 + 12\sqrt{3}}{4 - 27}$
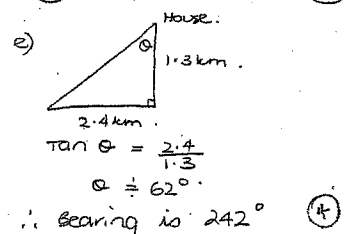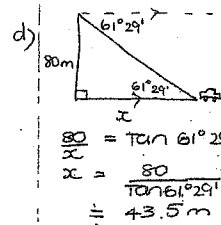$= \frac{14\sqrt{3} - 17}{-23}$
$= \frac{17 - 14\sqrt{3}}{23}$
$\therefore a = \frac{17}{23} \quad b = \frac{-14}{23}$ ⑤

Q3 a) i) $6x + 14 = 6 - x + 1$
$6x + 14 = 7 - x$
$6x = -7 - x$
$7x = -7$
$\underline{x = -1}$ ③

ii) $3y - 4 > 5y + 12$
$3y > 5y + 16$
$-2y > 16$
$y < -8$ ③

iii) $-4 \le 2x \le 2$
$-2 \le x \le 1$ ②

iv) $4(a-2) < 24 + 9a$
$4a - 8 < 24 + 9a$
$4a < 32 + 9a$
$-5a < 32$
$a > \frac{32}{-5}$ ③

v) $2x + 1 = 3x - 2$ or $2x + 1 = -(3x-2)$
$2x = 3x - 3$ $2x + 1 = -3x + 2$
$-x = -3$ $2x = -3x + 1$
$x = 3$ ✓ $5x = 1$
    $x = \frac{1}{5}$
    (not a solution) $|1\frac{2}{5}| \ne \frac{3}{5} - 2$
      $\ne -1\frac{2}{5}$ ④

vi) $2t - 3 \ge 5$ or $-(2t-3) \ge 5$
$2t \ge 8$ $2t - 3 \le -5$
$t \ge 4$ $2t \le -2$
   $t \le -1$
$\therefore t \le -1 , t \ge 4$ ④

b) i) $3a^2 - 14a + 8 = 0$
$3a^2 - 12a - 2a + 8 = 0$
$3a(a-4) - 2(a-4) = 0$
$(3a-2)(a-4) = 0$
$\therefore a = \frac{2}{3}, a = 4$ ④

ii) $x = \frac{-12 \pm \sqrt{144 - 4.4.1}}{2.4}$
$= \frac{-12 \pm \sqrt{128}}{8}$
$= \frac{-12 \pm 8\sqrt{2}}{8}$
$= \frac{-3 \pm 2\sqrt{2}}{2}$ ④

Q3 c) $4 - 2a - 2 = 0$
$2 - 2a = 0$
$2a = 2$
$a = 1$
$\therefore x^2 - x - 2 = 0$
$(x-2)(x+1) = 0$
$\therefore x = 2$ and $x = -1$.
$\therefore$ the other solution is $x = -1$. ③

d) i) $3a + 2b = 5$ — ①
$2a - b = -6$ — ②
mult. ② by 2
$4a - 2b = -12$
$3a + 2b = 5$ (+)
$7a = -7$
$\underline{a = -1}$
sub $a = -1$ into ①
$-3 + 2b = 5$
$2b = 8$
$\underline{b = 4}$ ④

ii) $y = x^2 + 4x$ — ①
$2x - y - 1 = 0$ — ②
sub ① into ②
$2x - (x^2 + 4x) - 1 = 0$
$2x - x^2 - 4x - 1 = 0$
$x^2 + 2x + 1 = 0$
$(x+1)(x+1) = 0$
$\underline{x = -1}$
sub $x = -1$ into ①
$y = 1 - 4$
$\underline{y = -3}$ ④

Q4 a) i) 0.621 ii) -1.397 ②

b) i) $34°12'$ ii) $45°35'$ ②

c) i) $\frac{x}{10.7} = \cos 61°50'$
$x = 10.7 \cos 61°50'$
$\doteqdot 5.1$ cm. ②

ii) $\sin \theta = \frac{5.8}{11.7}$
$\theta \doteqdot 29°43'$ ②

d)
$\frac{80}{x} = \tan 61°29'$
$x = \frac{80}{\tan 61°29'}$
$\doteqdot 43.5$ m. ③

e)
$\tan \theta = \frac{2.4}{1.3}$
$\theta \doteqdot 62°$
$\therefore$ bearing is $242°$ ④

Q5 a) i) $\frac{\sqrt{3}}{2} + \frac{1}{2}$ ii) $\left(\frac{1}{\sqrt{2}}\right)^2$ iii) $\frac{2}{\sin 30°}$

$= \frac{\sqrt{3}+1}{2}$ ③ $= \frac{1}{2}$ ② $= \frac{2}{\frac{1}{2}}$

$= 4$ ③

iv) $\frac{1+\sqrt{3}}{1-1\cdot\sqrt{3}}$ ②   v) $-\sin 45°$ vi) $\cos 30°$

$= \frac{1+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$ $= -\frac{1}{\sqrt{2}}$ ② $= \frac{\sqrt{3}}{2}$ ②

$= \frac{1+2\sqrt{3}+3}{1-3}$ vii) $\tan 45$ viii) $-\sin 60°$

$= \frac{4+2\sqrt{3}}{-2}$ $= 1$ ② $= -\frac{\sqrt{3}}{2}$ ②

$= -2-\sqrt{3}$ ④   b)

$\tan x = \frac{\sqrt{55}}{3}$

③
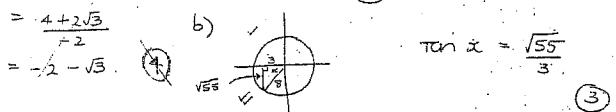
c) i) (1st + 2nd quad)   ii) (2nd + 3rd quad)

$\theta = 27°36'$ and $152°25'$ $\theta = 103°43'$ and $256°17$

②   ②

iii) $\sin \theta = \frac{\sqrt{3}}{2}$ iv) $0° \le 2\theta \le 720°$

(1st + 3rd quad)

(1st+2nd quad) $2\theta = 60°, 240°, 420°, 600$

$\theta = 60°$ and $120°$ ② $\theta = 30°, 120°, 210°, 300°$ ③
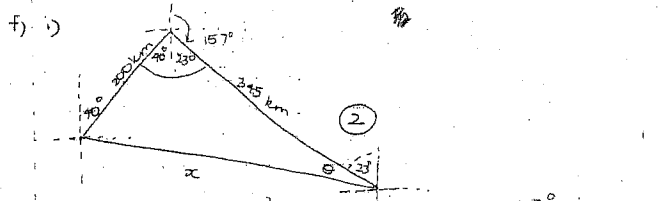
v) $\cos \theta = \pm\frac{1}{\sqrt{2}}$

(1st, 2nd, 3rd + 4th quad)

$\theta = 45°, 135°, 225°, 315°$ ③

Q6 a) i) $\frac{\sin\theta \cdot \cos\theta}{\sin\theta}$ ii) $3(1+\tan^2\theta)$ iii) $\cot\theta(1-\cos^2\theta)$

$= \cos\theta$ ② $= 3\sec^2\theta$ ② $= \frac{\cos\theta}{\sin\theta} \cdot \sin^2\theta$ ④

$= \sin\theta\cos\theta$

b) i) $\frac{1}{\cos^2\theta} - \sin^2\theta$ ii) $\frac{\cos\theta(\sin\theta+\cos\theta)}{1-\sin^2\theta}$

$= \sec^2\theta - \sin^2\theta$ $= \frac{\cos\theta(\sin\theta+\cos\theta)}{\cos^2\theta}$ ④

$= 1 + \tan^2\theta - \sin^2\theta$ $= \frac{\sin\theta+\cos\theta}{\cos\theta}$

$= \tan^2\theta + \cos^2\theta$ $= \sin\theta + \cos\theta = \tan\theta + 1$

Q6 c) i) $\frac{x}{\sin 43°} = \frac{10.7}{\sin 79°}$ ii) $x^2 = 8^2 + 10^2 - 2\times10\times8\times\cos6'$

$x = \frac{10.7\sin43°}{\sin 79°}$ ③ $x^2 = 107.2699\ldots$

$x = 7.4$ cm $x = 10.4$ m ③

d) $\cos\theta = \frac{3.6^2 + 4.2^2 - 3.8^2}{2\times3.6\times4.2}$ e) $A = \frac{1}{2}\times1.7\times1.5\times\sin65°$

$= \frac{16.16}{30.24}$ $= 1.2$ m² ②

$\theta = 57°42'$ ③

f) i)

②

ii) $x^2 = 200^2 + 345^2 - 2\times200\times345\times\cos63°$

$x^2 = 96374.31104$

$x = 310$ km ③

iii) $\cos\theta = \frac{310^2 + 345^2 - 200^2}{2\times310\times345}$

$= \frac{175125}{213900}$

$\theta = 35°$ ③

∴ Bearing $= 360° - 23° - 35° = 302°$

g)

$y = \cos x$

③