Year 11 **Mathematics** April, 2005

Task 2 Time: 90 mins

INSTRUCTIONS: 1. Attempt all questions.
2. Write your answers on your own paper.
3. All necessary working must be shown.
4. Marks will be deducted for careless or badly arranged work.

Question 1 (12 marks)

a) Find, correct to 3 decimal places. $\dfrac{3.24^2}{5.73 - 2.84}$ **2**

b) The base length x, of a square pyramid of volume V and perpendicular height h is given by $x = \sqrt{\dfrac{3v}{h}}$.

Find h, correct to two decimal places if $V = 750$ and $x = 16.32$ **2**

c) Simplify $|3 - 5|^2 - 2|-2 + 7|$ **2**

d) The speed of light is about 300 000 km/h. Write this speed in scientific notation. **1**

e) Solve $\dfrac{2x}{x-5} = \dfrac{3}{5}$ **3**

f) Solve $-1 \le 2x + 1 \le 5$ **2**

Question 2 (18 marks)

a) Factorise and simplify:

(i) $\dfrac{x-5}{5-x}$ (ii) $\dfrac{x^2 - 9}{x - 3}$ (iii) $\dfrac{2(x-3) - 4}{2}$ **5**

b) Expand and simplify $(x-1)(x^2 - 2x - 1)$ **2**

c) Factorise fully $2x^2 - 50y^4$ **2**

d) Find integers a and b such that $\dfrac{2}{2 - \sqrt{3}} = a + \sqrt{b}$. **3**

e) Simplify, giving exact answers:
(i) $\sqrt{45} + \sqrt{80}$ (ii) $(2\sqrt{11} - 5)^2$ **4**

f) Rationalise the denominator. $\dfrac{\sqrt{5}}{2\sqrt{2} - \sqrt{3}}$ **2**

Question 3   (21 marks)

a)   Simplify $\dfrac{4}{t-1}+\dfrac{t}{t^2-1}$ **2**

b)   Factorise   (i) $6x^2-29x+28$ (ii) x^4-16 **4**

c)   Solve $\left|\dfrac{x+1}{3}\right|=5$ **3**

d)   Solve, giving the answers correct to 2 decimal places: $2x^2-4x-3=0$ **3**

e)   Solve: (i) $|2x-1|=|x+5|$ **3**

(ii) $(x-2)(x-3)=12$ **3**

(iii) $x^2-y^2=4$ **3**
$x+y=1$

Question 4   (11 marks)

a)   Find, correct to 3 decimal places: (i) $\cos 65^0 16'$ (ii) $\tan 135^0 6'$ **2**

b)   Find θ correct to the nearest minute: (i) $\tan\theta=1.7$ (ii) $\cos\theta=\dfrac{1.6}{2.8}$ **2**

c)   Find the value of x,
correct to 2 decimal places.   **2**
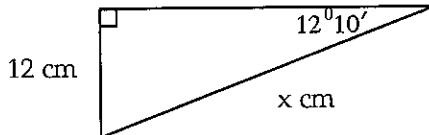
d)   A ship leaves port and sails on a bearing of 120^0 for 50 km.
(i)   Draw a sketch to show this information.   **1**
(ii)   How far east of the port is it at this time, correct to the nearest km?   **2**

e)   Sketch $y=\tan x$ for $0^0 \le x \le 360^0$ **2**

Question 5   (20 marks)

a)   Find the exact value of:
   (i) $\sin 150^0$ (ii) $\cos 45^0$ (iii) $\tan 330^0$ **3**

b)   Solve for $0^0 \leq \theta \leq 360^0$:
   (i) $\sin \theta = \dfrac{1}{2}$ (ii) $\tan \theta = -\sqrt{3}$ **6**

c)   Solve for $0^0 \leq \theta \leq 360^0$:
   (i) $4\sin^2 \theta = 3$ (ii) $\cos^2 \theta = 1 - 2\sin^2 \theta$ **6**
   (iii) $\tan 2\theta = \dfrac{1}{\sqrt{3}}$ **3**

d)   Simplify fully: $\dfrac{1}{\cos\theta} - \sin\theta \tan\theta$ **2**

Question 6   (12 marks)

a)   Simplify $\sec^2 \theta - \tan^2 \theta$ **2**

b)   Prove $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$ **3**

c)
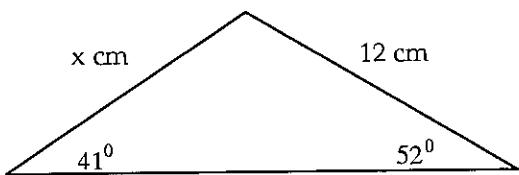
   (i)   Use the sine rule to find x, correct to 2 decimal places.   **2**
   (ii)   Find the area of the triangle, correct to the nearest cm^2.   **3**

d)
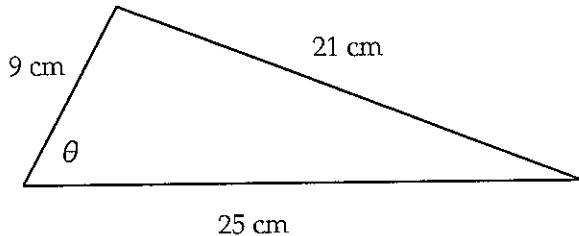
   Use the cosine rule to find the value of θ, correct to the nearest minute.   **2**

Question 7   (13 marks)

a)   Find x, correct to 2 decimal places   **2**

x cm

1.3 cm

130^0

2.9 cm

b)

A

BC = 4 metres
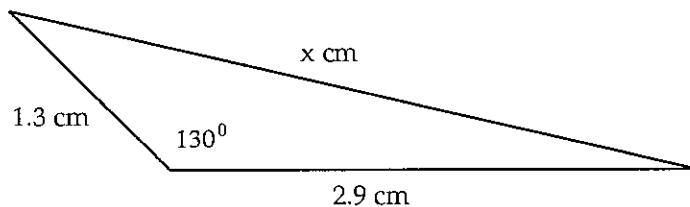
C 35^0 X 40^0 B
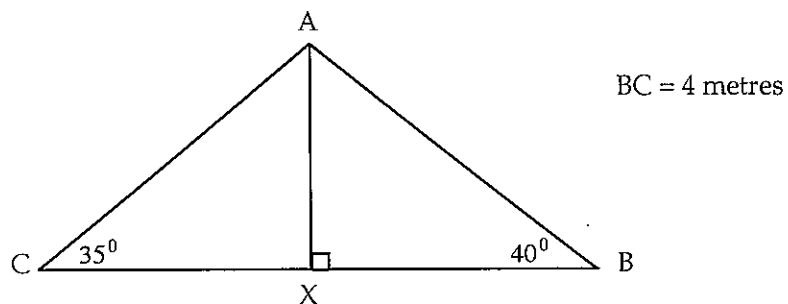
(i)   Use the sine rule in $\triangle ABC$ to find AC, correct to 2 decimal places.   **3**

(ii)   Find the length of AX, correct to 2 decimal places.   **2**

c)

N

15^0 30^0

K   L   M
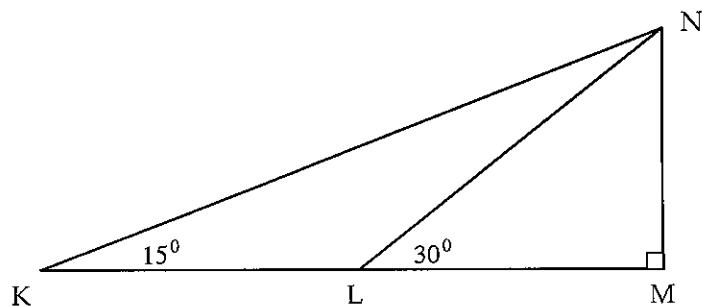
(i)   If $KL = 1$ metre, explain why LN also equals 1 metre.   **2**

(ii)   Use the sketch to deduce that: $\tan 15^0 = 2 - \sqrt{3}$ **4**

Question 1.

a) 3.632

b) $x = \sqrt{\dfrac{3V}{h}}$

$16.32 = \sqrt{\dfrac{3 \times 750}{h}}$

$266.3424 = \dfrac{2250}{h}$

$h = \dfrac{2250}{266.3424}$

$h = 8.45$

c) $|3-5|^2 - 2|-2+7|$

$= 4 - 2 \times 5$

$= -6$

d) 3×10^5

e) $\dfrac{2x}{x-5} = \dfrac{3}{5}$

$10x = 3x - 15$

$7x = -15$

$x = -\dfrac{15}{7}$ or $-2\dfrac{1}{7}$

f) $-1 \leq 2x+1 \leq 5$

$-2 \leq 2x \leq 4$

$-1 \leq x \leq 2$

Question 2

a) i) -1 ii) $\dfrac{(x-3)(x+3)}{x-3} = x+3$

iii) $\dfrac{2(x-3)-4}{2} = x-3-2$

$= x-5$

b) $x^3 - 2x^2 - x - x^2 + 2x + 1$

$= x^3 - 3x^2 + x + 1$

c) $2(x^2 - 25y^4) = 2(x - 5y^2)(x + 5y^2)$

d) $\dfrac{2}{2-\sqrt{3}} \times \dfrac{2+\sqrt{3}}{2+\sqrt{3}} = \dfrac{4+2\sqrt{3}}{4-3}$

$= 4 + \sqrt{12}$

$\therefore a = 4 \quad b = 12$

e) i) $\sqrt{45} + \sqrt{80}$

$= 3\sqrt{5} + 4\sqrt{5}$

$= 7\sqrt{5}$

ii) $(2\sqrt{11} - 5)^2 = 4 \times 11 - 20\sqrt{11} + 25$

$= 69 - 20\sqrt{11}$

f) $\dfrac{\sqrt{5}}{2\sqrt{2}-\sqrt{3}} \times \dfrac{2\sqrt{2}+\sqrt{3}}{2\sqrt{2}+\sqrt{3}}$

$= \dfrac{2\sqrt{10} + \sqrt{15}}{8-3}$

$= \dfrac{2\sqrt{10} + \sqrt{15}}{5}$

Question 3

a) $\dfrac{4}{t-1} + \dfrac{t}{(t-1)(t+1)}$

$= \dfrac{4(t+1) + t}{(t-1)(t+1)}$

$= \dfrac{5t+4}{t^2-1}$

b) i) $(3x-4)(2x-7)$

  ii) $(x^2-4)(x^2+4)$

   $= (x-2)(x+2)(x^2+4)$

c) $\dfrac{x+1}{3} = 5$ or $-\left(\dfrac{x+1}{3}\right) = 5$

$x+1 = 15$ $\qquad\dfrac{x+1}{3} = -5$

$x = 14$ $\qquad\qquad x+1 = -15$

$\qquad\qquad\qquad x = -16$

$\therefore x = 14 \text{ or } -16$

d) $x = \dfrac{-(-4) \pm \sqrt{16 - 4\times 2\times(-3)}}{4}$

$x = \dfrac{4 \pm \sqrt{40}}{4}$

$x = 2.58 \quad \text{or} \quad -0.58$

e)

i) $2x-1 = x+5$, $\quad -(2x-1) = x+5$

$x = 6$ $\qquad\qquad -2x+1 = x+5$

$\qquad\qquad\qquad -4 = 3x$

$\qquad\qquad\qquad x = -\dfrac{4}{3}$

ii) $(x-2)(x-3) = 12$

$x^2 - 5x + 6 = 12$

$x^2 - 5x - 6 = 0$

$(x-6)(x+1) = 0$

$\therefore x = -1 \text{ or } 6$

iii) $x^2 - y^2 = 4 \qquad y = 1-x$

$\therefore x^2 - (1-x)^2 = 4$

$x^2 - (1-2x+x^2) = 4$

$\cancel{x^2} - 1 + 2x - \cancel{x^2} = 4$

$2x = 5$

$x = \dfrac{5}{2}$

$y = -\dfrac{3}{2}$

Question 4

a) i) 0.418 ii) -0.997

b) i) $\theta = 59°32'$ ii) $55°9'$

c) $\sin 12°10' = \dfrac{12}{x}$

$x = \dfrac{12}{\sin 12°10'}$

$x = 56.94 \text{ cm}$

d)

i)

ii) $\cos 30 = \dfrac{d}{50}$

$\therefore d = 43 \text{ km}$

c)
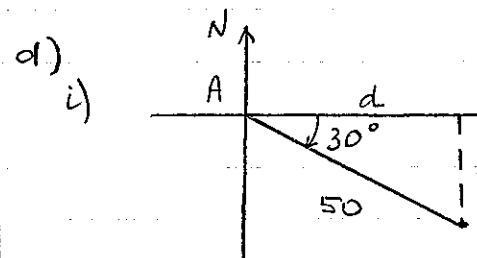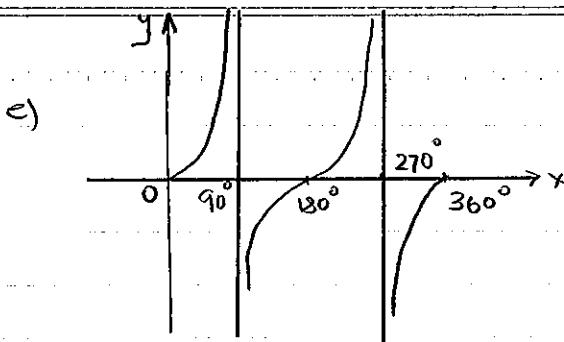
Question 5

a i) $\sin 150° = \frac{1}{2}$

ii) $\cos 45° = \frac{1}{\sqrt{2}}$

iii) $\tan 330° = -\frac{1}{\sqrt{3}}$

b) i) $\sin\theta = \frac{1}{2}$

acute angle $30°$

quadrants 1 and 2

$\therefore \quad \theta = 30°, 150°$

ii) $\tan\theta = -\sqrt{3}$

acute angle $60°$

quadrants 2 and 4

$\therefore \quad \theta = 120°, 300°$

c) i) $4\sin^2\theta = 3$

$\sin^2\theta = \frac{3}{4}$

$\sin\theta = \pm\frac{\sqrt{3}}{2}$

acute angle $60°$

quadrants 1, 2, 3 and 4

$\therefore \theta = 60°, 120°, 240°, 300°$

ii) $\cos^2\theta = 1 - 2\sin^2\theta$

$1 - \sin^2\theta = 1 - 2\sin^2\theta$

$\sin^2\theta = 0$

$\therefore \quad \theta = 0°, 180°, 360°$

ii) $\tan 2\theta = \frac{1}{\sqrt{3}}$

$0 \le 2\theta \le 720°$

acute $\angle: 30°$

$\therefore 2\theta = 30°, 210°, 390°, 570°$

$\therefore \theta = 15°, 105°, 195°, 285°$

d) $\frac{1}{\cos\theta} - \sin\theta\tan\theta$

$= \frac{1}{\cos\theta} - \sin\theta \cdot \frac{\sin\theta}{\cos\theta}$

$= \frac{1 - \sin^2\theta}{\cos\theta}$

$= \frac{\cos^2\theta}{\cos\theta}$

$= \cos\theta$

Question 6

a) $\sec^2\theta - \tan^2\theta = 1$

b) LHS:

$(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta$
$\quad + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta)$

$= 2 \quad$ RHS

c) i) $\frac{x}{\sin 52°} = \frac{12}{\sin 41°}$

$x = \frac{12\sin 52°}{\sin 41°}$

$x = 14.41$

c ii)
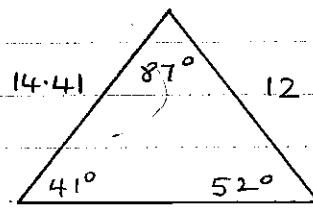
Area $= \frac{1}{2} \times 14\cdot41 \times 12 \times \sin 87°$

$= 86 \text{ cm}^2$

d) $\cos\theta = \dfrac{9^2 + 25^2 - 21^2}{2 \times 9 \times 25}$

$\cos\theta = 0\cdot58$

$\theta = 53° 55'$

Question 7

a) $x^2 = 1\cdot3^2 + 2\cdot9^2 - 2 \times 1\cdot3 \times 2\cdot9 \times \cos 130°$

$x^2 = 14\cdot946618\cdots$

$x = 3\cdot87 \text{ cm}$

b) i)
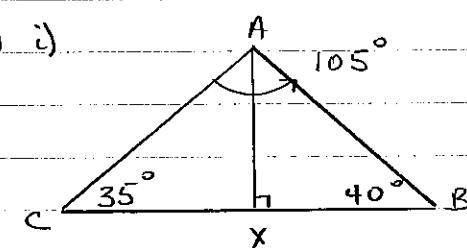
$\dfrac{AC}{\sin 40°} = \dfrac{4}{\sin 105°}$

$AC = \dfrac{4 \sin 40°}{\sin 105°}$

$AC = 2\cdot66 \text{ m}$

ii) $\sin 35° = \dfrac{AX}{2\cdot66}$

$AX = 2\cdot66 \times \sin 35°$

$AX = 1\cdot53 \text{ cm}$

c)
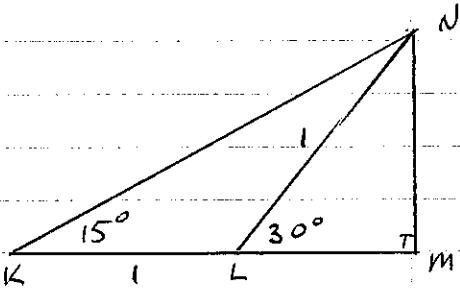
i) $\angle KNL = 15°$ (sum of interior opposite angles equals exterior angle of $\triangle KLN$)

$\therefore \triangle KLN$ is isosceles

$\therefore LN = 1 \text{ metre}$

ii) in $\triangle LNM$

• $\cos 30° = \dfrac{LM}{1}$

$\dfrac{\sqrt{3}}{2} = LM$

• $\sin 30° = \dfrac{MN}{1}$

$\dfrac{1}{2} = MN$

in $\triangle KNM$

$\tan 15° = \dfrac{MN}{KM}$

$= \dfrac{\dfrac{1}{2}}{1 + \dfrac{\sqrt{3}}{2}}$

$= \dfrac{1}{2 + \sqrt{3}} \times \dfrac{2 - \sqrt{3}}{2 - \sqrt{3}}$

$= \dfrac{2 - \sqrt{3}}{4 - 3}$

$\tan 15° = 2 - \sqrt{3}$