GIRRAWEEN HIGH SCHOOL

YEAR 11 - TASK 2

2006

MATHEMATICS

2 UNIT

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board-approved calculators may be used.
- Start each question on a *new* sheet of paper.

Question 1 (13 marks)

(a) Find $\sqrt[4]{1.6 \times 2.6}$ correct to 3 decimal places. **1**

(b) Evaluate $\dfrac{13^5}{11^6 + 17^4}$ correct to 3 significant figures. **1**

(c) Write 0.01072 in scientific notation. **1**

(d) Find the value of x if $\sqrt{18} + \sqrt{8} = \sqrt{x}$ **2**

(e) Classify each of these real numbers as rational.
$\sqrt{9}, \sqrt{10}, \sqrt[3]{15}, -0.16, \pi$ **2**

(f) Write each recurring decimal as a fraction in lowest terms.
(i) $0.3\dot{6}$ **3**

(ii) $0.\dot{2}9\dot{7}$ **3**

Question 2 (19 marks)

(a) Simplify.

(i) $\sqrt{500}$ **2**

(ii) $3\sqrt{2}+3\sqrt{8}-\sqrt{50}$ **2**

(b) Expand and simplify.

(i) $(2\sqrt{3}-1)(3\sqrt{3}+5)$ **2**

(ii) $(5\sqrt{2}-3)^2$ **3**

(c) Express with a rational denominator.

(i) $\dfrac{5}{2\sqrt{7}}$ **2**

(ii) $\dfrac{3}{\sqrt{11}+\sqrt{5}}$ **3**

(d) Solve for x.

(i) $|2x-5|=3$ **2**

(ii) $|2+4x|\geq 6$ **3**

Question 3 (19 marks)

(a) Expand and simplify

(i) $-4x^2(x+3)-2x^2(x-1)$ **2**

(ii) $(3x-5)(2x-3)$ **2**

(b) Factorise

(i) $36-25g^2$ **2**

(ii) $i^2+5i-36$ **2**

(iii) $3k^2-7k-6$ **3**

(iv) n^3+8 **3**

(v) $u^2w+vw-u^2x-vx$ **3**

(c) Find the values of p and q such that $\dfrac{\sqrt{5}}{\sqrt{5}-2}=p+q\sqrt{5}$. **2**

Question 4 (15 marks)

(a) Simplify

(i) $\frac{3a}{2b}+\frac{2a}{3b}$ **2**

(ii) $\frac{2}{x+1}-\frac{5}{x-4}$ **3**

(b) Factor where possible, then simplify

(i) $\frac{2x-2y}{x^2-y^2}$ **2**

(ii) $\frac{3x^2-19x-14}{9x^2-4}$ **3**

(c) Solve each linear equation

(i) $3(x+5)=17$ **2**

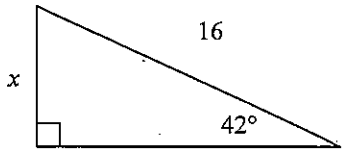
(ii) $\frac{a}{7}-4=\frac{a}{2}+11$ **3**

Question 5 (14 marks)

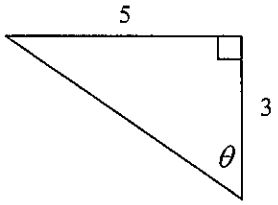
(a) Find correct to 4 decimal places, $\sin 38°24'$. **1**

(b) Find the acute angle θ, correct to the nearest minute, given that, $\cos\theta = \frac{1}{4}$. **2**

(c) Find, correct to two decimal places, the side marked x. **2**

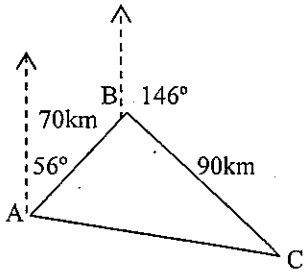


(d) Find, correct to the nearest minute, the angle θ. **2**



(e) A vertical pole stands on level ground. From a point on the ground 8 metres from its base, the angle of elevation of the top of the pole is 38°. Find the height of the pole, correct to the nearest centimeter. **2**

(f) A motorist drove 70 km from town A to town B on a bearing of 056°, and then drove 90km from town B to town C on a bearing of 146°.



(i) Explain why $\angle ABC = 90°$ **1**

(ii) How far apart are the towns A and C, correct to the nearest metre **2**

(iii) Find $\angle BAC$, and hence find the bearing of town C from A, correct to the nearest degree. **2**

Question 6 (23 marks)

(a) Find the exact value of

(i) $\tan 240^\circ$ **1**

(ii) $\sin 315^\circ$ **1**

(b) Simplify $\dfrac{1}{\cot\theta}$ **1**

(c) Prove the following trigonometric identities

(i) $\tan\theta \operatorname{cosec}\theta = \sec\theta$ **2**

(ii) $4\sec^2\theta - 3 = 1 + 4\tan^2\theta$ **2**

(d) Solve for $0^\circ \le x \le 360^\circ$.

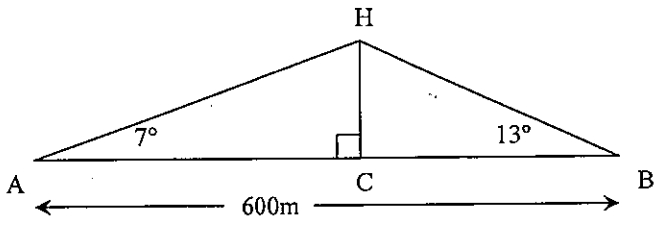
(i) $\sqrt{2}\sin x + 1 = 0$ **2**

(ii) $\cos 2x = \dfrac{1}{2}$ **3**

(iii) $\sin^2 x + \sin x = 0$ **3**

(e) A triangle has sides 7cm, 8cm and 10cm. Use the cosine rule to find its largest angle, and hence find the area of the triangle, correct to the nearest cm^2. **4**

(f) A helicopter is hovering above a straight, horizontal road AB of length 600 metres. The angles of elevation of H from A and B are 7° and 13° respectively. The point C lies on the road directly below H.



(i) Use the sine rule to show that $HB = \dfrac{600\sin 7^\circ}{\sin 160^\circ}$ **2**

(ii) Hence find the height CH of the helicopter above the road, correct to the nearest metre. **2**

Yr 11 – Mathematics – Task 2 (2006)

Q1 a) $\sqrt[4]{1.6 \times 2.6} = 1.428$ (1)

b) $\frac{13^5}{11^6 + 17^4} = 0.200$ (1)

c) $0.01072 = 1.072 \times 10^{-2}$ (1)

d) $\sqrt{18} + \sqrt{8} = 3\sqrt{2} + 2\sqrt{2}$
$= 5\sqrt{2}$ (2)

e) Rational no's. are
$\sqrt{9}$, -0.16 (2)

f) (i) $x = 0.3\dot{6}$
$10x = 3.666\ldots$
$100x = 36.666\ldots$
$100x - 10x = 36.66\ldots - 3.66\ldots$
$90x = 33$
$x = \frac{33}{90} = \frac{11}{30}$. (3)

(ii) $x = 0.\dot{2}\dot{9}\dot{7}$
$x = 0.297297\ldots$
$1000x = 297.297292\ldots$
$1000x - x = 297.297\ldots - 0.297\ldots$
$999x = 297$
$x = \frac{297}{999} = \frac{11}{37}$ (3)

Q2 a) i) $\sqrt{500} = \sqrt{100}\sqrt{5}$
$= 10\sqrt{5}$ (2)

ii) $3\sqrt{2} + 3\sqrt{8} - \sqrt{50}$
$= 3\sqrt{2} + 6\sqrt{2} - 5\sqrt{2}$
$= 4\sqrt{2}$. (2)

b) i) $(2\sqrt{3} - 1)(3\sqrt{3} + 5)$
$= 6\sqrt{9} + 10\sqrt{3} - 3\sqrt{3} - 5$
$= 18 + 7\sqrt{3} - 5$
$= 13 + 7\sqrt{3}$. (2)

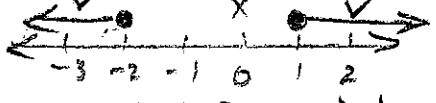
ii) $(5\sqrt{2} - 3)^2$
$= 25\sqrt{4} - 30\sqrt{2} + 9$
$= 59 - 30\sqrt{2}$ (3)

c) i) $\frac{5}{2\sqrt{7}} = \frac{5}{2\sqrt{7}} \times \frac{\sqrt{7}}{\sqrt{7}}$
$= \frac{5\sqrt{7}}{14}$. (2)

ii) $\frac{3}{\sqrt{11} + \sqrt{5}} \times \frac{\sqrt{11} - \sqrt{5}}{\sqrt{11} - \sqrt{5}}$
$= \frac{3\sqrt{11} - 3\sqrt{5}}{11 - 5}$
$= \frac{\sqrt{11} - \sqrt{5}}{2}$ (3)

d) i) $2x - 5 = 3$ $\quad 2x - 5 = -3$
$2x = 8$ $\quad 2x = 2$
$x = 4$ $\quad x = 1$ (2)

ii) $2 + 4x = 6$ $\quad 2 + 4x = -6$
$4x = 4$ $\quad 4x = -8$
$x = 1$ $\quad x = -2$



(3)

$\therefore x \leq -2, \quad x \geq 1$

Q5 a) $\sin 38°24' = 0.6211$. (1)

b) $\cos\theta = \frac{1}{4}$

$\theta = \cos^{-1}\left(\frac{1}{4}\right)$

$\theta = 75°31'$ (2)

c) $\sin 42° = \frac{x}{16}$

$x = 16\sin 42°$

$x = 10.7060897..$ (2)

$x = 10.71$ (to 2 dec pl.)

d) $\tan\theta = \frac{5}{3}$

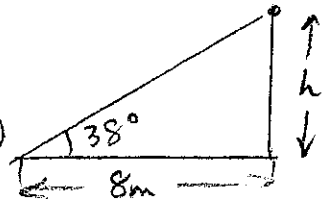
$\theta = \tan^{-1}(5/3)$

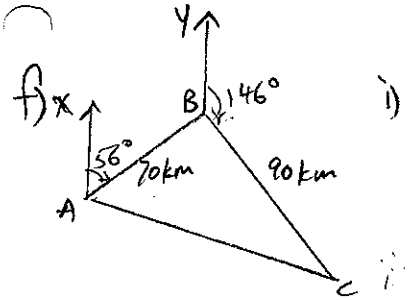
$\theta = 59°02'$ (2)

e) $\tan 38° = \frac{h}{8}$

$h = 8\tan 38°$

$h = 6.25$ m (2)



f) 

i) $\angle ABY = 124°$ (co-int. $\angle$s. // lines)

$\angle ABC = 360 - 124 - 146$ (revolution)

$\angle ABC = 90°$ (1)

ii) Since $\angle ABC = 90°$

$70^2 + 90^2 = AC^2$

$AC^2 = 13000$

$AC = \sqrt{13000}$ (2)

$AC \doteq 114.018$ km (to nearest metre)

iii) $\tan\angle BAC = \frac{90}{70}$

$\angle BAC = 52°$ (to nearest degree)

$\therefore$ Bearing A to C

$= 56 + 52$

$= 108°$ (2)

Q6 a) (i) $\tan 240° = \tan 60$

$= \sqrt{3}$. (1)

(ii) $\sin 315° = -\sin 45$

$= -\frac{1}{\sqrt{2}}$ (1)

b) $\frac{1}{\cot\theta} = \frac{1}{\frac{1}{\tan\theta}} = \tan\theta$ (1)

c) (i) $\tan\theta\,\text{cosec}\,\theta = \frac{\sin\theta}{\cos\theta} \cdot \frac{1}{\sin\theta}$

$= \frac{1}{\cos\theta}$

$= \sec\theta$. (2)

(ii) $4\sec^2\theta - 3 = 4(\tan^2\theta + 1) - 3$

$= 4\tan^2\theta + 4 - 3$

$= 1 + 4\tan^2\theta$ (2)

d) (i) $\sqrt{2}\sin x + 1 = 0$

$\sin x = -\frac{1}{\sqrt{2}}$

$x = 225°, 315°$ (2)

(ii) $\cos 2x = \frac{1}{2}$ $\quad 0 \le x \le 360$

$\therefore 0 \le 2x \le 720$

$\therefore 2x = 60°, 300°, 420°, 660°$

$x = 30°, 150°, 210°, 330°$ (3)

(iii) $\sin^2 x + \sin x = 0$

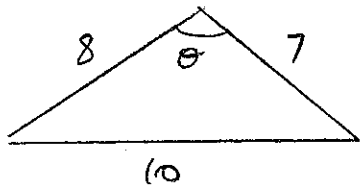
$\sin x(\sin x + 1) = 0$

$\therefore \sin x = 0$ or $\sin x = -1$

$\therefore x = 0, 180°, 360°$ $\quad x = 270°$

$\therefore x = 0, 180°, 270°, 360°$ (3)

(Q6) e)



$$\cos\theta = \frac{8^2+7^2-10^2}{2(8)(7)}$$

$$\cos\theta = \frac{13}{112}$$

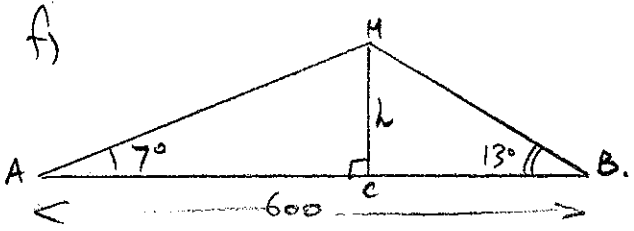
$$\theta = 83°20'04''$$

$$\therefore \text{Area} = \frac{1}{2}(8)(7)\sin 83°20'04''$$

$$= 27.81.\ cm^2$$

Area $\doteq 28\ cm^2$ (to nearest cm^2) (4)

f)



$\therefore \angle AHB = 180 - 13 - 7$

$= 160°$

i) $\therefore \frac{HB}{\sin 7°} = \frac{600}{\sin 160°}$

$\therefore HB = \frac{600\sin 7°}{\sin 160°}$ (2)

ii) $\therefore \sin 13° = \frac{h}{HB}$

$h = HB\sin 13°$

$h = \frac{600\sin 7°\sin 13°}{\sin 160°}$

$h = 48.09...$

$h \doteq 48\ m$ (to nearest metre) (2)

Q3 a) i) $-4x^2(x+3) - 2x^2(x-1)$

$= -4x^3 - 12x^2 - 2x^3 + 2x^2$

$= -6x^3 - 10x^2$ (2)

ii) $(3x-5)(2x-3)$

$= 6x^2 - 9x - 10x + 15$

$= 6x^2 - 19x + 15$ (2)

b) i) $36 - 25g^2 = (6-5g)(6+5g)$ (2)

ii) $i^2 + 5i - 36 = (i+9)(i-4)$ (2)

iii) $3k^2 - 7k - 6 = (3k+2)(k-3)$ (3)

$3k \times +2$, $k \times -3$ | $2k$, $-9k$ | $-7k$

iv) $n^3 + 8 = (n+2)(n^2 - 2n + 4)$ (3)

v) $u^2w + vw - u^2x - vx$

$= w(u^2+v) - x(u^2+v)$

$= (u^2+v)(w-x)$ (3)

c) $\frac{\sqrt{5}}{\sqrt{5}-2} \times \frac{(\sqrt{5}+2)}{(\sqrt{5}+2)} = \frac{5+2\sqrt{5}}{5-4}$

$\frac{5}{\sqrt{5}-2} = 5 + 2\sqrt{5}$.

$\therefore p = 5, \; q = 2$ (2)

Q4 a) i) $\frac{3a}{2b} + \frac{2a}{3b} = \frac{9a}{6b} + \frac{4a}{6b}$

$= \frac{13a}{6b}$. (2)

ii) $\frac{2}{x+1} - \frac{5}{x-4} = \frac{2(x-4) - 5(x+1)}{(x+1)(x-4)}$

$= \frac{2x - 8 - 5x - 5}{(x+1)(x-4)}$

$= \frac{-3x - 13}{(x+1)(x-4)}$ (3)

b) i) $\frac{2x - 2y}{x^2 - y^2} = \frac{2(x-y)}{(x+y)(x-y)}$

$= \frac{2}{x+y}$. (2)

ii) $\frac{3x^2 - 19x - 14}{9x^2 - 4} = \frac{(3x+2)(x-7)}{(3x+2)(3x-2)}$

$(3x \times +2)$, $(x \times -7)$

$= \frac{x-7}{3x-2}$ (3)

c) (i) $3(x+5) = 17$

$3x + 15 = 17$

$3x = 2$

$x = \frac{2}{3}$ (2)

(ii) $\frac{a}{7} - 4 = \frac{a}{2} + 11 \quad (\times 14)$

$2a - 56 = 7a + 154$

$5a = -210$

$a = -42$ (3)