GIRRAWEEN HIGH SCHOOL

EXAMINATION

2007

MATHEMATICS

Time allowed – 90 minutes

Year 11

HALF YEARLY

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.

- All necessary working should be shown in every question.
Marks may be deducted for careless or badly arranged work.

- Board-approved calculators may be used.

- Each question attempted is to be returned on a *separate* piece of paper clearly marked Question
1, Question 2, etc. Each piece of paper must show your name.

Question 1 (12 Marks)   **Marks**

(a) Find $(1.7)^{23} \div 0.034$ expressing your answer in scientific notation   **2**

    correct to 4 significant digits.

(b) Evaluate $\sqrt[3]{\dfrac{6.7+4.3}{9.6-1.5}}$ correct to 3 decimal places.   **2**

(c) $\sqrt{1800} = x\sqrt{2}$ Find the value of x.   **2**

(d) Write each of the following decimals as fractions in lowest terms

    (i) $0.6\dot{3}$ **3**

    (ii) $0.\dot{4}0\dot{4}$ **3**

Question 2 (24 Marks)

(a) Expand and simplify

    (i) $(\sqrt{3}+2\sqrt{7})(\sqrt{14}-\sqrt{6})$ **3**

    (ii) $(2\sqrt{7}-3\sqrt{2})^2$ **3**

(b) Express with a rational denominator

$$\dfrac{3\sqrt{2}-2\sqrt{3}}{5\sqrt{5}+\sqrt{7}}$$ **3**

(c) Factorise fully

    (i) $8-18x^2$ **2**

    (ii) $x^2-7x+12$ **2**

    (iii) $3x^2-10x+3$ **3**

    (iv) $8-27x^3$ **3**

(d) Simplify

    (i) $\dfrac{3a}{7}\times\dfrac{14}{2a^2}$ **2**

    (ii) $\dfrac{x^2-6x+8}{x-2}\times\dfrac{x^2-9}{x^2-7x+12}$ **3**

3

Question 3 (22 Marks)   **Marks**

   (a) Simplify

     (i) $\dfrac{3}{x^2-9}-\dfrac{2}{x-3}$ 3

     (ii) $\dfrac{3x+3y}{x^2-y^2}$ 2

   (b) Solve the following

     (i) $3(2x-5)=6-x$ 2

     (ii) $\dfrac{x-7}{2}\le 3+\dfrac{2x}{3}$ 3

     (iii) $|2x-3|=5$ 3

     (iv) $|3x-5|\ge 4$ 3

   (c) Solve the simultaneous equations

     (i) $2a+3b=4$

              $a-b=7$ 3

     (ii) $y=x^2-3x$

           $y+2x=2$ 3

Question 4 (12 Marks)

   (a) Find correct to 3 decimal places

     (i) $\tan 48^0 07'$ 1

     (ii) $\sin 69^0 31'$ 1

   (b) Find θ in degrees and minutes if $0\le\theta\le 90^0$ and
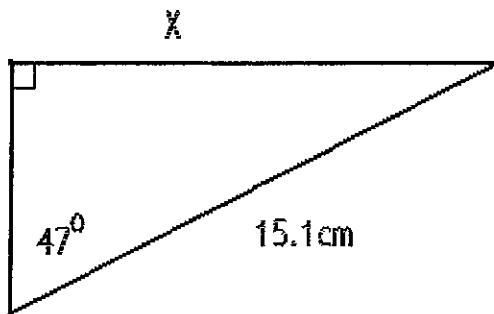
     (i) $\cos\theta=\dfrac{\sqrt{5}}{4}$ 2

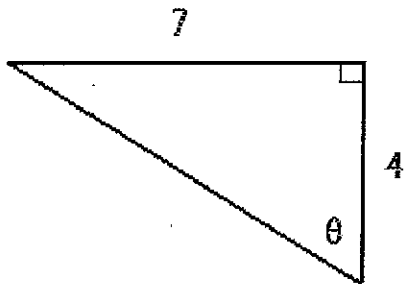   (ii) $\sec\theta=\dfrac{3}{2}$ 2

4

(c) (i) Find the value of x to one decimal place 3

(ii) Find θ to the nearest degree 3

Question 5 (16 Marks)

(a) Find the exact value of

    (i)   cos 30 + sin 30 1

    (ii) $\tan^2 60$ 1

    (iii)   2 sec 30 1

    (iv)   tan 240 1

    (v)   cos 135 1

    (vi)   2sin 30 cos 30 – sin 60 2

(b) Solve the following equations for θ , $0 \le \theta \le 360^0$

    (i) $\sin^2 \theta = \dfrac{1}{4}$ 3

    (ii) $\tan^2 \theta = \dfrac{1}{3}$ 3

    (iii) $6\cos^2 \theta + \cos\theta - 1 = 0$ 3

5

Question 6 (18 Marks)

(a) From the top of a tall building, the angle of depression to a point 60m

from and on the same level as the base of the building is 82^0.   **3**

Find the height of the building to the nearest metre.

(b) A yacht sails from Sydney (S) on a bearing of 060^0 for 70km to a point L.

It then sails on a bearing of 173^0 for 50km to H.

     (i)   Draw a neat sketch of the course showing all

             the given information   **3**

     (ii)   Explain why the $\angle SLH$ is 67^0 **2**

     (iii)   Using the Cosine Rule find the distance from H

             back to Sydney.   **4**

     (iv)   Using the Sine Rule find the angle $\angle LHS$ **4**

     (v)   Hence or otherwise find the bearing back to Sydney from H. **2**

Question 7 (9 Marks)

(a) Prove the following identities

     (i) $\cos^4 \theta(\tan^2 \theta +1) =1-\sin^2 \theta$ **2**

     (ii) $\dfrac{\sin \theta}{1-\cos\theta}+\dfrac{\sin \theta}{1+\cos\theta} = 2\sec \theta \cot \theta$ **3**

(b) Draw a graph of the function y = cos x. On the same axes graph the

function y = sec x, $0 \le x \le 180$ **4**

6

Q1 (12 MARKS)

a) $(1.7)^{23} \div 0.034 = 5872814.383$

$$= 5.873 \times 10^6 \quad \text{(2)}$$

b) $\sqrt[3]{\dfrac{6.7 + 4.3}{9.6 - 1.5}} = 1.10739 \quad \underline{CALC}$

$$= 1.107 \quad \text{(2)}$$

c) $\sqrt{1800} = \sqrt{900 \times 2}$

$$= 30\sqrt{2}$$

$$\therefore \quad x = 30 \quad \text{(2)}$$

i) $x = 0.6\dot{3}$

$10x = 6.3\dot{3}$

$9x = 5.7$

$x = 5\dfrac{7}{90}$

$$x = \dfrac{19}{30} \quad \text{(3)}$$

(ii) $x = 0.\dot{4}0\dot{4}$

$1000x = 404.\dot{4}0\dot{4}$

$x = 0.\dot{4}0\dot{4}$

$999x = 404$

$$x = \dfrac{404}{999} \quad \text{(3)}$$

Q2 (24 MARKS)

(a) (i) $(\sqrt{3} + 2\sqrt{7})(\sqrt{14} - \sqrt{6})$

$= \sqrt{42} - 3\sqrt{2} + 14\sqrt{2} - 2\sqrt{42}$

$= 11\sqrt{2} - \sqrt{42} \quad \text{(3)}$

(ii) $(2\sqrt{7} - 3\sqrt{2})^2 = 28 - 12\sqrt{14} + 18$

$= 46 - 12\sqrt{14} \quad \text{(3)}$

(b) $\dfrac{3\sqrt{2} - 2\sqrt{3}}{5\sqrt{5} + \sqrt{7}} \times \dfrac{5\sqrt{5} - \sqrt{7}}{5\sqrt{5} - \sqrt{7}}$

$= \dfrac{15\sqrt{10} - 3\sqrt{14} - 10\sqrt{15} + 2\sqrt{21}}{(5\sqrt{5})^2 - (\sqrt{7})^2}$

$= \dfrac{15\sqrt{10} - 3\sqrt{14} - 10\sqrt{15} + 2\sqrt{21}}{118} \quad \text{(3)}$

(c) (i) $8 - 18x^2 = 2(4 - 9x^2)$

$= 2(2 - 3x)(2 + 3x) \quad \text{(2)}$

(ii) $x^2 - 7x + 12 = (x - 3)(x - 4) \quad \text{(2)}$

(iii) $3x^2 - 10x + 3 = (3x - 1)(x - 3) \quad \text{(3)}$

(iv) $8 - 27x^3 = (2 - 3x)(4 + 6x + 9x^2) \quad \text{(3)}$

(d) (i) $\dfrac{3a}{7} \times \dfrac{14}{2a^2} = \dfrac{3}{a} \quad \text{(2)}$

(ii) $\dfrac{x^2 - 6x + 8}{x - 2} \times \dfrac{x^2 - 9}{x^2 - 7x + 12}$

$= \dfrac{(x - 4)(x - 2)(x - 3)(x + 3)}{(x - 2)(x - 3)(x - 4)}$

$= x + 3 \quad \text{(3)}$

Question 3   22 MARKS

(i) $\dfrac{3}{x^2-9} - \dfrac{2}{x-3}$

$= \dfrac{3 - 2(x+3)}{x^2-9}$

$= \dfrac{-2x-3}{x^2-9}$ ③

(ii) $\dfrac{3x+3y}{x^2-y^2} = \dfrac{3(x+y)}{(x-y)(x+y)}$

$= \dfrac{3}{x-y}$ ②

(i) $3(2x-5) = 6-x$

$6x - 15 = 6-x$

$7x = 21$

$x = 3$ ②

(ii) $\dfrac{x-7}{2} \le 3 + \dfrac{2x}{3}$

$3(x-7) \le 2(9+2x)$

$3x - 21 \le 18 + 4x$

$-39 \le x$

$x \ge -39.$ ③

iii) $|2x-3| = 5.$

$2x - 3 = 5$ OR $2x - 3 = -5.$

$2x = 8$ $2x = -2$

$x = 4$ $x = -1$ ③

iv) $|3x-5| \ge 4$

$3x - 5 \ge 4$ OR $3x - 5 \le -4$

$3x \ge 9$ $3x \le 1$

$x \ge 3$ $x \le \frac{1}{3}$ ③

(c) (i) $2a + 3b = 4$ ①
$a - b = 7$ ②

FROM ② $a = 7+b$

SUBST ① $2(7+b) + 3b = 4$

$14 + 5b = 4$

$b = -2$ ③

$\therefore a = 5$

(ii) $y = x^2 - 3x$ ①

$y = -2x + 2$ ②

$x^2 - 3x = -2x + 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = -1, 2$

$y = 4, -2$

$(-1, 4)$ $(2, -2)$.   ③

Question 4 . (12 MARKS)

(a)(i) $\tan 48°07' = 1.115.$ ①

(ii) $\sin 69°31' = 0.937.$ ①

(b) (i) $\cos\theta = \dfrac{\sqrt 5}{4}$ $\theta = 56°1'$ ②

(ii) $\sec\theta = \frac{3}{2}$ $\cos\theta = \frac{2}{3}$

$-\theta = 48°11'$ ②

(c) (i) $\sin 47 = \dfrac{x}{15.1}$

$x = 15.1 \sin 47$

$x = 11.0$ cm   ③

(ii) $A\tan\theta = 7/4$

$\theta = 60°$ ③

Question 5.   16 MARKS

a) (i) $\cos 30 + \sin 30$

$= \dfrac{\sqrt{3}}{2} + \dfrac{1}{2} = \dfrac{\sqrt{3}+1}{2}$. (1)

(ii) $\tan^2 60 = (\sqrt{3})^2$

$= 3.$ (1)

(iii) $2 \sec 30 = \dfrac{2}{\cos 30}$

$= \dfrac{2}{\sqrt{3}/2}$

$= \dfrac{4}{\sqrt{3}}$ (1)

(iv) $\tan 240 = \tan(180+60)$

$= \tan 60$

$= \sqrt{3}$ (1)

(v) $\cos 135 = \cos(180-45)$

$= -\cos 45$

$= -\dfrac{1}{\sqrt{2}} = -\dfrac{\sqrt{2}}{2}$ (1)

(vi) $2\sin 30 \cdot \cos 30 - \sin 60$

$= 2 \times \dfrac{1}{2} \times \dfrac{\sqrt{3}}{2} - \dfrac{\sqrt{3}}{2}$

$= 0.$ (2)

b) (i) $\sin^2 \theta = \dfrac{1}{4}$

$\sin \theta = \pm \dfrac{1}{2}$

$\theta = 30, 150, 210, 330.$ (3)

(ii) $\tan^2 \theta = \dfrac{1}{3}$

$\tan \theta = \pm \dfrac{1}{\sqrt{3}}$

$\theta = 30, 150, 210, 330.$ (3)

(iii) $6\cos^2 \theta + \cos\theta - 1 = 0$
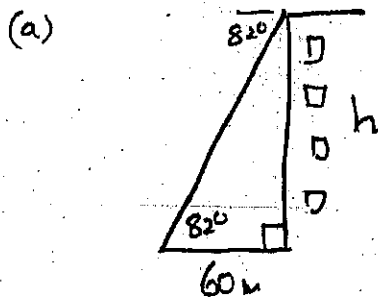
$(3\cos\theta - 1)(2\cos\theta + 1) = 0$

$\cos\theta = \dfrac{1}{3}$ or $-\dfrac{1}{2}$

$\theta = 70.5°, 289.5°$

$\theta = 120°, 240°.$ (3)

Question 6.   18 MARKS

(a)

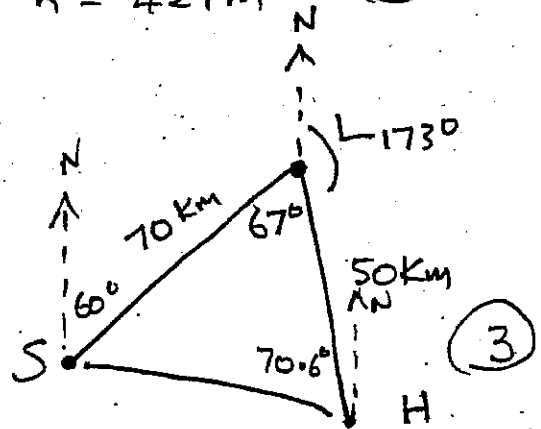$\tan 82 = \dfrac{h}{60}$

$h = 60 \tan 82$

$h = 426.9$

$h = 427m$ (3)

(b) (i)

(ii) $\angle SLN = 120°$ $\angle NLH = 173°$

$\therefore \angle SLH + 120° + 173° = 360°$

$\angle SLH = 67°$ (2)

(iii) $\ell^2 = h^2 + s^2 - 2hs \cos L$

$\ell^2 = 70^2 + 50^2 - 7000 \cos 67$

$\ell^2 = 4664.88$

$\ell = 68.3 km.$ (4)

(iv) $\dfrac{\sin H}{h} = \dfrac{\sin L}{\ell}$

$\dfrac{\sin H}{70} = \dfrac{\sin 67}{68.3}$

Question 6   cont.

$\therefore \sin H = 0.9434$

$H = 70.6°$ (4)

1) $\angle LHN = 7°$ $\angle LHS = 70°.6$

$\therefore$ Bearing H to Sydney

$= 360 - 77.6$

$= 282.4°$

Question 7 (9 marks)   (2)

1) (i) $\cos^4\theta(\tan^2\theta + 1) = LHS$

$= \cos^2\theta(\sin^2\theta + \cos^2\theta)$

$= \cos^2\theta$

$= 1 - \sin^2\theta = RHS$ (2)

(ii) $\dfrac{\sin\theta}{1-\cos\theta} + \dfrac{\sin\theta}{1+\cos\theta} = RHS$

$= \dfrac{\sin\theta(1+\cos\theta) + \sin\theta(1-\cos\theta)}{1 - \cos^2\theta}$

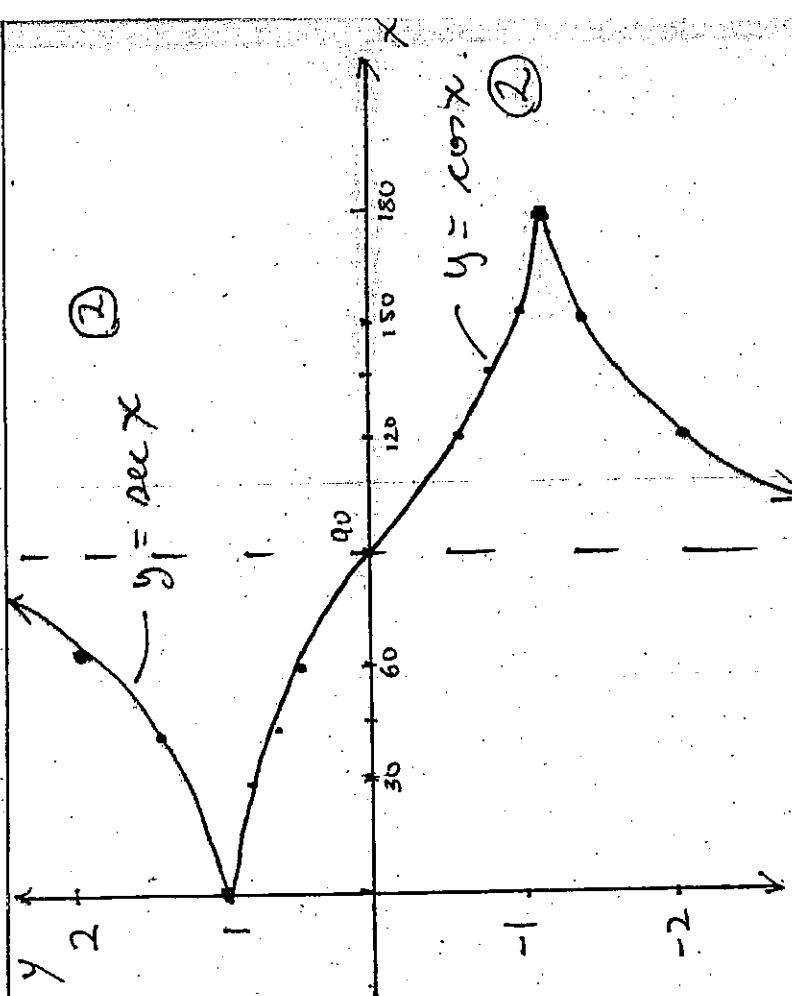$= \dfrac{\sin\theta + \sin\theta\cos\theta + \sin\theta - \sin\theta\cos\theta}{1 - \cos^2\theta}$

$= \dfrac{2\sin\theta}{\sin^2\theta}$

$= \dfrac{2}{\sin\theta}$

$= \dfrac{2\cos\theta}{\sin\theta\cos\theta}$

$= 2\cot\theta\sec\theta = RHS.$

(3)

7(b)   (2)

$y = \sec x$

$y = \cos x$ (2)