

**GIRRAWEE HIGH SCHOOL**

Year 11

**MATHEMATICS**

May 2008

Task 2

Time: 90 mins

**INSTRUCTIONS:**

- Attempt all questions
- Write your answers on your own paper.
- All necessary working must be shown.
- Marks will be deducted for careless or badly arranged work.

**Question 1 (12 marks)**

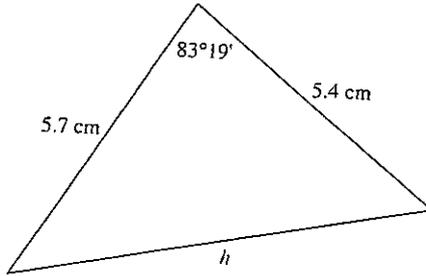
- (a) Find correct to 3 decimal places: 2  
(i)  $\sin 39^\circ 25'$                       (ii)  $\tan 218^\circ 43'$
- (b) Find  $\theta$  correct to the nearest minute: 2  
(i)  $\cos \theta = 0.827$                       (ii)  $\tan \theta = 1.056$
- (c) The angle of elevation of a tower is  $39^\circ 44'$  when measured at a point 100m from its base. Find the height of the tower, correct to 1 decimal place. 2
- (d) A plane flies north from Sydney for 560 km, then turns and flies west for 390 km. What is its bearing from Sydney, to the nearest degree. 3
- (e) If  $\sin \theta = \frac{4}{7}$  and  $\tan \theta < 0$ , find the exact value of  $\cos \theta$  and  $\tan \theta$ . 3

**Question 2 (28 marks)**

- (a) Find the exact value of : 3  
(i)  $\sin 300^\circ$                       (ii)  $\cos 225^\circ$                       (iii)  $\tan (-150^\circ)$
- (b) Solve for  $0^\circ \leq \theta \leq 360^\circ$   
(i)  $\cos \theta = \frac{1}{2}$                       (ii)  $\tan \theta = -\frac{1}{\sqrt{3}}$                       (iii)  $4 \sin^2 \theta = 1$   
(iv)  $\cos^2 \theta = 1 - 2 \sin^2 \theta$   
(v)  $\cos 2\theta = 0.4684$  (write answer to the nearest minute) 15
- (c) Simplify:  
(i)  $2 - 2 \cos^2 \theta$                       (ii)  $\sec^2 \theta - \tan^2 \theta$  4
- (d) Sketch  $y = \sin x$ ,  $-360^\circ \leq x \leq 360^\circ$  3
- (e) Prove the following identity:  
 $1 - 2 \sin^2 \theta = 2 \cos^2 \theta - 1$  3

**Question 3 (14 marks)**(a) Find the value of  $h$  correct to one decimal place.

3



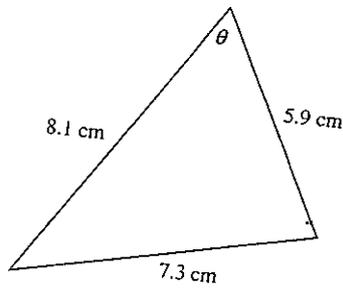
(b) Find

(i)  $\theta$  to the nearest minute.

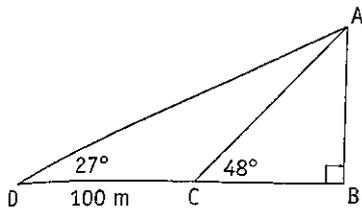
3

(ii) the area of  $\triangle ABC$  (correct to one decimal place)

3



(c)

(i) Show that  $AC = \frac{100 \sin 27^\circ}{\sin 21^\circ}$ 

2

(ii) Calculate the length of  $AB$  correct to one decimal place.

3

**Question 4** (18 marks)

(a) Solve:

(i)  $|y - 8| = 20$

(ii)  $|7x - 3| = |4x - 5|$

6

(b) Solve and graph on a number line:  $|3x + 2| > 4$ 

3

(c) Solve the quadratic inequality:  $2x^2 - 13x - 7 < 0$ 

3

(d) Solve:

(i)  $144^x = 12$

(ii)  $\left(\frac{1}{16}\right)^x = \frac{1}{2}$

6

**Question 5** (18 marks)

(a) Find the centre and radius of the circle

(i)  $(x + 2)^2 + (y - 3)^2 = 144$

2

(ii)  $x^2 + y^2 - 6x + 3y - 1 = 0$

4

(b) If  $f(x) = x^2 + 1$ , find

(i)  $f(-2)$

(ii)  $f(0)$

(iii)  $f(\sqrt{x})$

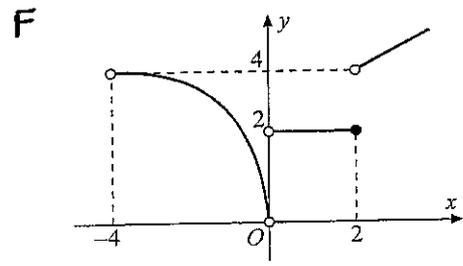
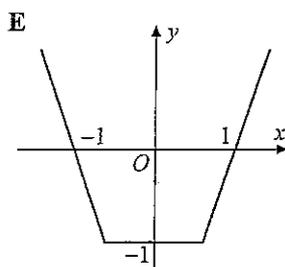
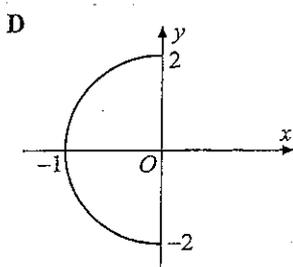
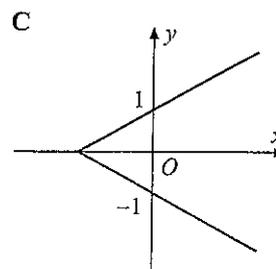
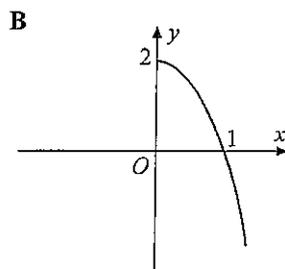
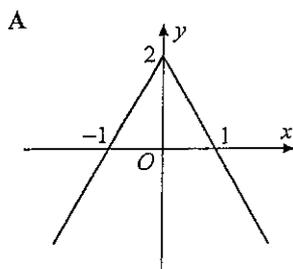
3

(c) Given  $f(x) = \frac{x}{x-1}$ . Find  $f\left(\frac{1}{x}\right)$  in the simplest form.

3

(d) Determine whether the following curves are functions or not. Indicate your answer by writing either yes or no.

6



**Question 6** (12 marks)

(a) Given  $f(x) = \frac{12}{ax+b}$ ,  $f(0) = 3$  and  $f(2) = -3$ . Find

the values of  $a$  and  $b$ .

4

(b) A function is defined by the rule

$$g(x) = \begin{cases} 4 & x \leq -2 \\ x^2 & -2 < x < 2 \\ -x + 6 & x \geq 2 \end{cases}$$

Evaluate  $g(-3) + g(1) - g(4)$ .

3

(c) Show that  $f(x) = x^3 - 16x$  is an odd function. Hence find  $f(x) - f(-x)$ .

5

**Question 7** (26 marks)

(a) For each of the following curves ( $\alpha$ ), ( $\beta$ ), ( $\gamma$ ) and ( $\delta$ )

- (i) Draw a neat sketch.
- (ii) Name the type of curve
- (iii) State the domain and range.

( $\alpha$ )  $y = (x-1)^2 + 1$

( $\beta$ )  $y = |x| - 3$

( $\gamma$ )  $y = 2^x$

( $\delta$ )  $y = \frac{1}{x+4}$

16

(b) Find the domain of the following:

(i)  $y = \sqrt{x-5}$

(ii)  $y = \sqrt{64-x^2}$

4

(c) Sketch the following regions.

(i)  $x^2 + y^2 < 25$

2

(ii)  $x + y \geq 0$  and  $y \leq 1 - x^2$

4

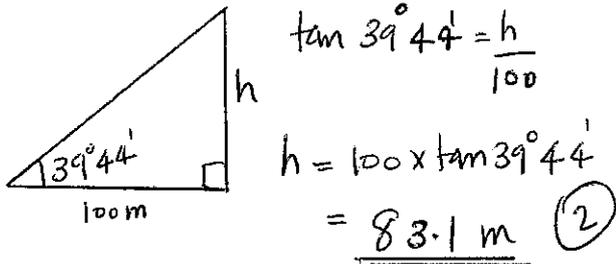
# Year 11 Mathematics Task 2, 2008 - Solutions

## Question 1 (12 marks)

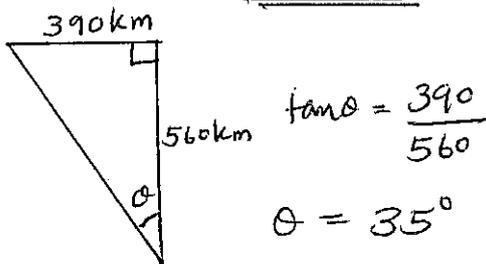
(a) (i) 0.635    (ii) 0.802    (2)

(b) (i)  $34^{\circ}12'$     (ii)  $46^{\circ}34'$     (2)

(c)



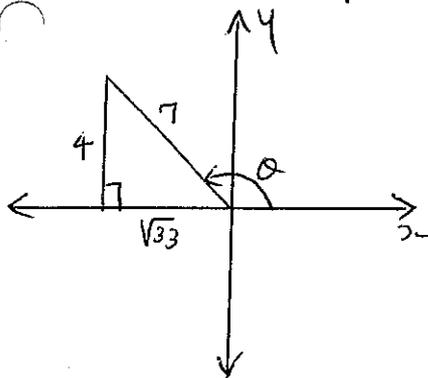
(d)



bearing =  $360^{\circ} - 35^{\circ}$

$= \underline{325^{\circ}}$     (3)

(e)  $\sin \theta$  is positive and  $\tan \theta$  is negative  $\Rightarrow \theta$  is in the 2nd quadrant



$\cos \theta = \underline{\underline{-\frac{\sqrt{33}}{7}}}$     (3)

$\tan \theta = \underline{\underline{-\frac{4}{\sqrt{33}}}}$

## Question 2 (28 marks)

(a) (i)  $\sin 300^{\circ} = -\sin 60$

$= -\frac{\sqrt{3}}{2}$     (1)

(ii)  $\cos 225^{\circ} = -\cos 45$

$= -\frac{1}{\sqrt{2}}$     (1)

(iii)  $\tan (-150^{\circ}) = -\tan 150^{\circ}$

$= \frac{1}{\sqrt{3}}$     (1)

(b) (i)  $\cos \theta = \frac{1}{2}$

$\theta = \underline{60^{\circ}, 300^{\circ}}$     (2)

(ii)  $\tan \theta = -\frac{1}{\sqrt{3}}$

$\theta = \underline{150^{\circ}, 330^{\circ}}$     (2)

(iii)  $4 \sin^2 \theta = 1$

$\sin^2 \theta = \frac{1}{4}$

$\sin \theta = \pm \frac{1}{2}$

$\underline{\sin \theta = \frac{1}{2}} \quad \theta = 30^{\circ}, 150^{\circ}$

$\underline{\sin \theta = -\frac{1}{2}} \quad \theta = 210^{\circ}, 330^{\circ}$

$\underline{\underline{\theta = 30^{\circ}, 150^{\circ}, 210^{\circ}, 330^{\circ}}}$     (4)

$$(iv) \cos^2 \theta = 1 - 2\sin^2 \theta$$

$$1 - \sin^2 \theta = 1 - 2\sin^2 \theta$$

$$2\sin^2 \theta - \sin^2 \theta = 1 - 1$$

$$\sin^2 \theta = 0$$

$$\sin \theta = 0$$

$$\theta = \underline{0^\circ, 180^\circ, 360^\circ} \quad (3)$$

$$(v) \cos 2\theta = 0.4684$$

$$\text{Let } u = 2\theta$$

$$\cos u = 0.4684, 0 \leq u \leq 720^\circ$$

$u$  is in the 1st or 4th quadrant  
acute  $\angle u = 62^\circ 4'$

$$360^\circ - u = 297^\circ 56'$$

$$u = 62^\circ 4', 297^\circ 56', 422^\circ 4', 657^\circ 56'$$

$$\theta = \frac{u}{2}$$

$$\theta = 31^\circ 2', 148^\circ 58', 211^\circ 2',$$

$$\underline{328^\circ 58'} \quad (4)$$

$$(c) (i) 2 - 2\cos^2 \theta$$

$$= 2(1 - \cos^2 \theta) \quad (2)$$

$$= 2\sin^2 \theta$$

$$(ii) \sec^2 \theta - \tan^2 \theta$$

$$= 1 + \tan^2 \theta - \tan^2 \theta$$

$$= 1 \quad (2)$$

$$(d) y = \sin x, -360^\circ \leq x \leq 360^\circ$$

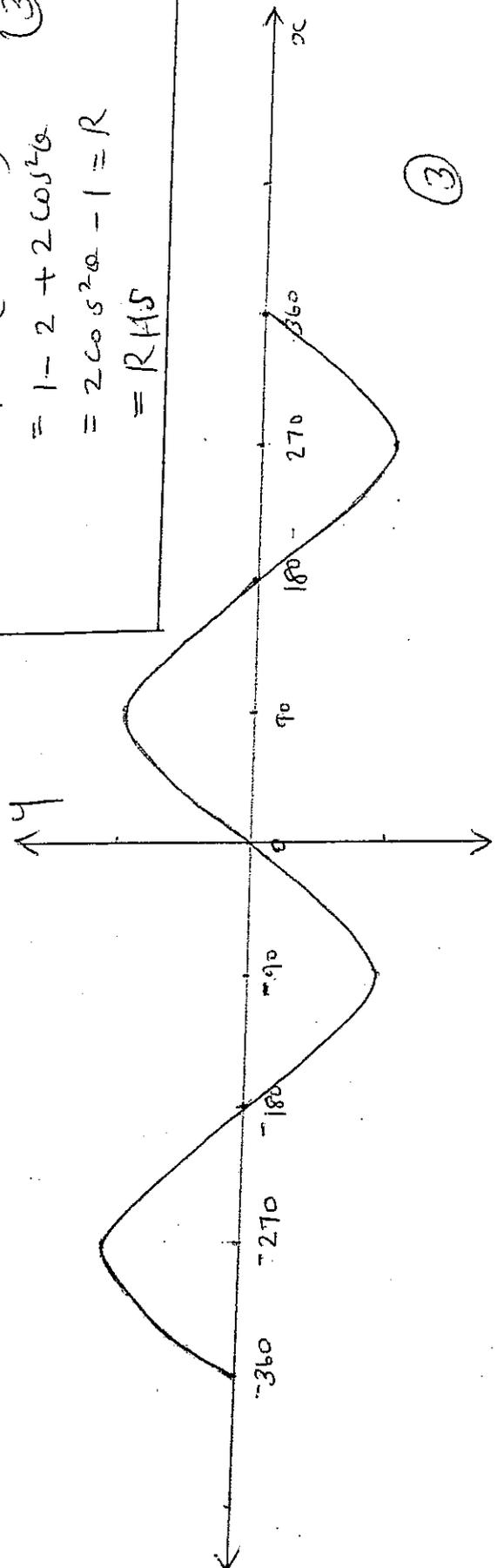
$$(e) \text{ LHS} = 1 - 2\sin^2 \theta$$

$$= 1 - 2(1 - \cos^2 \theta)$$

$$= 1 - 2 + 2\cos^2 \theta$$

$$= 2\cos^2 \theta - 1 = R$$

$$= R \text{HS}$$



### Question 3 (14 marks)

$$(a) h^2 = 5.7^2 + 5.4^2 - 2 \times 5.7 \times 5.4 \times \cos 83^\circ 19'$$

$$h = \underline{7.4 \text{ cm}} \quad (3)$$

$$(b)(i) \cos \theta = \frac{8.1^2 + 5.9^2 - 7.3^2}{2 \times 8.1 \times 5.9}$$

$$\theta = \underline{60^\circ 27'} \quad (3)$$

$$(ii) \text{Area} = \frac{1}{2} \times 8.1 \times 5.9 \times \sin 60^\circ 27'$$

$$= \underline{20.8 \text{ cm}^2} \quad (3)$$

$$(c)(i) \angle DCA = 180 - 48$$

$$= 132$$

$$\angle DAC = 180 - (132 + 27)$$

$$= 21^\circ$$

By sine rule in  $\triangle ACD$

$$\frac{AC}{\sin 27^\circ} = \frac{100}{\sin 21^\circ}$$

$$AC = \frac{100 \times \sin 27^\circ}{\sin 21^\circ} \quad (2)$$

$$(ii) \sin 48^\circ = \frac{AB}{AC}$$

$$AB = AC \sin 48^\circ$$

$$= \frac{100 \sin 27^\circ \sin 48^\circ}{\sin 21^\circ}$$

$$= \underline{94.1 \text{ m}} \quad (3)$$

Question 4 (18 marks)

(a)(i)  $|y-8|=20$

$y-8=20$  or  $-(y-8)=20$

$y-8=20$  or  $-y+8=20$

$y=28$  or  $y=-12$  (3)

(ii)  $|7x-3|=|4x-5|$

$7x-3=4x-5$  or  $7x-3=-(4x-5)$

$7x-4x=3-5$  or  $7x-3=-4x+5$

$3x=-2$  or  $11x=8$

$x = \frac{-2}{3}$  or  $\frac{8}{11}$  (3)

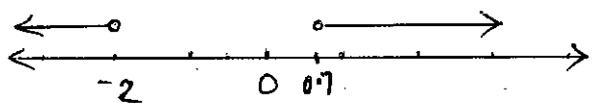
(b)  $|3x+2| > 4$

$3x+2 > 4$  or  $3x+2 < -4$

$3x > 2$  or  $3x < -6$

$x > \frac{2}{3}$  or  $x < -\frac{6}{3}$

$x > \frac{2}{3}$  or  $x < -2$  (3)

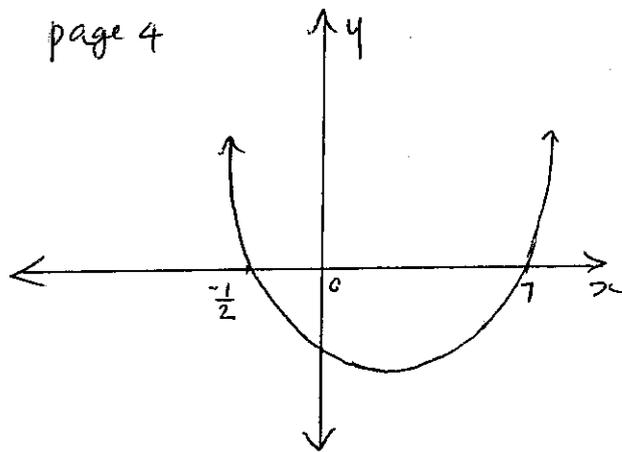


(c)  $2x^2-13x-7 < 0$

Factorise  $2x^2-13x-7=0$

$(x-7)(2x+1)=0$

$x=7$  or  $x=-\frac{1}{2}$



From the graph

$2x^2-13x-7 < 0$  (3)

when  $-\frac{1}{2} < x < 7$

d(i)  $144^x = 12$

$(12^2)^x = 12$

$12^{2x} = 12$

$2x = 1$  (3)

$x = \frac{1}{2}$

(ii)  $\left(\frac{1}{16}\right)^x = \frac{1}{12}$

$(2^{-4})^x = 2^{-1}$

$2^{-4x} = 2^{-1}$

$4x = 1$  (3)

$x = \frac{1}{4}$

(d) A. Yes B. Yes C. No

D. No E. Yes F. Yes (6)

Question 6 (12 marks)

(a)  $f(x) = \frac{12}{ax+b}$  ;  $f(0) = \frac{12}{b}$

Given  $f(0) = 3$

$$\frac{12}{b} = 3 \quad \therefore \underline{\underline{b = 4}}$$

$$f(2) = \frac{12}{2a+b} = \frac{12}{2a+4}$$

Given  $f(2) = -3$

$$\frac{12}{2a+4} = -3$$

$$-3(2a+4) = 12$$

$$-6a - 12 = 12 \quad (4)$$

$$a = \frac{-24}{6}$$

$$\underline{\underline{= -4}}$$

(b)  $g(-3) + g(1) - g(4)$

$$= 4 + 1 - 2$$

$$\underline{\underline{= 3}} \quad (3)$$

Question 5 (18 marks)

(a) (i)  $(x+2)^2 + (y-3)^2 = 144$

Centre =  $(-2, 3)$  radius = 12 (2)

(ii)  $x^2 + y^2 - 6x + 3y - 1 = 0$

$$x^2 - 6x + y^2 + 3y = 1$$

$$x^2 - 6x + 9 + y^2 + 3y + \left(\frac{3}{2}\right)^2 = 1 + 9 + \frac{9}{4}$$

$$(x-3)^2 + \left(y + \frac{3}{2}\right)^2 = \frac{49}{4}$$

$$(x-3)^2 + \left(y + \frac{3}{2}\right)^2 = \left(\frac{7}{2}\right)^2$$

Centre =  $\left(3, -\frac{3}{2}\right)$  (4)

radius =  $\underline{\underline{\frac{7}{2} \text{ units}}}$

(b)  $f(x) = x^2 + 1$

(i)  $f(-2) = (-2)^2 + 1$  (1)  
 $= 5$

(ii)  $f(0) = 1$  (1)

(iii)  $f(\sqrt{x}) = (\sqrt{x})^2 + 1$  (1)  
 $= x + 1$

(c)  $f(x) = \frac{x}{x-1}$

$$f\left(\frac{1}{x}\right) = \frac{\frac{1}{x}}{\frac{1}{x} - 1} = \frac{\frac{1}{x}}{\frac{1-x}{x}} \quad (3)$$
$$= \frac{1}{x} \times \frac{x}{1-x} = \underline{\underline{\frac{1}{1-x}}}$$

$$(c) f(x) = x^3 - 16x$$

$$f(-x) = (-x)^3 - 16(-x)$$

$$= -x^3 + 16x$$

$$= -(x^3 - 16x)$$

$$= -f(x)$$

$\therefore f(x)$  is an odd function

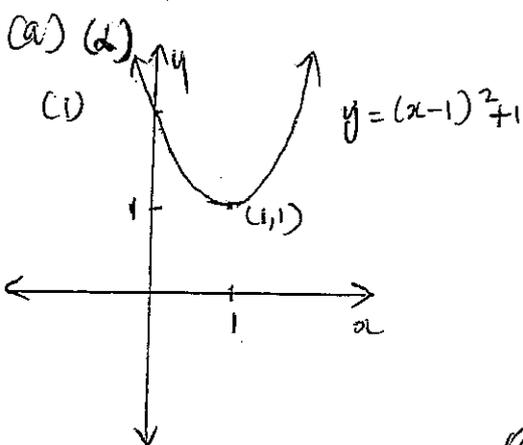
$$f(x) - f(-x) \quad (5)$$

$$= f(x) - (-f(x))$$

$$= f(x) + f(x)$$

$$= \underline{\underline{2f(x)}}$$

Question 7 (26 marks)



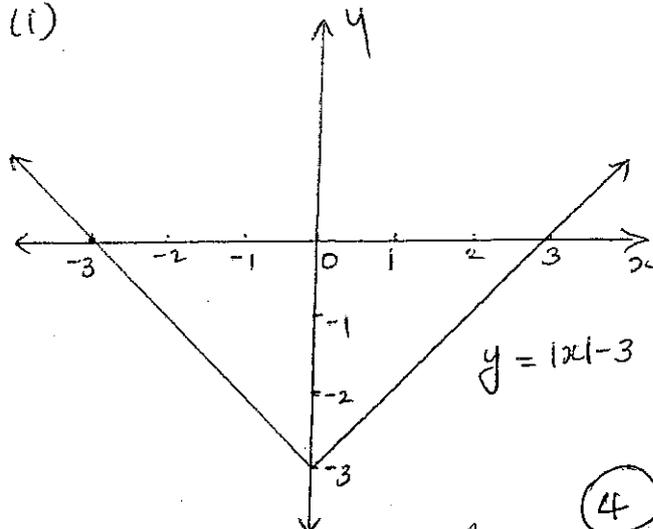
(iii) D: all real numbers

R: all real numbers,  $y \geq 1$

page 6

(4)

(B) (i)

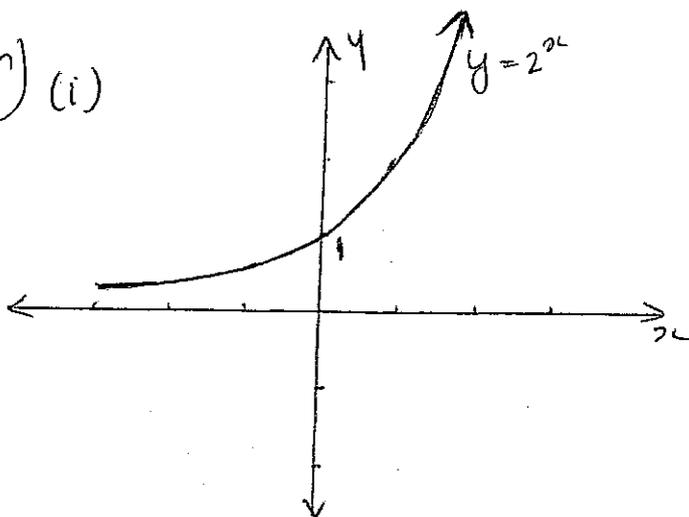


(ii) absolute value graph

(iii) D: all real numbers

R: all real numbers,  $y \geq -3$

(C) (i)

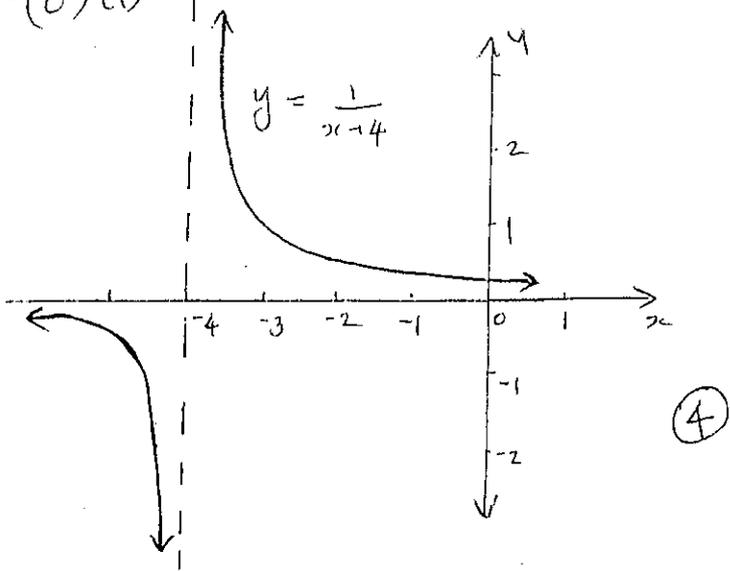


(ii) exponential graph

(iii) D: all real numbers (4)

R: all real numbers,  $y > 0$

(d) (i)



(ii) hyperbola

(iii) D: all real numbers,  $x \neq -4$

R: all real numbers,  $y \neq 0$

(b) (i)  $y = \sqrt{x-5}$

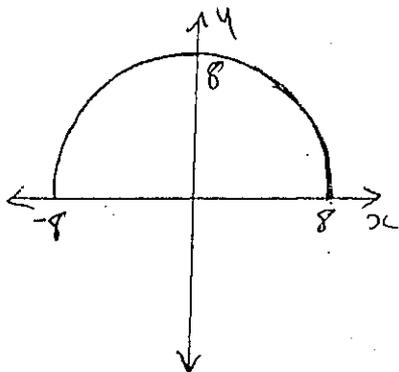
$x-5 \geq 0$

$x \geq 5$

(2)

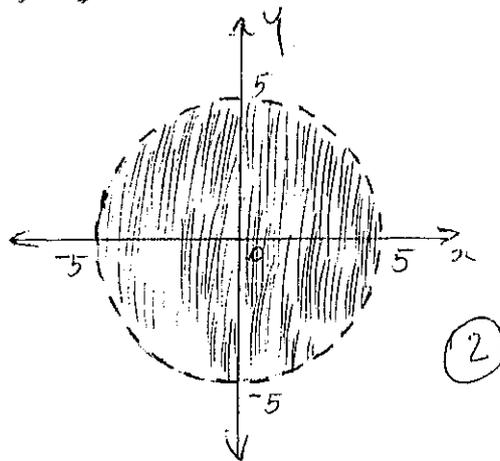
D: all real numbers,  $x \geq 5$

(ii)



D: all real numbers,  $-8 \leq x \leq 8$

(c) (i)



(ii)  $x+y \geq 0$

$y \geq -x$

