GIRRAWEEN HIGH SCHOOL

YEAR 11 – Task 2

2009

MATHEMATICS

Time allowed – 90 minutes

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.

- All necessary working should be shown in every question.
  Marks may be deducted for careless or badly arranged work.

- Board-approved calculators may be used.

- Start each question on a *new* sheet of paper.

QUESTION 1 (18 marks)   **Marks**

(a) Evaluate $\dfrac{7.63 - 4.82}{\sqrt{(8.2)^2 - 58.7}}$ correct to 3 decimal places.   **2**

(b) Write 0.00471825 in scientific notation correct to 3 significant figures.   **2**

(c) Evaluate $\left|-3\right| + \left|-2\right|^2 - \left|4 \times -5\right|$.   **2**

(d) Express $0.3\overset{\bullet}{2}\overset{\prime}{4}$ as a fraction in lowest terms.   **3**

(e) Given that $L = \dfrac{g}{4}\left(\dfrac{T}{\pi}\right)^2$, find the value of T when $L = 1.5$, $g = 9.8$ **3**

and $T > 0$. Give answer correct to 2 decimal places.

(f) Find the value of x if $\sqrt{52} + \sqrt{13} = \sqrt{x}$ **3**

(g) After GST of 10% the new price is \$12.50.   **3**
What was the price before GST?
(Round the answer to the nearest 10 cents).

QUESTION 2 (23 marks)   **Marks**

(a) Simplify:

    (i) $\sqrt{700}$ **2**

    (ii) $\sqrt{8} + \sqrt{32} - \sqrt{50}$ **3**

(b) Expand and simplify:

    (i) $\left(2\sqrt{5} - 1\right)\left(3\sqrt{5} + 4\right)$ **3**

    (ii) $\left(2\sqrt{5} - 3\right)^2$ **3**

(c) Express with a rational denominator :

    (i) $\dfrac{2}{3\sqrt{5}}$ **2**

    (ii) $\dfrac{6}{\sqrt{2} + \sqrt{5}}$ **3**

(d) Solve for x :

    (i) $|x - 3| = 7 - 2x$ **4**

    (ii) $|3x - 4| \le 2$ **3**

QUESTION 3 (21 marks)   **Marks**

(a) Expand and simplify $(4x+3y)(4x-3y)-(5x-3y)^2$.   **3**

(b) Factorise :

   (i) $16m^2-9$ **2**

   (ii) $x^2-2x-63$ **2**

   (iii) $3a^2-10a+8$ **2**

   (iv) $27-y^3$ **3**

   (v) $2ax+6a-8x-24$.   **3**

(c) Express with positive indices $a^{-3} \div 4a^{-4} \times 3a^5$.   **3**

(d) Find the values of x and y if $\dfrac{6}{\sqrt{3}+2}=x+y\sqrt{3}$.   **3**

QUESTION 4 (17 marks)

(a) Simplify:

   (i) $m+\dfrac{3}{m-1}$ **2**

   (ii) $\dfrac{7x-1}{4}-\dfrac{x+2}{3}$ **3**

(b) Simplify:

   (i) $\dfrac{3m^2+24}{12-3m^2}$ **2**

   (ii) $\dfrac{x^2+3x}{x-2}\times\dfrac{x^2-6x+8}{x^2-4x}$ **4**

(c) Solve:

   (i) $\dfrac{4x}{x+3}=\dfrac{2}{3}$ **3**

   (ii) $\dfrac{1}{3}(m+1)-\dfrac{1}{4}(m-2)=2$ **3**

QUESTION 5 (26 marks)   **Marks**

(a) Sketch the following graphs:

    (i) $y = |x| + 1$ 2

    (ii) $y = \dfrac{1}{x - 2}$ 2

    (iii) $x^2 + y^2 = 4$ 2

    (iv) $y = 2^x$ 2

(b) Which of the above graphs does not represent a function?   1

(c) State the domain and range of the following functions:

    (i) $y = x^2$ 2

    (ii) $y = \sqrt{4 - x^2}$ 2

    (iii) $y = \dfrac{1}{x - 2}$ 2

(d) A function is defined by the rule   2

$$f(x) = \begin{cases} 0 \text{ if } & x \le 0 \\ -1 \text{ if } & 0 < x < 2 \\ x \text{ if } & x \ge 2. \end{cases}$$

Evaluate $f(-1) + f(1) + f(5)$.

(e) Determine whether the function $y = 8x^3 - 7x - 7$ 3
    is even, odd, or neither. Justify your answer.

(f) Shade in the region on the number plane for which   3
    $x - 2y > 0$ and $2x + y > 0$ hold simultaneously.

(g) Find the centre and radius for the circle whose equation is   3
    $x^2 + y^2 - 4x - 14y + 17 = 0$.

QUESTION 6 (18 marks)   **Marks**

(a) Evaluate $\cos 58°19'$ correct to 3 decimal places.   2

(b) Find the value of θ in degrees and minutes if $\tan \theta = 0.348$ 2

(c) Find the value of $\angle BAC$ in degrees and minutes to the   3
   nearest minute.
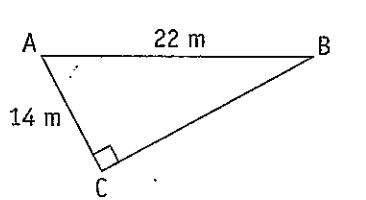
(d) Find the exact value of $\tan 300°$.   2

(e) Simplify $\sin B \cot B$.   2

(f) Prove that $\dfrac{1}{1-\sin\theta} - \dfrac{1}{1+\sin\theta} = \dfrac{2\sin\theta}{\cos^2\theta}$.   3

(g) Solve for $0° \le \theta \le 360°$

   (i) $2\cos\theta = \sqrt{3}$ 2

   (ii) $\tan\theta + 1 = 0$ 2

QUESTION 7 (13 marks)   **Marks**

(a) Amberville (A) is 35km from Bordernook (B) and on a bearing of 245°.
     Coopertown (C) is due south of Bordernook and on a bearing of 155°
     from Amberville.

    (i) Show this information on a diagram.   **3**

    (ii) Find the size of $\angle BAC$.   **2**

    (iii) How far is A from C? (Give answer to 1 decimal place).   **2**

(b) Use the sine rule to find the value of x.   **3**
     Give answer to 1 decimal place.
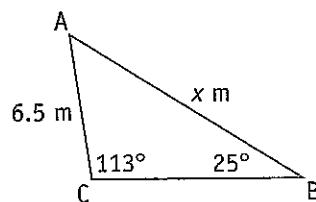
(c) Use the cosine rule to find the value of θ to the nearest whole degree.   **3**
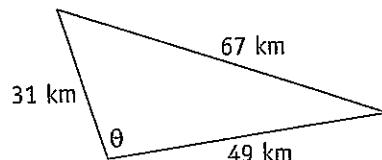
END OF TASK

Year 11, Mathematics, Task 2, 2009 Solutions

Question 1 (18 marks)

(a) $\dfrac{7.63 - 4.82}{\sqrt{8.2^2 - 58.7}} = 0.962$ ②

(b) 0.00471825
$= 4.72 \times 10^{-3}$ ②

(c) $|-3| + |-2|^2 - |4 \times -5|$
$= 3 + 4 - 20$
$= -13.$ ②

(d) Let $x = 0.324324\ldots$ (1)
$\therefore 1000x = 324.324324\ldots$ (2)
(2) − (1) gives
$999x = 324$
$\therefore x = \dfrac{324}{999}$
$x = \dfrac{12}{37}$ ③

(e) $L = \dfrac{g}{4}\left(\dfrac{T}{\pi}\right)^2$
$\therefore 1.5 = \dfrac{9.8}{4}\left(\dfrac{T}{\pi}\right)^2$
$\dfrac{6}{9.8} = \dfrac{T^2}{\pi^2}$
$\therefore T^2 = \dfrac{6\pi^2}{9.8}$ ③
$\therefore T = \pm 2.46$
$\therefore T = 2.46$ since $T > 0$

(f)
$\sqrt{52} + \sqrt{13} = \sqrt{x}$
$2\sqrt{13} + \sqrt{13} = \sqrt{x}$
$3\sqrt{13} = \sqrt{x}$
$\therefore \sqrt{117} = \sqrt{x}$
$\therefore x = 117.$ ③

(g) $110\% = \$12.50$
$\therefore 100\% = \dfrac{\$12.50}{110} \times 100$
$= \$11.36$ ③
$\therefore \$11.40.$

Question 2 (23 marks)

(a) (i) $\sqrt{700}$
$= \sqrt{100} \times \sqrt{7}$ ②
$= 10\sqrt{7}.$

(ii) $\sqrt{8} + \sqrt{32} - \sqrt{50}$
$= 2\sqrt{2} + 4\sqrt{2} - 5\sqrt{2}$
$= \sqrt{2}$ ③

(b) (i) $(2\sqrt{5} - 1)(3\sqrt{5} + 4)$
$= 6 \times 5 + 8\sqrt{5} - 3\sqrt{5} - 4$
$= 26 + 5\sqrt{5}.$ ③

(ii) $(2\sqrt{5} - 3)^2$
$= 4 \times 5 - 12\sqrt{5} + 9$
$= 29 - 12\sqrt{5}.$ ③

(c) (i) $\dfrac{2}{3\sqrt{5}} = \dfrac{2}{3\sqrt{5}} \times \dfrac{\sqrt{5}}{\sqrt{5}}$
$= \dfrac{2\sqrt{5}}{15}.$ ②

(ii) $\dfrac{6}{\sqrt{2} + \sqrt{5}} = \dfrac{6}{\sqrt{2} + \sqrt{5}} \times \dfrac{\sqrt{2} - \sqrt{5}}{\sqrt{2} - \sqrt{5}}$
$= \dfrac{6(\sqrt{2} - \sqrt{5})}{2 - 5}$
$= -2(\sqrt{2} - \sqrt{5})$ ③

(d) (i) $|x-3| = 7-2x$

$x-3 = 7-2x$ or $-(x-3) = 7-2x$

$3x = 10$ $\qquad -x+3 = 7-2x$

$\therefore x = \frac{10}{3}$ $\qquad x = 4$

Check: For $x = \frac{10}{3}$,

$LHS = |\frac{10}{3} - 3| = \frac{1}{3}$

$RHS = 7 - 2 \times \frac{10}{3} = \frac{1}{3}$

For $x = 4$,

$LHS = |4-3| = 1$.

$RHS = 7 - 2 \times 4 = -1$.

④

$\therefore x = \frac{10}{3}$ or $3\frac{1}{3}$ is the only solution.

(ii) $|3x-4| \leq 2$

$\therefore -2 \leq 3x-4 \leq 2$

$-2+4 \leq 3x \leq 2+4$

$2 \leq 3x \leq 6$ ③

$\therefore \frac{2}{3} \leq x \leq 2$

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<u>Question 3</u> (21 marks)

(a) $(4x+3y)(4x-3y) - (5x-3y)^2$

$= 16x^2 - 9y^2 - (25x^2 - 30xy + 9y^2)$

$= 16x^2 - 9y^2 - 25x^2 + 30xy - 9y^2$

$= -9x^2 - 18y^2 + 30xy$ ③

(b) (i) $16m^2 - 9$

$= (4m-3)(4m+3)$ ②

(ii) $x^2 - 2x - 63$

$= (x+7)(x-9)$ ②

(iii) $3a^2 - 10a + 8$

$= (3a-4)(a-2)$ ②

(iv) $27 - y^3$ ③

$= 3^3 - y^3$

$= (3-y)(9+3y+y^2)$

(v) $2ax + 6a - 8x - 24$

$= 2(ax + 3a - 4x - 12)$

$= 2[a(x+3) - 4(x+3)]$ ③

$= 2(a-4)(x+3)$

(c) $a^{-3} \div 4a^{-4} \times 3a^5$

$= \frac{a^{-3}}{4a^{-4}} \times 3a^5$ ③

$= \frac{1}{4a^{-1}} \times 3a^5 = \frac{3}{4}a^6$.

(d) $\frac{6}{\sqrt{3}+2} \times \frac{\sqrt{3}-2}{\sqrt{3}-2}$ ③

$= \frac{6(\sqrt{3}-2)}{3-4} = -6(\sqrt{3}-2)$

$= -6\sqrt{3} + 12$

$= y\sqrt{3} + x$

$\therefore x = 12, y = -6$.

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<u>Question 4</u> (17 marks)

(a) (i) $m + \frac{3}{m-1}$

$= \frac{m(m-1)+3}{m-1}$ ②

$= \frac{m^2 - m + 3}{m-1}$

(ii) $\frac{7x-1}{4} - \frac{x+2}{3}$

$= \frac{3(7x-1) - 4(x+2)}{12}$

$= \frac{21x - 3 - 4x - 8}{12}$

$= \frac{17x - 11}{12}$. ③

(b) (i) $\dfrac{3m^2+24}{12-3m^2}$

$= \dfrac{3(m^2+8)}{3(4-m^2)}$

$= \dfrac{m^2+8}{4-m^2}$ ②

(ii) $\dfrac{x^2+3x}{x-2} \times \dfrac{x^2-6x+8}{x^2-4x}$

$= \dfrac{x(x+3)}{x-2} \times \dfrac{(x-2)(x-4)}{x(x-4)}$

$= x+3$ ④

(c) (i) $\dfrac{4x}{x+3} = \dfrac{2}{3}$

$\therefore \quad 12x = 2(x+3)$

$12x = 2x+6$

$\therefore \quad 10x = 6$ ③

$\therefore \quad x = \tfrac{3}{5}$

(ii) $\tfrac{1}{3}(m+1) - \tfrac{1}{4}(m-2) = 2$

Multiply by 12 throughout,

$\therefore \quad 4(m+1) - 3(m-2) = 24$

$4m+4 - 3m+6 \quad = 24$

$m + 10 \quad = 24$ ③

$\therefore \quad m = 14$

Question 5 (26 marks)

(a)

(i)

$y = |x| + 1$ ②

(ii)

$y = \dfrac{1}{x-2}$ ②

(iii)

$x^2 + y^2 = 4$ ②

(iv)

$y = 2^x$ ②
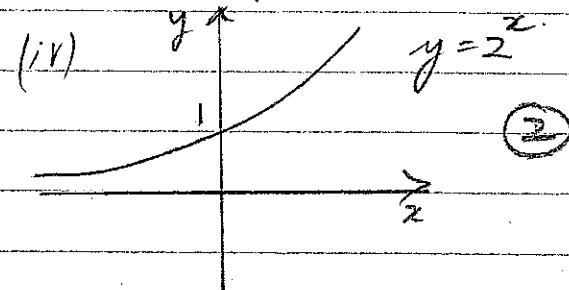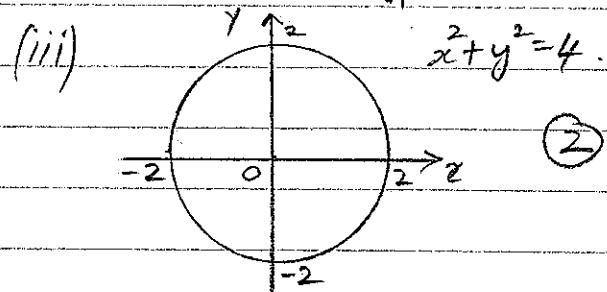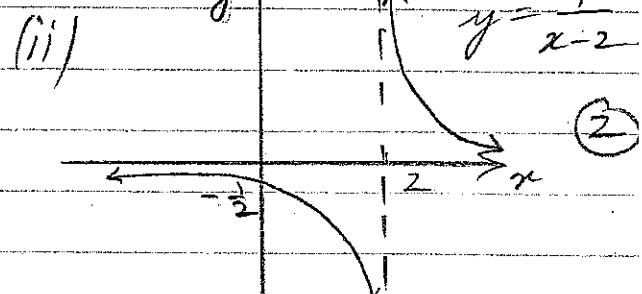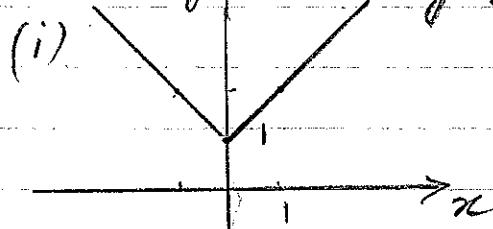
(b) (iii) above   ①

(c) (i) $y = x^2$

D: all real x

R: $y \geq 0$.   ②

(ii) $y = \sqrt{4-x^2}$

D: $-2 \leq x \leq 2$

R: $0 \leq y \leq 2$ ②

(iii) $y = \dfrac{1}{x-2}$

D: all real x, $x \neq 2$

R: all real y, $y \neq 0$.   ②

(d) $f(-1) + f(1) + f(5)$

$= 0 + -1 + 5$

$= 4$.   ②

(e) $f(x) = 8x^3 - 7x - 7$

$f(-x) = 8(-x)^3 - 7(-x) - 7$

$\qquad = -8x^3 + 7x - 7.$

$-f(x) = -8x^3 + 7x + 7.$ ③

Since $f(x) \neq -f(x)$ not even, and $-f(x) \neq f(-x)$ not odd, $\therefore$ neither.

(f) ③
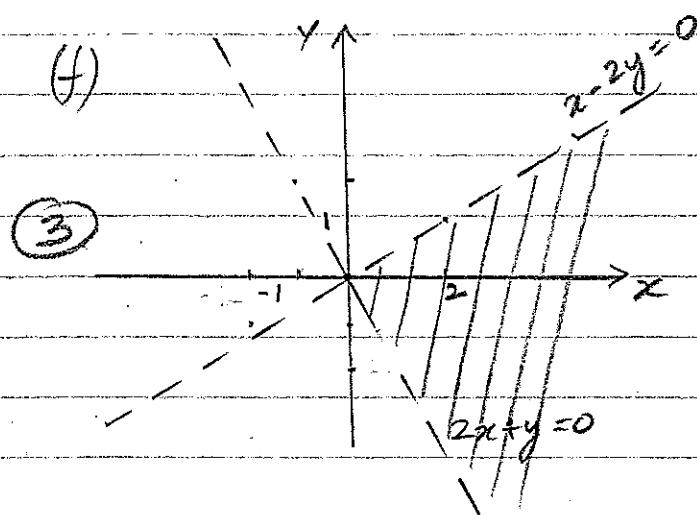
(g) $x^2 - 4x + y^2 - 14y = -17$

$(x^2 - 4x + 4) + (y^2 - 14y + 49) = \overset{53}{-17}$

$\therefore (x-2)^2 + (y-7)^2 = 36$ ③

$\therefore$ centre $(2, 7)$

radius $= 6$.

Question 6 (18 marks)

(a) $\cos 58°19' = 0.525$ ②

(b) $\tan \theta = 0.348$

$\therefore \theta = 19°11'$ ②

(c) $\cos \theta = \dfrac{14}{22}$

$\therefore \theta = 50°29'$ ③

(d) $\tan 300°$

$= \tan(360 - 60)$

$= -\tan 60° = -\sqrt{3}$ ②

(e) $\sin B \cot B$

$= \sin B \times \dfrac{\cos B}{\sin B}$ ②

$= \cos B.$

(f) $LHS = \dfrac{1}{1 - \sin \theta} - \dfrac{1}{1 + \sin \theta}$

$= \dfrac{1 + \sin \theta - 1 + \sin \theta}{(1 - \sin \theta)(1 + \sin \theta)}$

$= \dfrac{2 \sin \theta}{1 - \sin^2 \theta}$

$= \dfrac{2 \sin \theta}{\cos^2 \theta}$ ③

$= RHS.$

(g)

(i) $2 \cos \theta = \sqrt{3}$ $\dfrac{S \mid A}{T \mid C}$

$\therefore \cos \theta = \dfrac{\sqrt{3}}{2}$

$\therefore \theta = 30°, 330°$ ②

(ii) $\tan \theta + 1 = 0$

$\therefore \tan \theta = -1$ ②

$\therefore \theta = 135°, 315°$

Question 7 (13 marks)

(a)

(i) ③
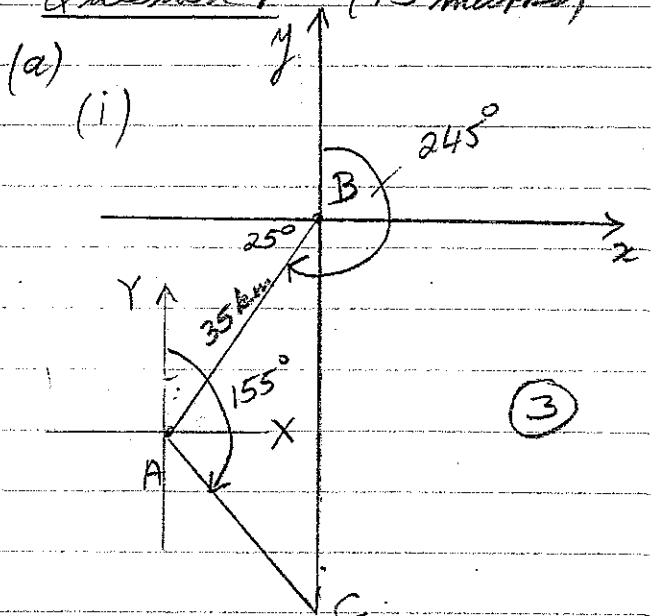
(ii) $\angle ABC = 245° - 180°$

$\qquad = 65°$

$\angle BAX = 25°, \therefore \angle BAY = 65°$

$\therefore \angle XAC = 155° - 90° = 65°$

$\therefore \angle BAC = \angle BAX + \angle XAC$

$\qquad = 25° + 65°$

$\qquad = 90°$ ②

(iii)

$\tan 65° = \dfrac{AC}{35}$

$\therefore AC = 35 \tan 65°$

$\qquad = 75 \cdot 1 \ km.$ ②

(b) $\dfrac{x}{\sin 113°} = \dfrac{6 \cdot 5}{\sin 25°}$

$\therefore x = \dfrac{6 \cdot 5}{\sin 25°} \times \sin 113°$

$x = 14 \cdot 2$ ③

(c) $67^2 = 31^2 + 49^2 - 2 \times 31 \times 49 \cos \theta$

$\therefore \cos \theta = \dfrac{31^2 + 49^2 - 67^2}{2 \times 31 \times 49}$

$\qquad = \dfrac{-1127}{3038}$

$\therefore \theta = 111 \cdot 775 \ldots°$ ③

$\qquad = 112°$