GIRRAWEEN HIGH SCHOOL

MATHEMATICS

Year 11   **April, 2010**

Task 2 Part A

Time Allowed: 45 minutes

INSTRUCTIONS:

- Attempt all questions.
- Write your answers on your own paper.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.

Question 1 (20 marks)

a) Find, correct to 4 significant places: **1**

$$\frac{3.24^2}{5.73-2.84}$$

b) If $\sqrt{5x}=5\sqrt{2}$, find the value of x.   **2**

c) Express 0.00205 in scientific notation.   **1**

d) Simplify, giving exact answers:
   (i) $\sqrt{20}+\sqrt{125}-\sqrt{45}$ (ii) $2\sqrt{3}(4\sqrt{2}-5\sqrt{3})$ (iii) $(\sqrt{5}+2\sqrt{3})^2$ **6**

e) Rationalise the denominator: $\dfrac{\sqrt{6}}{3\sqrt{6}-\sqrt{2}}$ **3**

f) Find the values of p and q if $\dfrac{\sqrt{5}}{\sqrt{5}-2}=p+q\sqrt{5}$ **2**

g) Find the exact value of $\dfrac{A^2B^3}{C^2}$ where $A=(\frac{5}{2})^2, B=(\frac{2}{3})^3, C=(\frac{5}{3})^3$ **3**

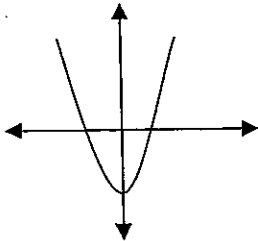h) Express $0.2\dot{3}\dot{5}$ as a fraction in its simplest form.   **2**

Question 2 (22 marks)
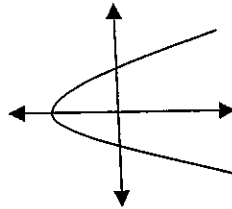
    a) Indicate whether the following curves are functions or not: **4**
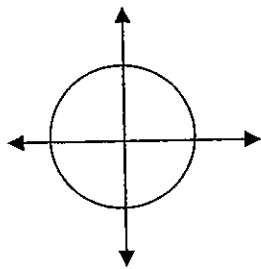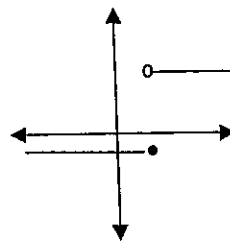
(i)

(ii)

(iii)

(iv)

b) For each of the following: **16**

    (i)   Sketch the curve

    (ii)   State whether they are functions

    (iii)   State the domain and range

(α) $x^2 + y^2 = 9$

(β) $y = 4^x$

(γ) $y = |x - 1|$

(δ) $y = \dfrac{1}{x+1}$

c) State the domain of each function: **2**

(i) $y = \sqrt{5-x}$ (ii) $y = \dfrac{1}{\sqrt{16-x^2}}$

Question 3 (34 marks)

a) Find the exact value of: 6

    (i) $\cos 135°$ (ii) $\tan 330°$ (iii) $\sin(-150°)$

b) Given that $\cos\theta = \dfrac{2}{5}$ and $\sin\theta < 0$, find the exact value of $\tan\theta$.   2

c) Simplify: 4

    (i) $\dfrac{\sin x}{\tan x}$ (ii) $\dfrac{1}{\cos x} - \sin x \tan x$

d) Prove the following identities: 9

    (i) $3 - 4\cos^2 x = 4\sin^2 x - 1$

    (ii) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

    (iii) $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2\tan x$

e) Sketch $y = \cos\theta,\ 0° \le \theta \le 360°$.   2

f) Solve for $0° \le \theta \le 360°$ 11

    (i) $2\cos\theta = \sqrt{3}$ (ii) $\operatorname{cosec}^2\theta - 2 = 0$

    (iii) $\tan 2\theta = \dfrac{1}{\sqrt{3}}$ (iv) $\sin^2\theta + \sin\theta = 0$

GIRRAWEEN HIGH SCHOOL

MATHEMATICS

Year 11   **April, 2010**

Task 2 Part B   **Time Allowed:** 45 minutes

INSTRUCTIONS:

- Attempt all questions.
- Write your answers on your own paper.
- All necessary working must be shown.
- Marks may be deducted for careless or badly arranged work.

Question 4 (31 marks)

a) Factorise:

(i) $16 - 25x^2$ 1

(ii) $3x^2 + 14x - 5$ 2

(iii) $x^3 - 64$ 2

b) Simplify:

(i) $\dfrac{a^4x^2 - b^4x^2}{ax + bx}$ 3

(ii) $\dfrac{3}{x+6} - \dfrac{1}{x+2}$ 3

(iii) $\dfrac{6x^2 + x - 2}{3x^2 - 3} \times \dfrac{4x^3 + 4x^2 - x - 1}{6x^2 + 7x + 2}$ 3

c) Solve:

(i) $\dfrac{2x-1}{3} = 1 - \dfrac{x-2}{5}$ 3

(ii) $2x - 3 < 5$ 2

(iii) $4 - 3x \geq 2 - 2x$ 2

(iv) $(x+1)(x-3) > 0$ 2

(v) $|x+3| = 5$ 2

(vi) $\left|\dfrac{3x+2}{5}\right| \leq 4$ 3

(vii) $|2m - 5| > 9$ 3

Question 5 (11 marks)

a) A function is defined by the rule:

$$f(x) = \begin{cases} -2 & \text{if } x \le -5 \\ 0 & \text{if } -5 < x < 2 \\ x & \text{if } x \ge 2 \end{cases}$$

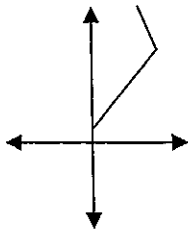Find $f(-6) + f(1) + f(6)$ **2**

b) Find the coordinates of the centre and the length of the radius of the circle
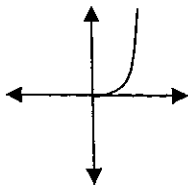
$$x^2 + y^2 + 6x - 10y + 18 = 0$$ **4**

c) Copy and complete this diagram to give an even function: **1**

d) Copy and complete this diagram to give an odd function: **1**

e) Determine whether the function $f(x) = 2x^4 + 3x$ is even or odd or neither.
    Show all your working.   **3**
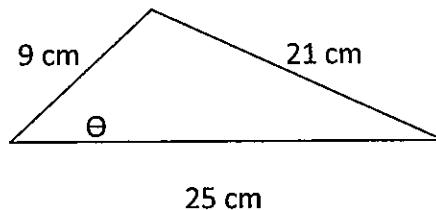
Question 6 (17 marks)

a) The angle of depression of a car, from the top of a 300m tower is $59^\circ02'$.   **2**
    How far from the tower is the car?

b) A bushwalker trekked west from town A for 40km and then 75km south.   **3**
    Find the bearing of the bushwalker from town A.

c)
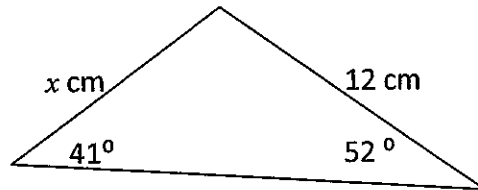
    (i)   Find the value of x, correct to 2 decimal places.   **2**

    (ii)   Find the area of the triangle.   **2**

d)

25 cm

    Find the value of Θ, correct to the nearest minute.   **2**

e) Sam drove from his home for 200 km on a bearing of 040°,
    then drove on a bearing of 157° for 345 km.

    (i)   Draw a diagram showing this information.   **1**

    (ii)   How far from home is Sam?   **2**

    (iii)   What is his bearing from home?   **3**

Year 11 Mathematics Task 2 2010

Solutions

Question 1 (20 marks)

a) 3.632 ①

b) $\sqrt{5x} = 5\sqrt{2}$
$= \sqrt{25 \times 2}$
$= \sqrt{50}$
$\therefore 5x = 50$ ②
$x = 10$

c) $0.00205 = 2.05 \times 10^{-3}$ ①

d) i) $\sqrt{20} + \sqrt{125} - \sqrt{45}$
$= 2\sqrt{5} + 5\sqrt{5} - 3\sqrt{5}$ ②
$= 4\sqrt{5}$

ii) $2\sqrt{3}(4\sqrt{2} - 5\sqrt{3})$
$= 8\sqrt{6} - 30$ ①

iii) $(\sqrt{5} + 2\sqrt{3})^2$
$= 5 + 4\sqrt{15} + 12$
$= 17 + 4\sqrt{15}$ ②

e) $\dfrac{\sqrt{6}}{3\sqrt{6} - \sqrt{2}} \times \dfrac{3\sqrt{6} + \sqrt{2}}{3\sqrt{6} + \sqrt{2}}$

$= \dfrac{18 + \sqrt{12}}{52}$

$= \dfrac{18 + 2\sqrt{3}}{52}$

$= \dfrac{9 + \sqrt{3}}{26}$ ③

f) $\dfrac{\sqrt{5}}{\sqrt{5} - 2} \times \dfrac{\sqrt{5} + 2}{\sqrt{5} + 2}$

$= \dfrac{5 + 2\sqrt{5}}{5 - 4}$

$= 5 + 2\sqrt{5} = p + q\sqrt{5}$ ②
$\therefore p = 5, q = 2$

g) $\dfrac{A^2 B^3}{C^2}$

$= \dfrac{\left[\left(\frac{5}{2}\right)^2\right]^2 \left[\left(\frac{2}{3}\right)^3\right]^3}{\left[\left(\frac{5}{3}\right)^3\right]^2}$

$= \dfrac{5^4}{2^4} \times \dfrac{2^9}{3^9} \times \dfrac{3^6}{5^2}$

$= \dfrac{2^5}{5^2 \times 3^3}$

$= \dfrac{32}{675}$ ③

h) $0.\dot{2}3\dot{5}$
$x = 0.235\,235\cdots$
$1000x = 235.235\cdots$
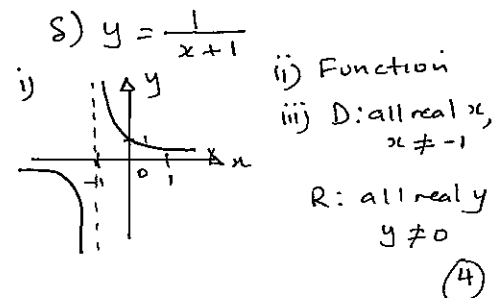$999x = 235$ ②
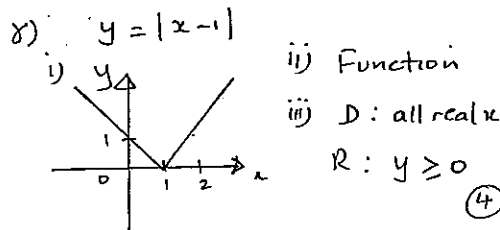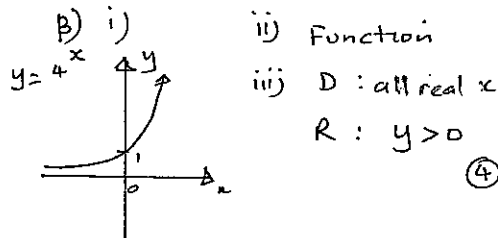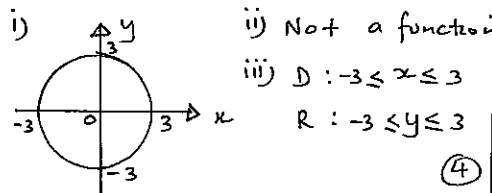$x = \dfrac{235}{999}$

Question 2 (22 marks)

a) i) function
ii) not a function
iii) not a function
iv) function. ④

b) α) $x^2 + y^2 = 9$

i)
ii) Not a function
iii) D : $-3 \leq x \leq 3$
R : $-3 \leq y \leq 3$ ④

β) i) $y = 4^x$

ii) Function
iii) D : all real x
R : $y > 0$ ④

γ) $y = |x - 1|$
i)
ii) Function
iii) D : all real x
R : $y \geq 0$ ④

δ) $y = \dfrac{1}{x+1}$
i)
ii) Function
iii) D: all real x, $x \neq -1$
R: all real y $y \neq 0$ ④

c) $y = \sqrt{5 - x}$ ①
D : $x \leq 5$

d) $y = \dfrac{1}{\sqrt{16 - x^2}}$ ①
D : $-4 < x < 4$

Question 3 (34 marks)

1) i) $\cos 135°$ (Q2)

$= \cos(180° - 45°)$

$= -\cos 45°$

$= \dfrac{-1}{\sqrt{2}}$ ②

ii) $\tan 330°$ (Q4)

$= \tan(360° - 30°)$

$= -\tan 30°$

$= \dfrac{-1}{\sqrt{3}}$ ②

iii) $\sin(-150°)$ Q3

$= \sin 210°$

$= \sin(180° + 30°)$

$= -\sin 30°$

$= -\dfrac{1}{2}$ ②

b) $\cos \theta = \dfrac{2}{5}$, $\sin \theta < 0$ [Q4]

$\tan \theta = -\dfrac{\sqrt{21}}{2}$ ②

c) i) $\dfrac{\sin x}{\tan x}$

$= \sin x \times \dfrac{\cos x}{\sin x}$ ②

$= \cos x$
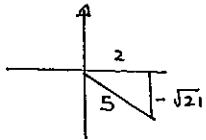
ii) $\dfrac{1}{\cos x} - \sin x \tan x$

$= \dfrac{1}{\cos x} - \sin x \cdot \dfrac{\sin x}{\cos x}$

$= \dfrac{1}{\cos x} - \dfrac{\sin^2 x}{\cos x}$

$= \dfrac{1 - \sin^2 x}{\cos x}$

$= \dfrac{\cos^2 x}{\cos x}$ ②
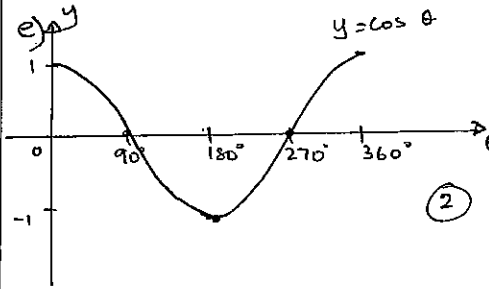
d) i) $3 - 4\cos^2 u \bigcirc \sin^2 u - 1$

LHS $= 3 - 4\cos^2 u$

$= 3 - 4(1 - \sin^2 u)$

$= 3 - 4 + 4\sin^2 u$

$= 4\sin^2 u - 1 =$ RHS ③

ii) $(\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2 = 2$

LHS $= (\sin\theta + \cos\theta)^2 + (\sin\theta - \cos\theta)^2$

$= \sin^2\theta + 2\sin\theta\cos\theta + \cos^2\theta + \sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta$

$= 2\sin^2\theta + 2\cos^2\theta$

$= 2(\sin^2\theta + \cos^2\theta)$

$= 2 =$ RHS ③

iii) $\dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x} = 2\tan x$

LHS $= \dfrac{\cos x}{1 - \sin x} - \dfrac{\cos x}{1 + \sin x}$

$= \dfrac{\cos x(1 + \sin x) - \cos x(1 - \sin x)}{(1 - \sin x)(1 + \sin x)}$

$= \dfrac{\cos x + \cos x \sin x - \cos x + \cos x \sin x}{(1 - \sin x)(1 + \sin x)}$

$= \dfrac{2\cos x \sin x}{1 - \sin^2 x}$

$= \dfrac{2\sin x \cos x}{\cos^2 x}$

$= \dfrac{2\sin x}{\cos x}$

$= 2\tan x =$ RHS ③

Question 3 (con²)

e)

$y = \cos\theta$ ②

f) i) $2\cos\theta = \sqrt{3}$

$\cos\theta = \dfrac{\sqrt{3}}{2}$ Q1, 4

$[\cos 30° = \dfrac{\sqrt{3}}{2}]$

$\theta = 30°, (360 - 30°)$

$= 30°, 330°$ ②

ii) $\csc^2\theta - 2 = 0$

$\csc^2\theta = 2$

$\dfrac{1}{\sin^2\theta} = 2$

$\sin^2\theta = \dfrac{1}{2}$

$\sin\theta = \pm\dfrac{1}{\sqrt{2}}$ Q1,2,3,4

$[\sin 45° = \dfrac{1}{\sqrt{2}}]$

$\theta = 45°, (180° - 45°)(180° + 45°), (360° - 45°)$

$\theta = 45°, 135°, 225°, 315°$ ③

iii) $\tan 2\theta = \dfrac{1}{\sqrt{3}}$ Q1,3 $\quad 0° \le \theta \le 360°$

$0° \le 2\theta \le 720°$

$[\tan 30° = \dfrac{1}{\sqrt{3}}]$

$2\theta = 30°, 210°, 390°, 570°$ ③

$\theta = 15°, 105°, 195°, 285°$

iv) $\sin^2\theta + \sin\theta = 0$

$\sin\theta(\sin\theta + 1) = 0$

$\sin\theta = 0 \quad$ or $\quad \sin\theta = -1$

$\theta = 0°, 180°, 360° \qquad \theta = 270°$

$\theta = 0°, 180°, 270°, 360°$ ③

Question 4 (31 marks)

a) i) $16 - 25x^2$

   $= (4-5x)(4+5x)$ ①

   ii) $3x^2 + 14x - 5$ $\quad x -15$
   $\quad\quad\quad\quad\quad\quad\quad + 14$

   $= 3x^2 + 15x - x - 5$

   $= 3x(x+5) - 1(x+5)$ ②

   $= (3x-1)(x+5)$

   iii) $x^3 - 64 = x^3 - 4^3$

   $= (x-4)(x^2 + 4x + 16)$ ②

b) i) $\dfrac{a^4 x^2 - b^4 x^2}{ax + bx}$

   $= \dfrac{x^2(a^4 - b^4)}{x(a+b)}$

   $= \dfrac{x(a^2+b^2)(a^2-b^2)}{(a+b)}$

   $= \dfrac{x(a^2+b^2)(a+b)(a-b)}{(a+b)}$ ③

   $= x(a-b)(a^2+b^2)$

   ii) $\dfrac{3}{x+6} - \dfrac{1}{x+2}$

   $= \dfrac{3(x+2) - (x+6)}{(x+6)(x+2)}$

   $= \dfrac{3x+6 - x - 6}{(x+6)(x+2)}$

   $= \dfrac{2x}{(x+6)(x+2)}$ ③

   iii) $\dfrac{6x^2 + x - 2}{3x^2 - 3} \times \dfrac{4x^3 + 4x^2 - x - 1}{6x^2 + 7x + 2}$

   $= \dfrac{(2x-1)(3x+2)}{3(x+1)(x-1)} \times \dfrac{(4x^2-1)(x+1)}{(2x+1)(3x+2)}$

   $= \dfrac{(2x-1)(2x-1)(2x+1)}{3(x-1)(2x+1)}$ ③

   $= \dfrac{(2x-1)^2}{3(x-1)}$

c) i) $\dfrac{2x-1}{3} = 1 - \dfrac{x-2}{5}$ $(\times 5)$

   $5(2x-1) = 15 - 3(x-2)$

   $10x - 5 = 15 - 3x + 6$

   $13x = 26$ ③
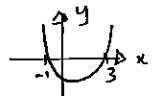
   $x = 2$

   ii) $2x - 3 < 5$

   $2x < 8$ ②

   $x < 4$

   iii) $4 - 3x \geq 2 - 2x$

   $-x \geq -2$ ②

   $x \leq 2$

   iv) $(x+1)(x-3) > 0$

   $x < -1, \ x > 3$ ②

   v) $|x+3| = 5$

   $x + 3 = 5 \quad \text{or} \quad x + 3 = -5$

   $x = 2, -8$ ②

   vi) $\left|\dfrac{3x+2}{5}\right| \leq 4$

   $-4 \leq \dfrac{3x+2}{5} \leq 4$

   $-20 \leq 3x + 2 \leq 20$

   $-22 \leq 3x \leq 18$ ③

   $-\dfrac{22}{3} \leq x \leq 6$

   vii) $|2m - 5| > 9$

   $2m - 5 > 9 \quad \text{or} \quad 2m - 5 < -9$

   $2m > 14 \quad\quad\quad\quad 2m < -4$

   $m > 7 \quad\quad\quad\quad\quad m < -2$

   $m < -2, \ m > 7$ ③

Question 5 (marks)

a) $f(-6) + f(1) + f(6)$
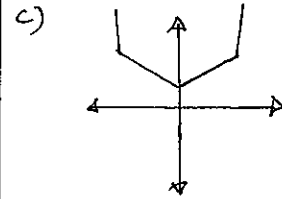
   $= -2 + 0 + 6$ ②

   $= 4$

b) $x^2 + y^2 + 6x - 10y + 18 = 0$

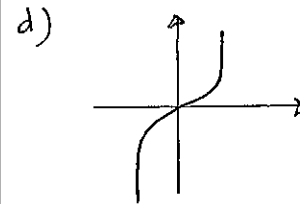 $x^2 + 6x + 9 + y^2 - 10y + 25 = -18 + 9 + 25$

   $(x+3)^2 + (y-5)^2 = 16$

   Centre $= (-3, 5)$

   radius $= 4$ units ④

c)

   ①

d)

   ①

e) $f(x) = 2x^4 + 3x$

   $f(-x) = 2(-x)^4 + 3(-x)$

   $= 2x^4 - 3x$

   $f(-x) \neq f(x) \ \therefore \text{not even}$

   $-f(x) = -2x^4 - 3x$

   $f(-x) \neq -f(x) \ \therefore \text{not odd}$

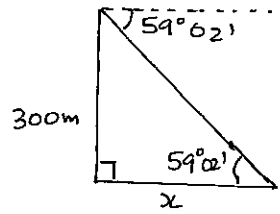 $\therefore f(x) = 2x^4 + 3x$ is
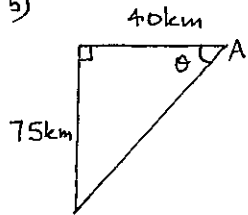
   neither even nor odd. ③

Question 6 (17 marks)

a)

$$\tan 59°02' = \frac{300}{x}$$

$$x = \frac{300}{\tan 59°02'}$$

$$= 180m$$

b)

$$\tan\theta = \frac{75}{40}$$

$$\theta = 61°56'$$

Bearing $= 270° - 61°56'$

$$= 208°04'$$

c)

i) $$\frac{x}{\sin 52°} = \frac{12}{\sin 41°}$$

$$x = \frac{12 \sin 52}{\sin 41}$$

$$= 14.41 \, cm$$

ii) $$A = \frac{1}{2} ab\sin C$$

$$= \frac{1}{2} \times 12 \times 14.41 \sin 87°$$

$$= 86.36 \, cm^2$$

d) $$\cos\theta = \frac{9^2 + 25^2 - 21^2}{2 \times 9 \times 25}$$

$$= 0.58\dot{8}$$

$$\theta = 53°55'$$

e)

i)
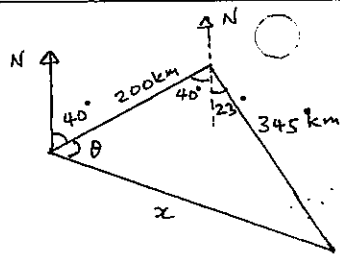
ii) $$x^2 = 200^2 + 345^2 - 2 \times 200 \times 345 \cos 63°$$

$$x = 310 \, km$$

iii) $$\frac{\sin\theta}{345} = \frac{\sin 63}{310}$$

$$\sin\theta = \frac{345 \sin 63}{310}$$

$$\theta = 82°34'$$

$\therefore$ Bearing $= 40° + 82°34'$

$$= 122°34'$$