Gosford High School

Year 11

2007
Preliminary
Higher School Certificate

Mathematics

Assessment Task 2

Time Allowed - 60 minutes
(plus 5 minutes reading time)

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that

* 'bald' answers may not gain full marks.
* untidy and/or poorly organised solutions may not gain full marks.

QUESTION 1 (12 marks)

(i) 35% of Jamiel's weekly wage is spent on rent. Find his weekly wage if his rent is \$128.80. (1)

(ii) Write $0\cdot\dot{7}\dot{1}\dot{5}$ as a fraction in simplest form (2)

(iii) Find the value of a given that $\sqrt{45}+\sqrt{80}=a\sqrt{5}$ (2)

(iv) Simplify $\dfrac{24+\sqrt{72}}{6}$ (1)

(v) $\dfrac{2}{7},\ \sqrt{7},\ -10,\ 0\cdot6,\ 4\dfrac{1}{2}\%,\ \sqrt{-4},\ \pi,\ |16|$

From the above set of numerals list the set of Rational numbers (1)

(vi) Simplify $\sqrt{(-6)^2}-|-6|$ (1)

(vii) Find the value of $\dfrac{\sqrt{21\cdot68-14\cdot9}}{2\cdot4\pi}$ correct to 2 significant figures (1)

(viii) Simplify $x-\dfrac{1}{x}$ if $x=\dfrac{1}{\sqrt{2}-1}$ (3)

QUESTION 2 (12 marks)

(i) Factorise (a) $2a^3+16$ (2)

(b) $1-2a-24a^2$ (1)

(ii) Solve (a) $8p^3+64=0$ (1)

(b) $\dfrac{3}{2a}+\dfrac{1}{a}=4-\dfrac{5}{3a}$ (2)

(c) $|2x+1|<7$ (2)

(iii) Find the exact roots of $2m^2+3m-1=0$ (2)

(iv) Make x the subject of the equation $4x-3xy-12=0$ (2)

QUESTION 3 (12 marks)

(i) Simplify $\frac{3}{x^2-x-2}-\frac{4}{x^2-2x-3}$ (3)

(ii) Simplify $\frac{xm^{-1}+mx^{-1}}{x^2+m^2}$ (3)

(iii) Simplify $\frac{8^{2n}\times 27^{n}}{18\times 12^{3n}}$ (3)

(iv) Solve equation $2x-3=\sqrt{33-2x}$ (3)

QUESTION 4 (12 marks)

(i) Plot the points A(–4, 5), B(4,–1) and C(5, 3) on a number plane.
Label the axes and the points clearly and neatly on your diagram. (2)

(ii) Find the midpoint of the interval CA. (1)

(iii) Find the length of the interval AB. (1)

(iv) Find the gradient of AB (1)

(v) Find the equation of the line AB, writing your answer in general form. (2)

(vi) On your diagram draw the perpendicular from C to the line AB (1)

(vii) Find the perpendicular distance from the point C to the line AB. (2)

(viii) Find the area of the triangle ABC. (2)

QUESTION 5 (12 marks)

The line RT has equation $4x + 3y - 6 = 0$ and the line QT has equation $2x - 4y + 3 = 0$.

(i) Find the gradient of the line RT. (1)

(ii) Find the angle of inclination of the line RT (answer to the nearest degree). (2)

(iii) Find the equation of the line that is perpendicular to RT and passing through the point $(-3, 1)$ (2)

(iv) The line QT cuts the x and y axes at M and N respectively. Find the coordinates of M and N. (2)

(v) The point $(-17, k)$ lies on the line QT. Find the value of k. (2)

(vi) Find the coordinates of the point T. (3)

QUESTION 6 (12 marks)

(i) Graph on a number plane the line $y = 2 - \frac{x}{4}$. (2)

(ii) On your diagram shade the region that simultaneously satisfies the inequalities $y \leq 2 - \frac{x}{4}$,

$x + 2 > 0$ and

$y \geq 0$ (4)

(iii) On a new diagram sketch the region satisfying $0 \leq x + y < 4$ (3)

(iv) The line $3x - 4y + f = 0$ is 2 units from the point A $(1,-3)$. Find the value(s) of f. (3)

QUESTION 1

2007

(i) $0.35 \times \text{Wage} = \$128.80$

$$\text{Wage} = \frac{\$128.80}{0.35}$$

$$= \$368$$

(ii) Let $x = 0.7\dot{1}\dot{5}$

$\therefore 100x = 71.5\dot{1}\dot{5}$

$$99x = 70.8$$

$$x = \frac{70.8}{99}$$

$$x = \frac{708}{990}$$

$$x = \frac{118}{165}$$

(iii) $\sqrt{45} + \sqrt{80} = a\sqrt{5}$

$$3\sqrt{5} + 4\sqrt{5} = a\sqrt{5}$$

$$7\sqrt{5} = a\sqrt{5}$$

$\therefore a = 7$

(iv) $$\frac{24+\sqrt{72}}{6} = \frac{24+6\sqrt{2}}{6}$$

$$= \frac{6(4+\sqrt{2})}{6}$$

$$= 4+\sqrt{2}$$

v) $\frac{2}{7}, -10, 0.6, 4\frac{1}{2}\%, |16|$

vi) $6 - 6 = 0$

vii) 0.35

viii) $$x = \frac{1}{\sqrt{2}-1} \times \frac{\sqrt{2}+1}{\sqrt{2}+1}$$

$$= \sqrt{2}+1$$

$$x - \frac{1}{x} = \sqrt{2}+1-(\sqrt{2}-1)$$

$$= \sqrt{2}+1-\sqrt{2}+1$$

$$= 2$$

QUESTION 2

i) a) $2a^3+16 = 2(a^3+8)$

$$= 2(a+2)(a^2-2a+4)$$

b) $1-2a-24a^2$

$$= (1-6a)(1+4a)$$

OR.

$$= -(6a-1)(4a+1)$$

ii) a) $8p^3 = -64$

$$p^3 = -8$$

$$p = -2$$

b) $$\frac{3}{2a} + \frac{1}{a} = 4 - \frac{5}{3a}$$

$$9 + 6 = 24a - 10$$

$$25 = 24a$$

$$\frac{25}{24} = a$$

c) $|2x+1| < 7$

$$-7 < 2x+1 < 7$$

$$-8 < 2x < 6$$

$$-4 < x < 3$$

(iii) $2m^2+3m-1=0$

$$m = \frac{-3 \pm \sqrt{9-4(2)(-1)}}{4}$$

$$= \frac{-3 \pm \sqrt{17}}{4}$$

(iv) $4x - 3xy = 12$

$$x(4-3y) = 12$$

$$x = \frac{12}{4-3y}$$

QUESTION 3

i) $$\frac{3}{(x-2)(x+1)} - \frac{4}{(x-3)(x+1)}$$

$$= \frac{3(x-3)-4(x-2)}{(x-2)(x+1)(x-3)}$$

$$= \frac{3x-9-4x+8}{(x-2)(x+1)(x-3)}$$

$$= \frac{-x-1}{(x-2)(x+1)(x-3)}$$

$$= \frac{-(x+1)}{(x-2)(x+1)(x-3)}$$

$$= \frac{-1}{(x-2)(x+1)(x-3)}$$

ii) $$= \frac{\frac{x}{m} + \frac{m}{x}}{x^2+m^2}$$

Multiply top & bottom by xm

$$= \frac{x^2+m^2}{xm(x^2+m^2)}$$

$$= \frac{1}{xm}$$

Part (iii) at end

IV) $(2x-3)^2 = 33-2x$

$$4x^2-12x+9 = 33-2x$$

$$4x^2-10x-24 = 0$$

$$2x^2-5x-12 = 0$$

$$(2x+3)(x-4) = 0$$

$$x = -\frac{3}{2}, 4$$

But when $x = -\frac{3}{2}$

L.H.S. $= -6$, R.H.S $= 6$

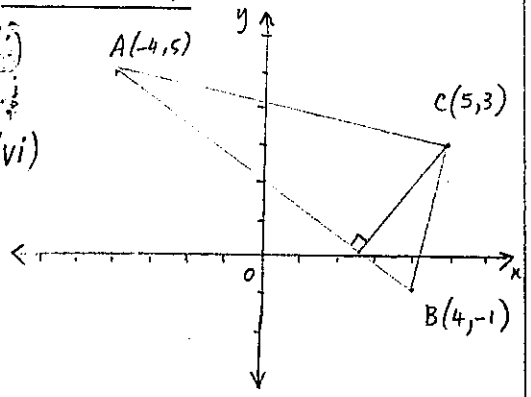
$\therefore x = -\frac{3}{2}$ is not a solution

ie $x = 4$ is only soluho

QUESTION 4

(i)

(vi)



(ii) Midpoint $= \left(\frac{5+(-4)}{2}, \frac{3+5}{2}\right)$

$= \left(\frac{1}{2}, 4\right)$

(iii) $AB = \sqrt{(4-(-4))^2 + (-1-5)^2}$

$= \sqrt{8^2 + (-6)^2}$

$= \sqrt{64+36}$

$= \sqrt{100}$

$= 10$

(iv) $m_{AB} = \frac{-1-5}{4-(-4)}$

$= \frac{-6}{8}$

$= -\frac{3}{4}$

(v) Equation of AB is

$y+1 = -\frac{3}{4}(x-4)$

$4y+4 = -3x+12$

$3x+4y-8 = 0$

vii) $d = \frac{|ax_1+by_1+c|}{\sqrt{a^2+b^2}}$

$= \frac{|3(5)+4(3)-8|}{\sqrt{3^2+4^2}}$

$= \frac{|19|}{5}$

$= \frac{19}{5}$ or $3\frac{4}{5}$

viii) Area of $\Delta ABC = \frac{1}{2}bh$

$= \frac{1}{2} \times 10 \times \frac{19}{5}$

$= 19$ sq units.

QUESTION 5

i) $3y = -4x + 6$

$y = -\frac{4x}{3} + 2$

Gradient of RT is $-\frac{4}{3}$

ii) Let θ be angle of inclination

$\therefore \tan\theta = -\frac{4}{3}$

$\theta = 127°$

iii) Equation of perpendicular is $y-1 = \frac{3}{4}(x+3)$

$4y-4 = 3x+9$

$3x-4y+13 = 0$

(iv) at M, $y = 0$

$\therefore 2x = -3$

$x = -\frac{3}{2} \rightarrow M\left(-\frac{3}{2}, 0\right)$

at N, $x = 0$

$\therefore -4y = -3$

$y = \frac{3}{4} \rightarrow N\left(0, \frac{3}{4}\right)$

v) $(-17, k)$ satisfies

$\therefore 2(-17) - 4k + 3 = 0$

$-34 - 4k + 3 = 0$

$-4k = 31$

$k = -\frac{31}{4}$ or $-7\frac{3}{4}$

vi) $4x + 3y - 6 = 0 \ldots$ (RT)

$4x - 8y + 6 = 0 \ldots$ (QT)

$\therefore 11y - 12 = 0$

$11y = 12$

$y = \frac{12}{11}$

$\therefore 2x - 4\left(\frac{12}{11}\right) + 3 = 0$

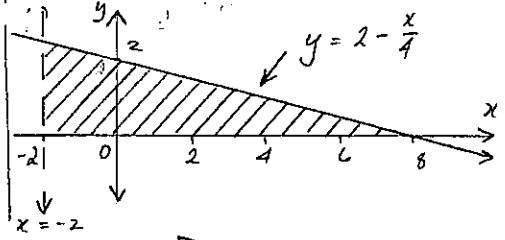
$22x - 48 + 33 = 0$

$22x = 15$

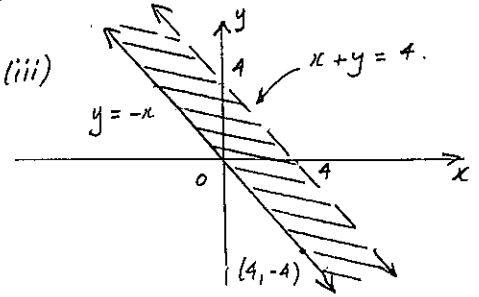
$x = \frac{15}{22}$

T is the point $\left(\frac{15}{22}, \frac{12}{11}\right)$

QUESTION 6



(iii)



(iv) Perp. distance = 2

$\therefore \frac{|3(1)+(-4)(-3)+f|}{\sqrt{3^2+4^2}} = 2$

$\therefore \frac{|15+f|}{5} = 2$

$|15+f| = 10$

$15+f = \pm 10$

$f = -5, -25$

Question 3 (iii) (sorry)

$\frac{8^{2n} \times 27^{n}}{18 \times 12^{3n}} = \frac{(2^3)^{2n} \times (3^3)^{n}}{18 \times (2^2 \times 3)^{3n}}$

$= \frac{2^{6n} \times 3^{3n}}{18 \times 2^{6n} \times 3^{3n}}$

$= \frac{1}{18}$