Gosford High School

Year 11

2008
Preliminary
Higher School Certificate

Mathematics

Assessment Task 2

Time Allowed - 60 minutes
(+ 5 minutes Reading Time)

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that
*   'bald' answers may not gain full marks.
*   untidy and/or poorly organised solutions may not gain full marks.

Question 1   (15 marks)

(a)   Evaluate $\left|-5\right|-\left|-8\right|$ (1)

(b)   Write $\dfrac{7}{11}$ correct to 2 decimal places   (1)

(c)   Find $\dfrac{\sqrt{573\cdot6+114\cdot9}}{3\pi}$ correct to the nearest whole number   (1)

(d)   In 2007 Council rates increased by $7\dfrac{1}{2}\%$. The new rate for a property is $1735.
What was the old rate for this property? (answer to the nearest dollar)   (2)

(e)   Write $0\cdot0384$ correct to 2 significant figures   (1)

(f)   Simplify $3\sqrt{32}-6\sqrt{18}+2\sqrt{24}$ (2)

(g)   Find a if $\dfrac{4\sqrt{2}\times\sqrt{12}}{2\sqrt{6}\times3\sqrt{8}}=a\sqrt{2}$ (3)

(h)   If $a=\sqrt{2}-1$ find the exact value of $a+a^{-1}$ (3)

Question 2   (12 marks)

(a)   Factorise each of the following

            (i) $10a^2 + 3a - 4$ (1)

            (ii) $8x^3 + 512$ (1)

            (ii) $x(3x - 7) - (3x - 7)^2$ (2)

            (iii) $p^2 - q^2 - p + q$ (2)

(b)   Expand and simplify

            (i) $(3y - 4x)^2$ (1)

            (ii) $(2m - 1)(4m^2 + 2m + 1)$ (1)

(c)   Simplify $\dfrac{6}{x^2 - 9} - \dfrac{x - 6}{x^2 - 3x - 18}$ (3)

Question 3   (12 marks)

(a)   Solve

            (i) $6x^2 = 9x$ (2)

            (ii) $2(\dfrac{1}{a} + 4) = 5 - \dfrac{2}{a}$ (2)

            (iii) $|4x - 11| = 5x + 2$ (3)

            (iv) $x = \sqrt{3x + 4}$ (3)

(b)   Solve $2t^2 = 3t + 8$ writing your answer in surd form.   (2)

Question 4   (12 marks)

(a)   Graph $\{x : -1 < x \le 2\}$ on a number line.   (1)

(b)   Solve

      (i) $(2x + 3)(x - 4) \ge 0$ (2)

      (ii) $|4x - 3| > 13$ (2)

      (iii) $\dfrac{x + 2}{-3} \ge \dfrac{x - 5}{2}$ (2)

(c)   Solve the following pairs of simultaneous equations

      (i) $\begin{aligned} 2c - 3d &= 4 \\ 3c + d &= 17 \end{aligned}$ (2)

      (ii) $\begin{aligned} xy &= -4 \\ 2x + y &= 2 \end{aligned}$ (3)

Question 5   (9 marks)

(a)   The equation $x^2 - 2ax - a = 0$ has one root at $x = -5$. Find the value of a.   (1)

(b)   Solve $27^x = \dfrac{1}{81}$ (2)

(c)   Show that the equation $2x^2 - 5x + 8 = 0$ has no real solutions   (2)

(d)   Use the formula $S = \dfrac{n}{2}\left[2a + (n - 1)d\right]$ to find the value of d when $n = 9, a = -6$ and $S = 540$ (2)

(e)   Use the method of 'Completion of the Square' to solve the equation $x^2 - 8x + 9 = 0$ (2)

Solutions to Assessment 2

a) $|-5| - |-8| = 5 - 8$

$$= -3$$

b) $\frac{7}{11} = 0.6363....$

$$= 0.64 \text{ to } 2 \text{ d.p.}$$

c) $\frac{\sqrt{(573.6 + 114.9)}}{(3\pi)} = 2.784..$

$$= 3 \text{ to the nearest whole number}$$

d) $107\frac{1}{2}\%$ of Old Rate = New Rate

$1.075 \times$ Old Rate $= \$1735$

$$\text{Old Rate} = \frac{\$1735}{1.075}$$

$$= \$1613.953...$$

$$= \$1614 \text{ (to the nearest dollar)}$$

e) $0.0384 = 0.038$ (to 2 sign. figures)

f) $3\sqrt{32} - 6\sqrt{18} + 2\sqrt{24} = 12\sqrt{2} - 18\sqrt{2} + 4\sqrt{6}$

$$= 4\sqrt{6} - 6\sqrt{2}$$

g) $\dfrac{4\sqrt{2} \times \sqrt{12}}{2\sqrt{6} \times 3\sqrt{8}} = \dfrac{4\sqrt{24}}{6\sqrt{48}}$

$$= \frac{2}{3\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$$

$$= \frac{3}{3}\sqrt{2}$$

$$\therefore a = \frac{1}{3}$$

h) $a + a^{-1} = a + \frac{1}{a}$

$$= \sqrt{2} - 1 + \frac{1}{\sqrt{2} - 1}$$

$$= \sqrt{2} - 1 + \frac{1}{\sqrt{2} - 1} \times \frac{\sqrt{2} + 1}{\sqrt{2} + 1}$$

$$= \sqrt{2} - 1 + \frac{\sqrt{2} + 1}{1}$$

$$= \sqrt{2} - 1 + \sqrt{2} + 1$$

$$= 2\sqrt{2}$$

2. a) i) $10a^2 + 3a - 4 = (5a + 4)(2a - 1)$

ii) $8x^3 + 512 = 8(x^3 + 64)$

$$= 8(x + 4)(x^2 - 4x + 16)$$

(iii) $x(3x - 7) - (3x - 7)^2 = (3x - 7)(x - (3x - 7))$

$$= (3x - 7)(x - 3x + 7)$$

$$= (3x - 7)(7 - 2x)$$

(iv) $p^2 - q^2 - p + q = (p - q)(p + q) - 1(p - q)$

$$= (p - q)((p + q) - 1)$$

$$= (p - q)(p + q - 1)$$

b) i) $(3y - 4x)^2 = 9y^2 - 24xy + 16x^2$

ii) $(2m - 1)(4m^2 + 2m + 1) = 8m^3 + 4m^2 + 2m - 4m^2 - 2m - 1$

$$= 8m^3 - 1$$

c) $\dfrac{6}{x^2 - 9} - \dfrac{x - 6}{x^2 - 3x - 18} = \dfrac{6}{(x-3)(x+3)} - \dfrac{x-6}{(x-6)(x+3)}$

$$= \frac{6}{(x-3)(x+3)} - \frac{1}{x+3}$$

$$= \frac{6 - 1(x-3)}{(x-3)(x+3)}$$

$$= \frac{9 - x}{}$$

3. a) (i) $6x^2 = 9x$

$$6x^2 - 9x = 0$$

$$3x(2x - 3) = 0$$

$$x = 0, \frac{3}{2}$$

(ii) $2\left(\frac{1}{a} + 4\right) = 5 - \frac{2}{a}$

$$\frac{2}{a} + 8 = 5 - \frac{2}{a}$$

$$\frac{4}{a} = -3$$

$$4 = -3a$$

$$a = -\frac{4}{3}$$

(iii) $|4x - 11| = 5x + 2$

$\therefore 4x - 11 = 5x + 2$ or $4x - 11 = -(5x + 2)$

$-13 = x$ $4x - 11 = -5x - 2$

Test $|-63| \neq -63$ $9x = 9$

$\therefore$ Not a solution $x = 1$

Test $|-7| = 7$

Is a solution

$\therefore x = 1$ only

3. (iv) $x = \sqrt{3x+4}$

$x^2 = 3x + 4$

$x^2 - 3x - 4 = 0$

$(x-4)(x+1) = 0$

$\therefore x = 4, -1$

Test $x = 4$ Test $x = -1$

$4 = \sqrt{16}$ $-1 = \sqrt{1}$

Is a solution   Is not a solution

$x = 4$ is the only solution

b) $2t^2 - 3t - 8 = 0$

$t = \dfrac{3 \pm \sqrt{(-3)^2 - 4(2)(-8)}}{4}$

$t = \dfrac{3 \pm \sqrt{9 + 64}}{4}$

$t = \dfrac{3 \pm \sqrt{73}}{4}$

(iii) $\dfrac{x+2}{-3} \geqslant \dfrac{x-5}{2}$

Multiply both sides by (-6)

$2(x+2) \leq -3(x-5)$

$2x + 4 \leq -3x + 15$

$5x \leq 11$

$x \leq 2\frac{1}{5}$

c)   i) $3c + d = 17$

$d = 17 - 3c \quad \ldots (1)$

$2c - 3d = 4 \quad \ldots (2)$

subst. (1) into (2)

$2c - 3(17 - 3c) = 4$

$2c - 51 + 9c = 4$

$11c = 55$

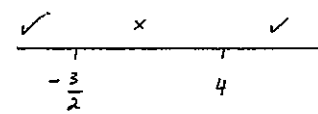$c = 5$

$\therefore d = 17 - 3(5)$

$d = 2$

$\therefore c = 5, d = 2$

---

b)   i) $(2x+3)(x-4) \geqslant 0$

Solve $(2x+3)(x-4) = 0$

$\therefore x = -\frac{3}{2}, 4$

✓	×	✓
$-\frac{3}{2}$		4

Test $x = -2$; $6 \geqslant 0$ True

$x = 0$; $-12 \geqslant 0$ False

$x = 5$; $13 \geqslant 0$ True

$\therefore x \leq -\frac{3}{2}$ or $x \geqslant 4$

ii) $|4x - 3| > 13$

$\therefore 4x - 3 > 13$ or $4x - 3 < -13$

$4x > 16$ $4x < -10$

$x > 4$ $x < -2\frac{1}{2}$

Test $x = 5$, $|17| > 13$ True

$x = -3$ $|-15| > 13$ True

$\therefore x < -2\frac{1}{2}$ or $x > 4$

4.   c)   (ii) $y = -\frac{4}{x} \quad \ldots (1)$

$2x + y = 2 \quad \ldots (2)$

Subst. (1) into (2)

$\therefore 2x + \left(-\frac{4}{x}\right) = 2$

$2x^2 - 4 = 2x$

$2x^2 - 2x - 4 = 0$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$\therefore x = 2, -1$

when $x = 2$, $y = -2$

when $x = -1$, $y = 4$

5.   a) $x = -5$ satisfies

$\therefore (-5)^2 - 2a(-5) - a = 0$

$25 + 10a - a = 0$

$\therefore 9a = -25$

$a = \dfrac{-25}{9}$

$a = -2\frac{7}{9}$

b) $27^{?} = \frac{1}{81}$

$(3^3)^x = 3^{-4}$

$3^{3x} = 3^{-4}$

$3x = -4$

$x = \frac{-4}{3}$

c) $b^2 - 4ac = (-5)^2 - 4(2)(8)$

$\qquad = 25 - 64$

$\qquad = -39 < 0$

$\therefore$ Equation has no real solutions

d) $S = \frac{n}{2}\left[2a + (n-1)d\right]$

$540 = \frac{9}{2}\left[2(-6) + 8d\right]$

$540 = \frac{9}{2}\left[8d - 12\right]$

$540 = 9(4d - 6)$

$60 = 4d - 6$

$4d = 66$

$d = 16\frac{1}{2}$

e) $x^2 - 8x \cdots$

$x^2 - 8x = -9$

$x^2 - 8x + 16 = -9 + 16$

$(x - 4)^2 = 7$

$x - 4 = \pm\sqrt{7}$

$x = 4 \pm \sqrt{7}$