Gosford High School

Year 11

2009
Preliminary
Higher School Certificate

Mathematics

Assessment Task 2

Time Allowed - 60 minutes

Remember to start each new question on a new page
Students must answer questions using a blue/black pen and/or a sharpened B or HB pencil.
Approved scientific calculators may be used
Students need to be aware that
*   'bald' answers may not gain full marks.
*   untidy and/or poorly organised solutions may not gain full marks.

Question 1 (13 marks)   **START A NEW PAGE**

(a)   Find the value $\sqrt{6\cdot714\times10^{-3}}$ giving your answer correct to 3 significant figures   (1)

(b)   A pump empties water from a tank at a rate of 300L/h.

   (ii)   How long will it take to empty a tank containing 750L?   (1)

   (ii)   How much water will it pump in 1 hour 40 minutes?   (1)

(c)   Simplify $\dfrac{2\sqrt{24}-\sqrt{54}}{2\sqrt{3}}$, writing your answer in exact form   (2)

(d)   Find a and m if $a+m\sqrt{3}=(3\sqrt{6}-5\sqrt{2})^2$ (3)

(e)   The average distance from the Earth to the Sun is $1\cdot50\times10^8\,km$.
How long would it take light travelling at $3\cdot0\times10^8\,m/s$ to reach the Earth from the Sun.
Give your answer correct to the nearest minute.   (2)

(f)   Simplify $\dfrac{7}{10-6\sqrt{2}}$ expressing your answer with a rational denominator.   (3)

Question 2 (13 marks)   **START A NEW PAGE**

(a)   Expand and simplify $(4x+3y)(2x-5y)$ (1)

(b)   Factorise $8x^3-1$ (1)

(c)   Factorise fully $36m^2-4$ (2)

(d)   Simplify $\dfrac{1}{2n^2+n}+\dfrac{1}{2n^2+3n+1}$ (3)

(e)   Factorise fully then simplify $\dfrac{a^4-1}{a^3-a^2-1+a}$ (3)

(f)   Simplify $\dfrac{6^{4n}\times9^{1-n}}{12^{2n}}$ (3)

Question 3 (18 marks)　　　**START A NEW PAGE**

(a) Solve $1 \cdot 2a - 0 \cdot 36 = 2 \cdot 4a$　　　(2)

(b) Solve $4 - 3y > \dfrac{y}{2}$　　　(2)

(c) Solve $(x-1)^2 = 2$　　　(2)

(d) If -3 is a root of the equation $x^2 + kx - 6 = 0$, find the value of k.　　　(2)

(e) Graph on a number line the set of real values of x satisfying $x > 2$ OR $x \le -1$　　　(2)

(f) Solve $2x^2 - 6x - 1 = 0$ writing your answers correct to 2 decimal places　　　(3)

(g) If $S = \dfrac{a(1-r^n)}{1-r}$ find the exact value of S when $a = 32$, $n = 8$ and $r = -\dfrac{1}{2}$　　　(2)

(h) Solve simultaneously $\begin{cases} 3x + 7y = 27 \\ 5x + 2y = 16 \end{cases}$　　　(3)

Question 4 (16 marks)　　　**START A NEW PAGE**

(a) Solve $|2x - 1| \ge 3$　　　(2)

(b) Solve $9^x = \dfrac{1}{27}$　　　(2)

(c) Solve $2x^2 - 5x \le 0$　　　(3)

(d) Solve $|2x - 4| = 7x + 1$　　　(3)

(e) Solve simultaneously $\begin{cases} y - 2x = 1 \\ x^2 + y^2 = 10 \end{cases}$　　　(4)

(f) Make y the subject of the formula $y^2 - 4xy + 1 = 0$　　　(2)

Question 1

a) 0.0819

b) i) 2.5 hours

   ii) $500 L$

c) $\dfrac{2\sqrt{24} - \sqrt{54}}{2\sqrt{3}} = \dfrac{4\sqrt{6} - 3\sqrt{6}}{2\sqrt{3}}$

$= \dfrac{\sqrt{6}}{2\sqrt{3}}$

$= \dfrac{\sqrt{2}}{2}$

d) R.H.S. $= 54 + 50 - 30\sqrt{12}$

$= 104 - 60\sqrt{3}$

$\therefore a = 104, \ m = -60$

e) Time $= \dfrac{\text{Distance}}{\text{Speed}}$

$= \dfrac{1.50 \times 10^{11}}{3 \times 10^8}$ seconds

$= 500$ seconds

$\doteqdot 8$ minutes

(to nearest minute)

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f) $\dfrac{7}{10 - 6\sqrt{2}} = \dfrac{7}{10 - 6\sqrt{2}} \times \dfrac{10 + 6\sqrt{2}}{10 + 6\sqrt{2}}$

$= \dfrac{7(10 + 6\sqrt{2})}{100 - 72}$

$= \dfrac{14(5 + 3\sqrt{2})}{28}$

$= \dfrac{5 + 3\sqrt{2}}{2}$

Question 2

a) $8x^2 - 14xy - 15y^2$

b) $(2x - 1)(4x^2 + 2x + 1)$

c) $4(9m^2 - 1) = 4(3m - 1)(3m + 1)$

d) $\dfrac{1}{n(2n+1)} + \dfrac{1}{(2n+1)(n+1)}$

$= \dfrac{n+1 + n}{n(2n+1)(n+1)}$

$= \dfrac{2n+1}{n(2n+1)(n+1)}$

$= \dfrac{1}{n(n+1)}$

$a^2(a-1) + 1(a-1)$

$= \dfrac{(a-1)(a+1)(a^2+1)}{(a^2+1)(a-1)}$

$= a + 1$

f) $\dfrac{(3 \times 2)^{4n} \times (3^2)^{1-n}}{(2^2 \times 3)^{2n}}$

$= \dfrac{3^{4n} \times 2^{4n} \times 3^{2-2n}}{2^{4n} \times 3^{2n}}$

$= \dfrac{3^{2n} \times 3^{2-2n}}{1}$

$= 3^2$

$= 9$

Question 3

a) $-0.36 = 1.2a$

$\quad a = -0.3$

b) $8 - 6y \geq y$

$\quad 8 > 7y$

$\quad y < \dfrac{8}{7}$
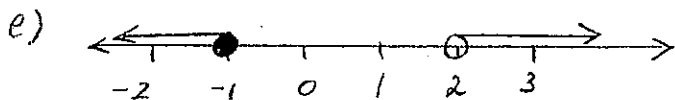
c) $x - 1 = \pm\sqrt{2}$

$\quad x = 1 \pm \sqrt{2}$

d) $(-3)^2 + k(-3) - 6 = 0$

$\quad 9 - 3k - 6 = 0$

$\quad 3 = 3k$

$\quad k = 1$

e)

f) $x = \dfrac{6 \pm \sqrt{36 - 4(2)(-1)}}{4}$

$\quad x = \dfrac{6 \pm \sqrt{44}}{4}$

$\quad x \doteq 3.16, -0.16$

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g) $S = \dfrac{32\left(1 - \left(-\frac{1}{2}\right)^8\right)}{1 + \frac{1}{2}}$

$\quad S = \dfrac{2}{3} \times 32 \times \left(1 - \dfrac{1}{256}\right)$

$\quad = \dfrac{2}{3} \times 32 \times \dfrac{255}{256}$

$\quad = 21\dfrac{1}{4}$

h) $6x + 14y = 54$

$\quad 35x + 14y = 112$

$\quad 29x = 58$

$\quad x = 2$

$\quad \therefore 6 + 7y = 27$

$\quad 7y = 21$

$\quad y = 3$

Question 4

a) $2x - 1 \geq 3 \quad \underset{=}{or} \quad 2x - 1 \leq -3$

$\quad 2x \geq 4 \qquad\qquad 2x \leq -2$

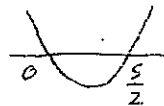$\quad x \geq 2 \qquad\qquad x \leq -1$

b) $(3^2)^x = 3^{-3}$

$\quad 2x = -3$

$\therefore x = -\dfrac{3}{2}$

c) $x(2x - 5) \leq 0$

$\quad \therefore 0 \leq x \leq \dfrac{5}{2}$

d) $2x - 4 = 7x + 1 \quad \& \quad 2x - 4 = -7x - 1$

$\quad -5 = 5x \qquad\qquad 9x = 3$

$\quad -1 = x \qquad\qquad x = \dfrac{1}{3}$

Does not satisfy $\qquad$ Satisfies

e) $y = 1 + 2x$

$\quad \therefore x^2 + (1 + 2x)^2 = 10$

$\quad x^2 + 1 + 4x + 4x^2 = 10$

$\quad 5x^2 + 4x - 9 = 0$

$\quad (5x + 9)(x - 1) = 0$

$\quad \therefore x = -\dfrac{9}{5}, 1$

$\quad y = -\dfrac{13}{5}, 3$

f) $y = \dfrac{4x \pm \sqrt{(4x)^2 - 4(1)(1)}}{2}$

$\quad y = \dfrac{4x \pm \sqrt{16x^2 - 4}}{2}$

$\quad = 2x \pm \sqrt{4x^2 - 1}$