PRELIMINARY

GOSFORD HIGH SCHOOL
MATHEMATICS
Assessment task 3: June 2010
Paper 1

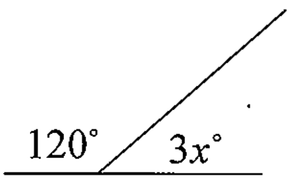
Start each question on a new page

Time: 60 minutes.

Question 1. (2 marks each)

Find the value of the pronumeral. (no reasoning is required).

a)



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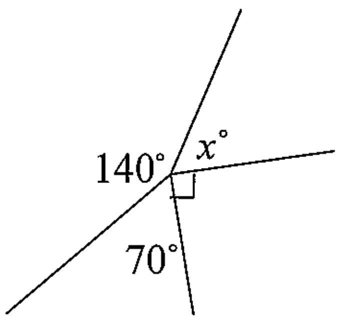

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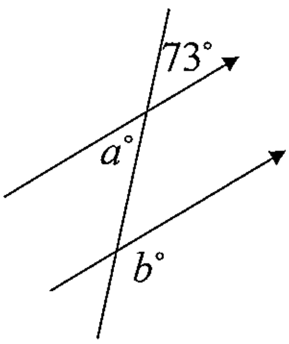

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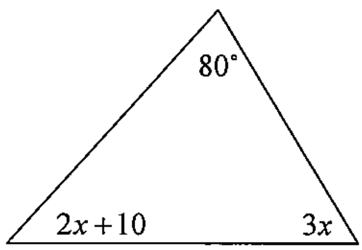

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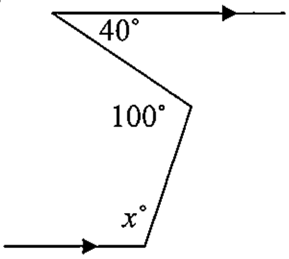

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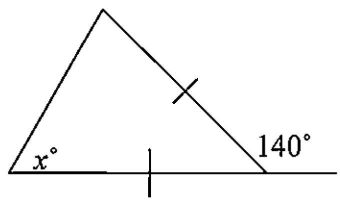
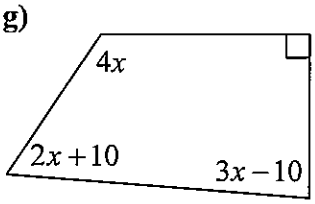


g)



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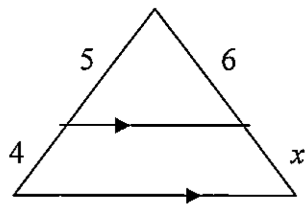
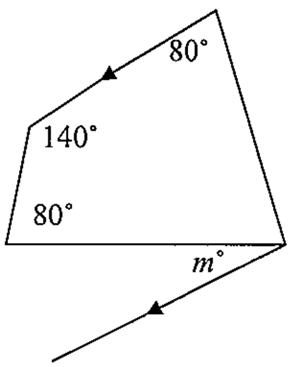
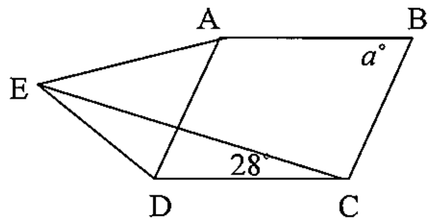


i)



j)

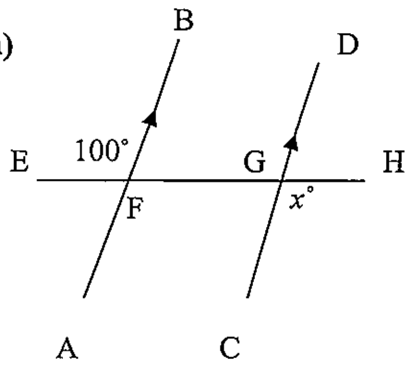


ABCD is a rhombus
$\triangle$DEA is equilateral

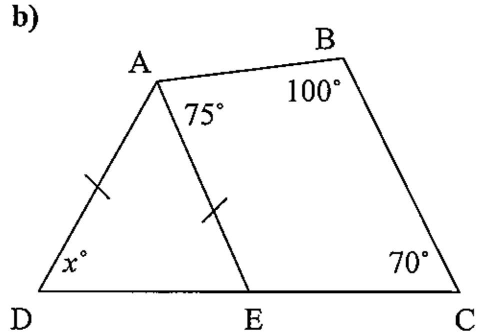
Question 2. (new page)

Find the value of the pronumeral giving full reasoning for your answer.

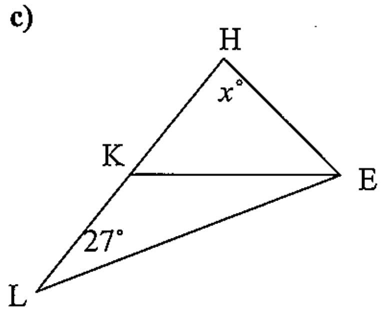
a) **(2)**



b) **(3)**

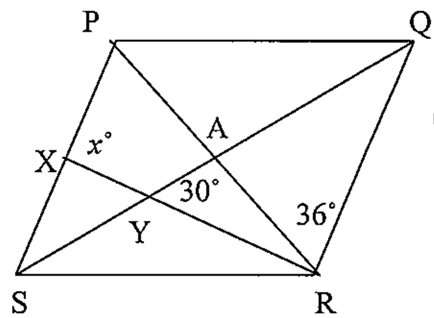


c) **(3)**



Given HK = HE
and KL = KE

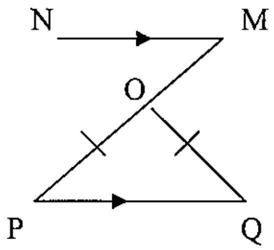
d) **(3)**



Given PQRS is a rhombus.

Question 3 (new page)

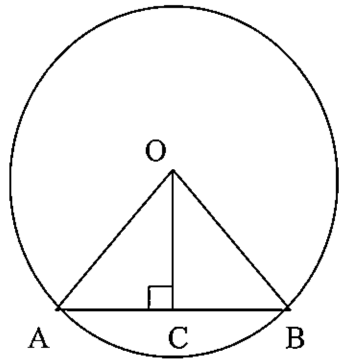
a)



(2)

Given NM is parallel to PQ and OP = OQ. Prove angle NMO equals angle OQP.

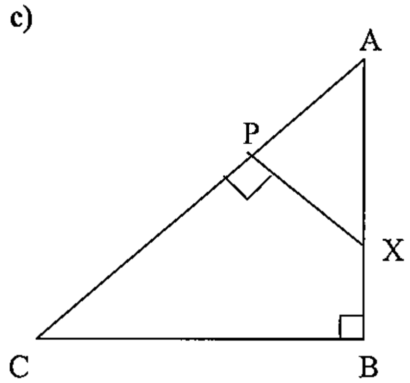
b)



(4)

If O is the centre of the circle and OC is perpendicular to AB. Prove, by use of congruent triangles, that OC bisects AB.

c)

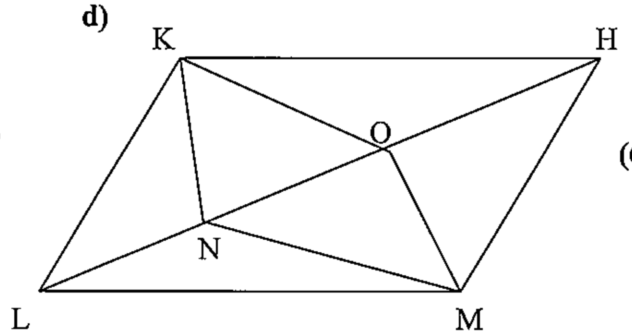


(5)

i) Show that triangle PAX is similar to triangle BAC.

ii) If AX and BC are both 5*cm* and AP is 4*cm* find PX and AC.

d)



(6)

Given HKLM is a parallelogram and OH = HM and LN = LK.

i) Show triangle LKN is congruent to triangle OHM.

ii) Hence show KNMO is a parallelogram.

Question 4. (new page)

a) If α and β are the roots of the quadratic equation $x^2 - 3x - 5 = 0$ find the value of:

i) $\alpha + \beta$

ii) $\alpha \beta$

iii) $\alpha^2 \beta + \alpha \beta^2$ **(8)**

iv) $\alpha^2 + \beta^2$

v) $\left(\alpha - \dfrac{1}{\beta}\right)\left(\beta - \dfrac{1}{\alpha}\right)$

b) Write a quadratic equation whose roots are:

$3+\sqrt{2}$ and $3-\sqrt{2}$ **(2)**

c) Solve

i) $x^4 - 3x^2 - 4 = 0$

ii) $9^x - 12.3^x + 27 = 0$ **(6)**

d) For the quadratic equation $x^2 - (k-4)x + 3k = 0$ find the value of k if the product of the roots is twice the sum of the roots. **(2)**

e) Find the value of A, B and C so that:

$$2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C.$$ **(3)**

f) By using the substitution $m = \dfrac{x^2}{x+1}$, or otherwise, solve

$$\frac{x^2}{x+1} = 2 - \frac{x+1}{x^2}$$ **(4)**

GOSFORD HIGH SCHOOL
MATHEMATICS
Assessment task 3: June 2010
Paper 2

Start each question on a new page

Time: 60 minutes.

Question 1.

For the points $A(0,2)$, $B(5,4)$ and $C(1,-1)$

i)	Plot their coordinates on a number plane.	**(2)**
ii)	Find the exact length of the interval joining B to C.	**(2)**
iii)	Find the midpoint of the interval joining A to C.	**(2)**
iv)	Find the gradient of the line passing through the points B and C.	**(2)**
v)	Show that the equation of the line passing through the points B and C is $5x-4y-9=0$.	**(2)**
vi)	Find the x intercept of the line in part (v)	**(1)**
vii)	Find the perpendicular distance from A to the line $5x-4y-9=0$.	**(2)**
viii)	Find the coordinates of D, such that $ABCD$ forms a parallelogram.	**(1)**
ix)	Find the area of the parallelogram $ABCD$.	**(1)**
x)	If A is the mid point of the points C and E, find the coordinates of E.	**(2)**
xi)	Find the equation of the line that passes through A and makes an angle of 135° with the positive direction of the x axis.	**(2)**

Question 2.(start a new page)

a) Find the equation of the line that passes through the point (2,3) and is parallel to the line $y=3x-4$. **(2)**

b) Find the equation of the line, in general form, that passes through the point $(2,-1)$ and is perpendicular to the line $5x-3y+9=0$. **(4)**

c) Find the equation of the line passing through the intersection of the lines whose equations are $3x-4y-1=0$ and $2x+3y-5=0$ and the point (1,2). **(3)**

Question 3. (start a new page)

a) Show that the line $3x+4y-20=0$ is a tangent to the circle $x^2+y^2=16$. **(3)**

b) Find the shortest distance between the lines $3x-4y+9=0$ and $3x-4y-20=0$. **(3)**

Question 4. (start a new page)

a) Find the value of a and b if $3\sqrt{5}-\dfrac{1}{\sqrt{5}-2}=a+b\sqrt{5}$ **(3)**

b) Solve $x^2 \le 5x$ **(3)**

c) Solve $|x-1| \ge 4$ **(2)**

d) Factorise $a^2-b^2+2bc-c^2$ **(2)**

e) Find x and y if $\dfrac{4^x}{16}=8^{x+y}$ and $2^{2x+y}=128$ **(4)**

Assessment Task 3: Paper 1

Q1) a) $3x = 60$
$x = 20°$

b) $x = 360 - (90+70+140)$
$= 60°$

c) $a = 73°$, $b = 107°$

d) $2x+10+3x = 100$
$5x = 90$
$x = 18$

e) $x + 60 = 180$
$x = 120°$

f) $x + x = 140$
$x = 70°$

g) $9x = 270$
$x = 30°$

h) $\frac{x}{6} = \frac{4}{5}$
$x = 4\frac{4}{5}$

i) $60 + m + 80 = 180$
$m = 40°$

j) $28 + 28 + 60 + a = 180$
$a = 64$

Q2) a) $\angle DGF = 100°$ (corresponding angles in parallel lines $AB \parallel CD$)
$x = 100°$ vertically opposite angles are equal.

b) $\angle AEC = 115°$ angle sum of a quadrilateral $= 360°$.
$\angle AED = 65°$ straight angle.
$x = 65°$ base angles of an isosceles Δ are equal

c) $\angle LEK = 27°$ base angles of an isosceles Δ are equal
$\angle HKE = 54°$ exterior angle of a triangle equals the sum of the two opposite interior angles.
$\angle HEK = 54°$ base angles of an isosceles triangle are equal.
$x = 72°$ angle sum of a Δ = 180°

d) $\angle XYA = 150°$ straight angle.
$\angle XPA = 36°$

alternate angles in parallel lines $PS \parallel OR$ opposite sides of a rhombus
$\angle PAY = 90°$ diagonals of a rhombus intersect a right angles.
$x = 84°$ angle sum of a quadrila[teral] $= 360°$

Q3)
a) $\angle NMO = \angle OPQ$ alternate angles in parallel lines.
$\angle OPQ = \angle OQP$ base angles of an isosceles Δ are equ[al]
∴ $\angle NMO = \angle OQP$.

b) ΔOCA, ΔOCB
OA = OB radii of same circle
OC common
$\angle OCA = \angle OCB = 90°$
∴ ΔOCA ≡ ΔOCB R[HS]
∴ AC = BC corresponding sides in congruent Δ's
∴ C mid pt of AB
∴ OC bisects AB.

c) i) $\Delta PAX, \Delta BAC$

$\angle A$ common

$\angle APX = \angle ABC$ given

$\therefore \Delta PAX \parallel\!| \Delta BAC$. (AAA)

ii) $PX^2 = 5^2 - 4^2$

$PX = 3$.

$\frac{AC}{5} = \frac{5}{3}$

$AC = \frac{25}{3}$

d) $KL = HM$

opposite sides of a parallelogram are equal.

$\therefore LN = OH$

$\angle KLN = \angle MHO$

alternate angles in parallel lines $KL \parallel HM$.

$\therefore \Delta LKN \equiv \Delta OHM$. (SAS)

ii) $KN = OM$

corresponding sides in congruent Δ's.

$\angle LNK = \angle HOM$ (congruent Δ's)

$\therefore \angle KNO = \angle MON$

straight angle

$\therefore KN \parallel OM$ alternate angles equal

$\therefore KNMO$ parallelogram opposite sides equal and parallel.

Q4)

a) $x^2 - 3x - 5 = 0$

i) $\alpha + \beta = 3$

ii) $\alpha\beta = -5$

iii) $\alpha^2\beta + \alpha\beta^2$

$= \alpha\beta(\alpha+\beta)$

$= -5 \times 3$

$= -15$

iv) $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 9 + 10$

$= 19$

v) $\left(\alpha - \frac{1}{\beta}\right)\left(\beta - \frac{1}{\alpha}\right)$

$= \alpha\beta - 1 - 1 + \frac{1}{\alpha\beta}$

$= -5 - 2 - \frac{1}{5}$

$= -7\frac{1}{5}$

b) $x^2 - (3+\sqrt{2}+3-\sqrt{2})x + (3+\sqrt{2})(3-\sqrt{2}) = 0$

$= x^2 - 6x + 7 = 0$

c) i) $x^4 - 3x^2 - 4 = 0$

$(x^2-4)(x^2+1) = 0$

$x^2 = 4$, $x^2 = -1$

$x = \pm 2$ no solution

ii) $9^x - 12.3^x + 27 = 0$

$(3^2)^x - 12.3^x + 27 = 0$

$(3^x)^2 - 12.3^x + 27 = 0$

let $a = 3^x$

$a^2 - 12a + 27 = 0$

$(a-9)(a-3) = 0$

$a = 9$, $a = 3$

$3^x = 9$ $3^x = 3$

$\therefore x = 2, 1$.

d) $x^2 - (k-4)x + 3k =$

$\alpha\beta = 2(\alpha+\beta)$

$3k = 2(k-4)$

$3k = 2k - 8$

$k = -8$

e) $2x^2 - 7x - 4 \equiv A(x+2)^2 + B(x+2) + C$

equating co-efficients of x^2: $A = 2$.

let $x = -2$: $18 = C$

let $x = -1$: $5 = A + B + C$

$5 = 2 + B + 18$

$B = -15$.

$\therefore A = 2, B = -15, C = 18$

f) $m = 2 - \frac{1}{m}$

$m^2 = 2m - 1$

$m^2 - 2m + 1 = 0$

$(m-1)^2 = 0$

$m = 1$.

$\frac{x^2}{x+1} = 1$

$x^2 = x + 1$

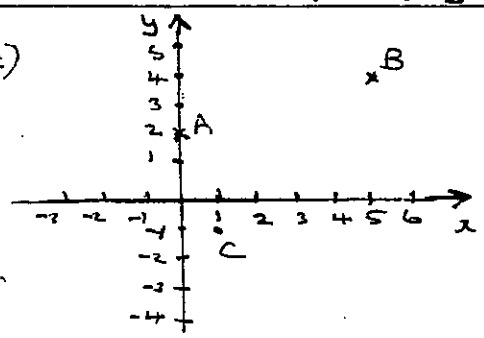
$x^2 - x - 1 = 0$

$x = \frac{1 \pm \sqrt{1+4}}{2}$

$x = \frac{1 \pm \sqrt{5}}{2}$.

Assessment task 3. PAPER 2.

i)



ii) $BC = \sqrt{(5-1)^2+(4+1)^2}$

$= \sqrt{16+25}$

$= \sqrt{41}$

iii) $\left(\frac{0+1}{2}, \frac{2-1}{2}\right)$

$= \left(\frac{1}{2}, \frac{1}{2}\right)$

iv) $m = \frac{4+1}{5-1}$

$= \frac{5}{4}$

v) $y-4 = \frac{5}{4}(x-5)$

$4y-16 = 5x-25$

$5x-4y-9=0$

vi) $y=0$:

$5x-9=0$

$x = \frac{9}{5}$

vii) $(0,2)$

dist $= \left|\frac{5\times 0 + -4\times 2 - 9}{\sqrt{5^2+4^2}}\right|$

$= \frac{17}{\sqrt{41}}$

viii) $D\ (-3,-6)$

ix) $\sqrt{41} \times \frac{17}{\sqrt{41}}$

$= 17$ sq units

x) $E\ (x,y)$

$\frac{x+1}{2} = 0 \quad \frac{y-1}{2} = 2$

$E\ (-1, 5)$

xi) $(0,2)$ $m = \tan 135°$

$= -1$

$y-2 = -1(x-0)$

$y = 2-x$.

$x+y-2=0$.

Q2) a) $y-3 = 3(x-2)$

$y-3 = 3x-6$

$y = 3x-3$.

b) $m = \frac{5}{3}$

$\therefore \perp$ gradient $= -\frac{3}{5}$

$y+1 = -\frac{3}{5}(x-2)$

$5y+5 = -3x+6$

$3x+5y-1=0$

c)

$3x-4y-1+k(2x+3y-5)=0$

passes through $(1,2)$

$3-8-1+3k=0$

$3k=6$

$k=2$.

$3x-4y-1+2(2x+3y-5)=$

$3x-4y-1+4x+6y-10=$

$7x+2y-11=0$

Q3)

a) tangent if dista[nce] from centre of the circle equals the radius (4)

$(0,0)$ $3x+4y-20=$

$d = \left|\frac{0+0-20}{\sqrt{3^2+4^2}}\right|$

$= \frac{20}{5}$

$= 4$

= radius of the circle. $\therefore$ a tange[nt]

$3x-4y+9=0$,

$3x-4y-20=0$

$(0,-5)$ lies on the 2nd line.

$\therefore d = \left|\frac{3\times 0 + -4\times -5 + 9}{\sqrt{3^2+4^2}}\right|$

$= \frac{29}{5}$ units.

Q4) $3\sqrt{5} - \frac{1}{\sqrt{5}-2} = a + b\sqrt{5}$

LHS

$= 3\sqrt{5} - \frac{1}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$

$= 3\sqrt{5} - \frac{\sqrt{5}+2}{5-4}$

$= 3\sqrt{5} - \sqrt{5} - 2.$

$= 2\sqrt{5} - 2$

$\therefore a = -2, \ b = 2.$

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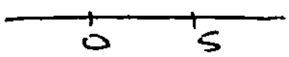
b) $x^2 \leq 5x$

$x^2 - 5x \leq 0$

$x(x-5) \leq 0$



test $x = 1$. $1 - 5 \leq 0$ T.

$\therefore \ 0 \leq x \leq 5$

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c) $|x-1| \geq 4$

$4 \leq x - 1$ or $x - 1 \leq -4$

$5 \leq x$ or $x \leq -3$.

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d) $a^2 - b^2 + 2bc - c^2$

$= a^2 - (b^2 - 2bc + c^2)$

$= a^2 - (b-c)^2$

$= (a + (b-c))(a - (b-c))$

$= (a+b-c)(a-b+c)$

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e) $\frac{4^x}{16} = 8^{x+y}$

$4^x = 16 . 8^{x+y}$

$2^{2x} = 2^4 . 2^{3x+3y}$

$2^{2x} = 2^{3x+3y+4}$

$\therefore 2x = 3x + 3y + 4$

$x + 3y = -4$ --- (1)

$2^{2x+y} = 128$

$2^{2x+y} = 2^7$

$\therefore 2x + y = 7$ ---- (2)

(1)×2. $2x + 6y = -8$ -- (3)

(3)−(2) $5y = -15$

$y = -3$

Sub $y = -3$ into (1)

$x - 9 = -4$

$x = 5.$
