YEAR 11 Mathematics
Preliminary Course
Assessment Task 2
June 2011
Paper 1

1. There are 3 sections.
2. Answer each question on your own paper showing all necessary working
3. Start each section on a new page
4. Calculators may be used

Topic	Mark
1. Section 1 (Quadratic Polynomial)	/18
2. Section 2 (Quadratic Polynomial)	/14
3. Section 3 (Geometry- angles, polygons)	/26

TOTAL **/58**

Section 1 Paper 1
(Quadratic Polynomial)

1) Solve the equation $2x^2 - 5x + 1 = 0$ leaving your answer in surd form.

2) Determine if the roots of the quadratic equation $9x^2 - 6x + 1 = 0$ are real or unreal, rational or irrational, equal or unequal.

3) Without solving, show that the roots of the quadratic equation $2x^2 - 4x + 9 = 0$ are unreal.

4) For what values of r does the equation $rx^2 - (3r + 1)x + r = 0$ have equal roots?

5) Without sketching, show that the quadratic function $f(x) = 2x^2 - 3x + 7$ lies entirely above the x-axis.

6) For what values of r will the quadratic expression $rx^2 - 4x - 7$ be negative definite?

7) Find a quadratic function $f(x) = ax^2 + bx + c$, which is positive for $\sqrt{2} < x < 3\sqrt{2}$ only.

8) Find a quadratic equation for which the sum of the roots is -10 and their product is 24.

9) Find the value of r for which the roots of the quadratic equation $3x^2 - 4x + r = 0$ are reciprocals of one another.

10) If $x = 3$ is one root of the equation $x^2 + 5x - r^2 = 0$, find the other root and the value of r.

Section 2 Paper 1
(Quadratic Polynomial)

1) Let α and β be the roots of the equation $x^2 + 5x - 2 = 0$. Without solving, find the value of:
   a. $\alpha + \beta$; b. $\alpha\beta$; c. $\alpha^2\beta + \alpha\beta^2$; d. $\alpha^3 + \beta^3$

2) Solve $x^4 - x^2 - 2 = 0$

3) Solve $9^x - 4(3^x) + 3 = 0$

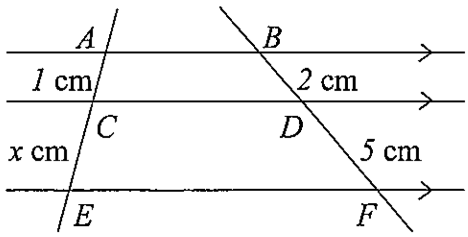
4) Rewrite the expression $x^2 - 4x - 3$ in the form $a(x - 1)^2 + b(x + 2)$

5) Find the values of m for which $12 + 4m - m^2 > 0$

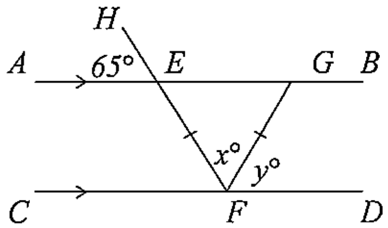
Section 3 Paper 1
(Geometry – Angles and polygons)

Reasons **NOT** required for questions 1 to 9

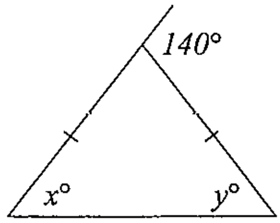
1) Use the information in the diagram given below to find the value of x.



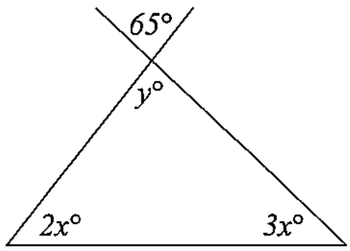
2) In the diagram given below, $AB \parallel CD$ and $EF = GF$. Find the values of x and y if $\angle AEH = 65°$.



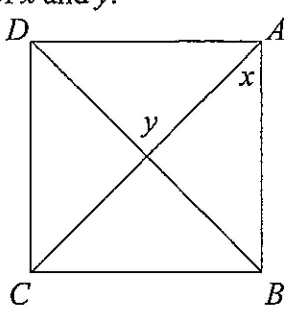
3) Use the information in the figure given below to find the values of x and y.



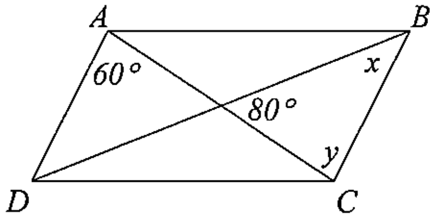
4) Use the information in the figure given below to find the values of x and y.



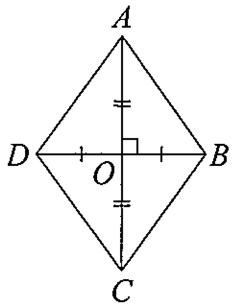
5) *ABCD* is a square. Find the values of x and y.



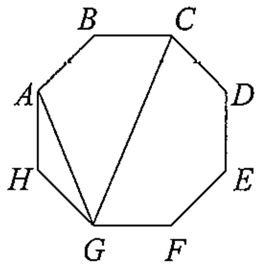
6) *ABCD* is a parallelogram. Find the values of x and y.



7) Find the area and perimeter of the rhombus *ABCD*, given that $AO = 8$ cm and $BO = 6$ cm.



8)

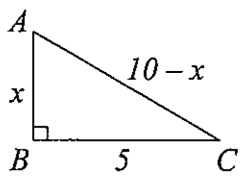


NOT TO SCALE

ABCDEFGH is regular octagon.

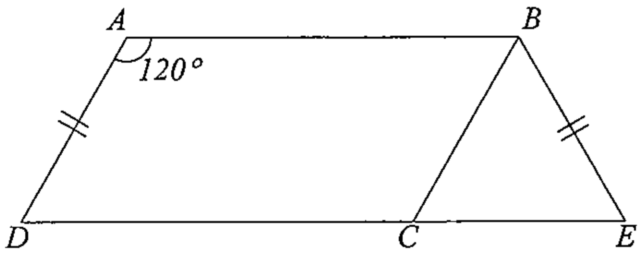
i. Calculate the size of $\angle ABC$.
ii. Calculate the size of $\angle GAH$.
iii. Using (i), or otherwise, calculate the size of $\angle CGF$.
iv. Hence, calculate the size of $\angle AGC$.

9)



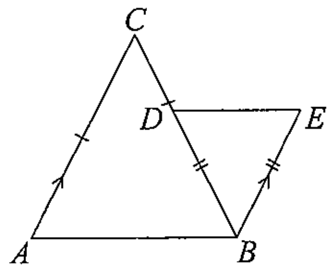
In the diagram $\angle ABC$ is a right angle. Find the value of x.

10)



The diagram shows a parallelogram $ABCD$ with $\angle DAB = 120°$. The side DC is produced to E so that $AD = BE$. Prove that ΔBCE is equilateral.

11)



In the diagram above, $AC = BC$, $BD = BE$ and $AC \parallel BE$. Prove that $AB \parallel DE$.

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Topic	Mark
1. Section 1 (Linear Functions)	/18
2. Section 2 (Linear Functions)	/14
3. Section 3 (Geometry- Congruency, similarity)	/20

TOTAL **/52**

Section 1 Linear Functions

1) Express the equation of the straight line $y = 2 - \frac{x}{3}$ in general form.

2) Write down the gradient and y-intercept of the line $5x - 3y + 2 = 0$.

3) Show that the point $(-3, -7)$ lies on the line $5x - 2y + 1 = 0$.

4) Sketch the graph of $y = 2x + 1$, indicating clearly where the line cuts the x and y axes.

5) Sketch the graph of $y = 3$.

6) Find the gradient of the straight line passing through the points $(-5, 6)$ and $(-2, 3)$.

7) Find (to 1 decimal place) the gradient of the straight line with an inclination of $97°$ with the positive direction of the x axis.

8) Find the length of the interval joining $(4, 3)$ and $(7, 7)$.

9) Find the co-ordinates of the mid-point of the interval joining $(2, 1)$ and $(4, 7)$.

10) The mid-point of the interval joining $(2x + 1, 1 + y)$ and $(4y + 1, 3 + y)$ is $(5, 1)$. Find x and y.

11) Find the equation of the straight line (in gradient-intercept form) which passes through $(2, 3)$ and has a gradient of 1.

12) Find the equation of the straight line (in general form) which passes through the points $(3, 2)$ and $(5, 3)$.

Section 2 Linear Functions

1) Find the equation of the straight line passing through $(-3, 1)$ and parallel to $6x - 12y + 5 = 0$.

2) Find the equation of the straight line passing through $(5, 2)$ and the point of intersection of the lines $3x - y - 5 = 0$ and $3x + y - 7 = 0$.

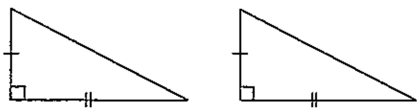
3) Find the perpendicular distance of the point $(5, 4)$ from the straight line $3x + 4y + 9 = 0$.

4) Show (without sketching on the number plane) that the points $(-2, 2)$ and $(3, -2)$ lie on opposite sides of the straight line $2x - y + 2 = 0$.

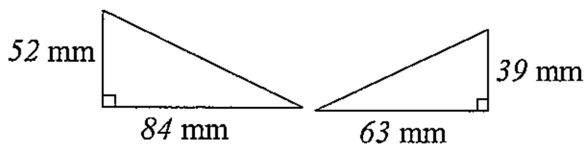
5) $P(1, 2)$ is a point on the line $3x + y - 5 = 0$. Find the coordinates of the points on this line so that their distance from P is $\sqrt{10}$ units.

Section 3 Geometry (Congruence and similarity)

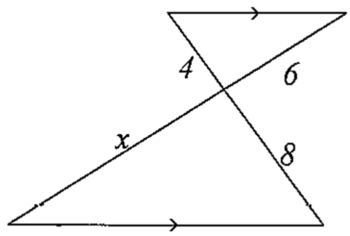
1) The triangles below are congruent. State the condition for the congruency (*SSS*, *SAS*, *AAS* or *RHS*).



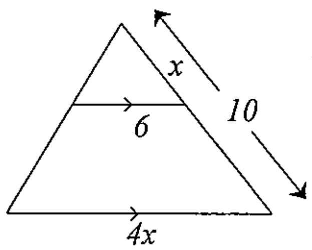
2) Are the triangles below similar? Give a reason for your answer.



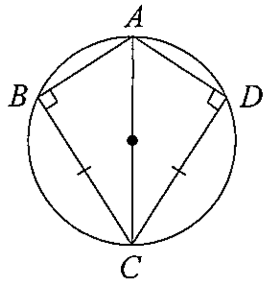
3) In the diagram given below, find the value of x. (All lengths are in cm.)



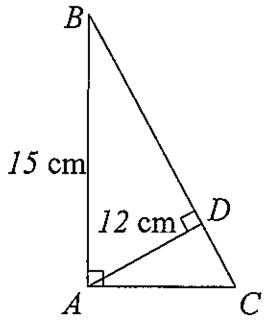
4) In the diagram given below, find the value of x. (All lengths are in cm.)



5) Prove that $\Delta ABC \equiv \Delta ADC$.



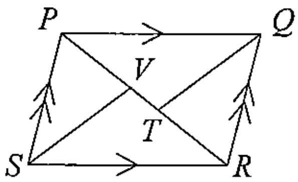
6) ΔABC is right-angled at A and AD is drawn perpendicular to BC. $AB = 15$ cm and $AD = 12$ cm. Copy the given diagram onto your answer sheet.



NOT TO SCALE

i. Show that $BD = 9$ cm.
ii. Prove that ΔABC is similar to ΔDBA.
iii. Hence find the length of AC.

7)



NOT TO SCALE

$PQRS$ is a parallelogram. TQ bisects $\angle PQR$ and VS bisects $\angle PSR$.
i. Copy this diagram onto your answer booklet.
ii. State why $\angle PQR = \angle PSR$.
iii. Prove that ΔPVS and ΔRTQ are congruent.
iv. Hence find the length of TV if $PR = 20$ cm and $TR = 8$ cm

Year 11 (Paper 1)
Mathematics (Solutions)

Q1/ $x = \dfrac{5 \pm \sqrt{25 - 4 \times 1 \times 2}}{4}$

$= \dfrac{5 \pm \sqrt{17}}{4}$ (1)

Q2/ $\Delta = 36 - 4 \times 9 \times 1$

$= 0$ $\therefore$ Roots are real, rational & equal (1)

Q3/ $\Delta = 16 - 4 \times 9 \times 2$

$= -$

$\therefore$ $\Delta < 0$ Roots are unreal (1)

Q4/ when $\Delta = 0$

$(3r+1)^2 - 4r^2 = 0$

$9r^2 + 6r + 1 - 4r^2 = 0$

$5r^2 + 6r + 1 = 0$

$5r \quad 1$

$r \quad 1$

$(5r+1)(r+1) = 0$

$r = -\frac{1}{5}$ or -1 (2)

25) $a = 2 \quad \therefore a > 0$

$b^2 - 4ac = 9 - 4 \times 2 \times 7$

$=$ (2)

$\therefore \Delta < 0$

$\therefore$ positive definite

$\therefore$ lies entirely above x axis

26/ $r < 0$ and $\Delta < 0$

$\Delta = 16 - 4 \times r \times -7$

$\Delta = 16 + 28r$

$16 + 28r < 0$

$28r < -16$

$r < \frac{-16}{28}$ (2)

$r < -\frac{4}{7}$

$\therefore \quad r < -\frac{4}{7}$ to be neg def.

Q7/ $x^2 - (\sqrt{2} + 3\sqrt{2})x + \sqrt{2} \cdot 3\sqrt{2} = 0$

$x^2 - 4\sqrt{2}x + 6 = 0$

But concave down (2)

so

$-x^2 + 4\sqrt{2}x - 6 = 0$

Q8/ $x^2 + 10x + 24 = 0$ (2)

Q9/ $\alpha \times \frac{1}{\alpha} = \frac{r}{3}$

$1 = \frac{r}{3}$ (2)

$r = 3$

10/ $9 + 15 - r^2 = 0$

$r^2 = 24$

$r = \pm 2\sqrt{6}$

$3 + \beta = -5$ (3)

$\beta = -8$

$\therefore$ other root is -8

Section 2 Paper 1

1/ (a) -5 (1)

(b) -2 (1)

(c) $\alpha\beta(\alpha+\beta)$

-2×-5

$= 10$ (2)

(d) $(\alpha+\beta)(\alpha^2 - \alpha\beta + \beta^2)$

$(\alpha+\beta)\left[(\alpha+\beta)^2 - 2\alpha\beta - \alpha\beta\right]$

$(\alpha+\beta)\left[(\alpha+\beta)^2 - 3\alpha\beta\right]$

$(-5)\left[25 - 3 \times -2\right]$ (2)

$= -155$

2/ $x^4 - x^2 - 2 = 0$

x^2 -2

x^2 $+1$

$(x^2 - 2)(x^2 + 1) = 0$ (2)

$x = \pm\sqrt{2}$ no real solutions for $x^2 + 1 = 0$

4/ Let $u = 3^x$

$u^2 - 4u + 3 = 0$

$(u-3)(u-1) = 0$

$u = 3, 1$

$3^x = 3$ and $3^x = 1$

$x = 1$ $\quad x = 0$

5/ $x^2 - 4x - 3 \equiv a(x^2 - 2x + 1) + bx + 2b$

$\equiv ax^2 - 2ax + a + bx + 2b$

$\therefore a = 1$

$\equiv x^2 - 2x + 1 + bx + 2b$

$\therefore 2b + 1 = -3$

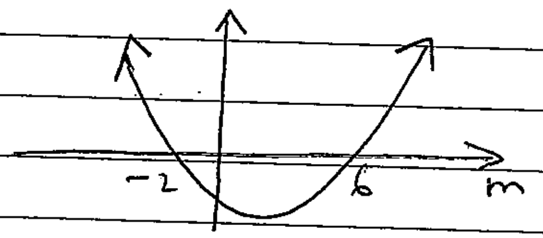
$2b = -4$

$b = -2$

$\therefore x^2 - 4x - 3 \equiv (x-1)^2 - 2(x+2)$

6/ $m^2 - 4m - 12 < 0$

$(m-6)(m+2) < 0$



$\therefore -2 < m < 6$

Geometry

1/ $\frac{x}{1} = \frac{5}{2}$

$x = 2\frac{1}{2}$ (2)

2/ $x = 50°$, $y = 65°$ (2)

3/ $x = y = 70°$ (2)

4/ $y = 65°$

$5x = 115$

$x = 23°$ (2)

5/ $x = 45°$

$y = 90°$ (2)

6/ $y = 60°$

$x = 40°$ (2)

7/ Perimeter $= 40\text{cm}$

Area $= 96\text{cm}^2$ (2)

8/ (i) $\frac{6 \times 180}{8} = 135°$

$\therefore \angle ABC = 135°$ (1)

(ii) $\frac{180 - 135}{2} = 22\frac{1}{2}°$ (1)

(iii) $67\frac{1}{2}°$ (1)

(iv) $135 - 22\frac{1}{2} - 67\frac{1}{2} = 45$ (1)

9/ $(10 - x)^2 = x^2 + 25$

$100 - 20x + \cancel{x^2} = \cancel{x^2} + 25$

$75 = 20x$ $x = 3.75$ (2)

10/ $\angle DCB = 120°$ (opp $\angle$'s parallelogram =)
$\angle BCE = 60°$ (adj supp $\angle$)
$AD = BC$ (opp sides parallelogram =)
$\therefore BC = BE$ (given $AD = BE$)
$\therefore \triangle BCE$ is isosceles
as $\angle BCE = \angle BEC = 60°$
$\therefore$ BCE is equilateral

11/ Let $\angle ACB = \alpha$
$\therefore \angle CB\cancel{A} = \alpha$ (alt $\angle$'s $AC \parallel BE$)
$\angle BDE = \frac{180-\alpha}{2}$ (base $\angle$'s = isoc. $\triangle$ BDE)

$\angle CBA = \frac{180-\alpha}{2}$ (base $\angle$'s = isoc. $\triangle$ ACB)

$\therefore \angle BDE = \angle CBA$ and are in alt. positions
$\therefore AB \parallel DE$

Paper two

Section 1 Linear Functions

1) $3y = 6 - x$

$x + 3y - 6 = 0$ (1)

2) $3y = 5x + 2$

$y = \frac{5}{3}x + \frac{2}{3}$ gradient $= \frac{5}{3}$

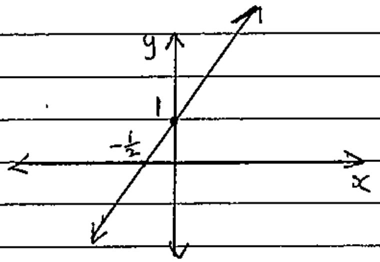
y intercept $= \frac{2}{3}$ (2)

3) $5(-3) - 2(-7) + 1 = 0$

$-15 + 14 + 1 = 0$

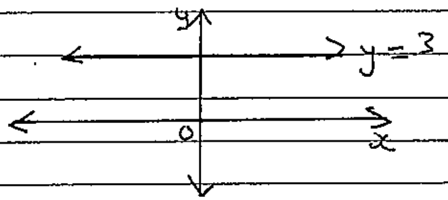
$0 = 0$ (1)

4)



(2)

5)



(1)

6) $m = \frac{6-3}{-5+2}$

$= \frac{3}{-3}$ (1)

$m = -1$

7) $m = \tan 97°$

$m = -8{\cdot}1$ (2)

8) $d = \sqrt{(4-7)^2 + (3-7)^2}$

$= \sqrt{(-3)^2 + (-4)^2}$ (1)

$= \sqrt{9+16}$

$= 5$

9) $\left(\frac{2+4}{2}, \frac{1+7}{2}\right) = (3, 4)$ (1)

10) $\frac{2x+1+4y+1}{2} = 5$

$2x + 4y + 2 = 10$

$2x + 4y = 8$

$x + 2y = 4$ ......(1)

$\frac{1+y+3+y}{2} = 1$ (2)

$2y + 4 = 2$

$2y = -2$

$y = -1$

Sub $y = -1$ into (1)

$x - 2 = 4$

$x = 6$

11) $y - 3 = 1(x - 2)$

$y - 3 = x - 2$ (2)

$y = x + 1$

12) $m = \frac{-1}{-2}$ $y - 2 = \frac{1}{2}(x - 3)$

$2y - 4 = x - 3$

$m = \frac{1}{2}$ $0 = x - 2y + 1$ (2)

13) $y = -5x + \cancel{x}$

∴ line parallel has a gradient of -5 (1)

14) $m_{\perp} = 3$

Section 2 Linear Functions

1) $12y = 6x + 5$

$y = \frac{1}{2}x + \frac{5}{12}$

$y - 1 = \frac{1}{2}(x + 3)$

$2y - 2 = x + 3$

$0 = x - 2y + 5$ (2)

OR

$y = \frac{x}{2} + \frac{5}{2}$

2) $\left.\begin{array}{l} 3x + y = 7 \\ 3x - y = 5 \end{array}\right\}$ Add

$6x = 12$

$x = 2$

$6 - y = 5$

$y = 1$

$m = \frac{1-2}{2-5}$

$= \frac{-1}{-3}$

$= \frac{1}{3}$

$y - 1 = \frac{1}{3}(x - 2)$ (3)

$3y - 3 = x - 2$

$0 = x - 3y + 1$

OR

$y = \frac{x}{3} + \frac{1}{3}$

3) $d = \frac{|(3)(5) + (4)(4) + 9|}{\sqrt{9 + 16}}$

$= \frac{|15 + 16 + 9|}{5}$

$= \frac{40}{5}$ (2)

$= 8$ units

4) For $(-2, 2)$

d_1 (numerator) $= (2)(-2) + (-1)(2) + 2$

$= -4 - 2 + 2$

$= -4$

For $(3, -2)$

d_2 (numerator) $= (2)(3) + (-1)(-2) + 2$

$= 6 + 2 + 2$

$= 10$ (3)

As the numerator (without absolute value) of the $\perp$ distance formula gives values opposite in sign, the points are on opposite sides

5) $\sqrt{(x-1)^2 + (y-2)^2} = \sqrt{10}$

$(x-1)^2 + (y-2)^2 = 10$

$x^2 - 2x + 1 + y^2 - 4y + 4 = 10$

$x^2 - 2x + y^2 - 4y = 5$ --- (1)

For $3x + y - 5 = 0$

$y = -3x + 5$

$y = 5 - 3x$ --- (2)

Sub (2) into (1)

$x^2 - 2x + (5-3x)^2 - 4(5-3x) = 5$

$x^2 - 2x + 25 - 30x + 9x^2 - 20 + 12x = 5$

$10x^2 - 20x = 0$

$10x(x-2) = 0$

$x = 0, 2$

$y = 5, -1$

(4)

$\therefore$ Points are $(0,5)$ and $(2,-1)$

Section 3 Geometry

1) SAS (1)

2) $\frac{52}{39} = 1\frac{1}{3}$ $\frac{84}{63} = 1\frac{1}{3}$

$\therefore$ Δ's are similar as two sides in ratio, included $\angle$ equal (2)

3) $\frac{4}{8} = \frac{6}{x}$

$4x = 48$ (1)

$x = 12$

4) $\frac{x}{10} = \frac{6}{4x}$

$4x^2 = 60$

$x^2 = 15$ (2)

$x = \sqrt{15}$

5) $BC = DC$ (given)

$\angle ABC = \angle ADC$ (given)

AC is common (2)

$\therefore \Delta ABC \equiv \Delta ADC$ (RHS)

6) (i) $(BD)^2 = 15^2 - 12^2$

$(BD)^2 = 81$ (1)

$BD = 9$

(ii) $\angle BAC = \angle BDA = 90$ (given)

$\angle ABC = \angle ABD$ (common $\angle$) (3)

$\therefore \Delta ABC \parallel\!| \Delta DBA$ (equiangular)

(iii) $\frac{15}{9} = \frac{AC}{12}$

$9AC = 180$ (2)

$AC = 20$ cm

7) (ii) opp $\angle$s parallelogram are equal (1)

(iii) $\angle PSV = \angle TQR$ (bisected equal angles of parallelogram)

$\angle SPV = \angle QRT$ (alt $\angle$s $PS \parallel QR$)

$PS = QR$ (opp sides of parallelogram are equal) (4)

$\therefore \Delta PVS \equiv \Delta RTQ$ (AAS)

(iv) $20 - 16 = 4$ cm (1)