GOSFORD HIGH SCHOOL

NAME:

Task 2 June 2012
YEAR 11 Preliminary MATHEMATICS

Multiple Choice	/ 7
Part A Geometry	/16
Part B Linear Functions	/16
Part C Quadratic Polynomials	/16
Total	/55

General Instructions:

- Reading time – 5minutes
- Working time – 80 minutes

- Write using black or blue pen.
- Board-approved calculators may be used
- All necessary working should be shown in every question.

Total marks: - 120

- Attempt ALL Questions
- Answer Multiple choice questions on paper provided.
- Part A, B and C on your paper (start each part on a new sheet).

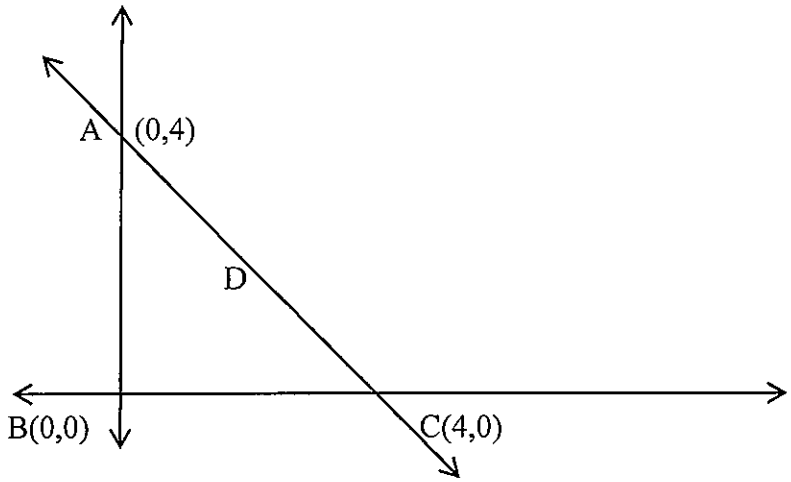
Multiple Choice Questions

Answers to the multiple choice on the answer sheet provided. (1 Mark Each)

1) A quadrilateral has equal diagonals that bisect each other at right angles, it is a

(A) Rectangle
(B) Rhombus
(C) Isosceles Trapezium
(D) Square

2) The following figure shows an isosceles triangle ABC. If D is the mid-point of AC, what is the length of AD?



(A) $2\sqrt{2}$

(B) $\sqrt{2}$

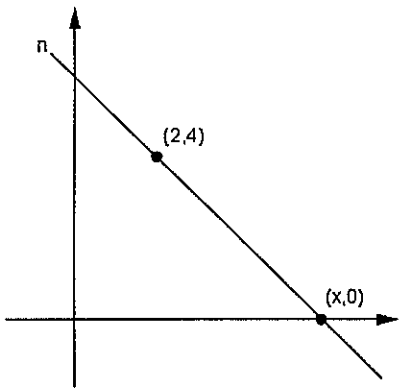
(C) $2+\sqrt{2}$

(D) $2-\sqrt{2}$

3) Two functions are defined as: $f(x) = 3x^2 - 4$ *and* $g(y) = y^2 - 2y$. How many values of the variable a satisfy the equation $(a) = g(2a)$?

(A) One
(B) Two
(C) Three
(D) None

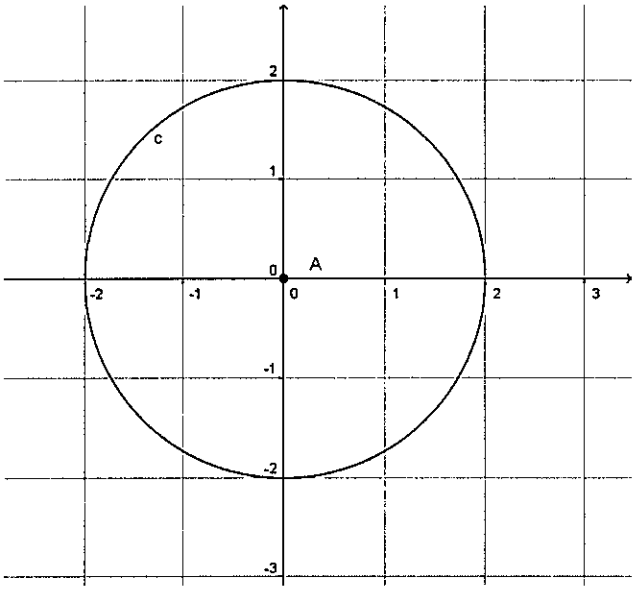
4)



In the above figure, the slope of line n is -1 . Find the value of x

(A) 3
(B) 4
(C) 5
(D) 6

5)



The above figure shows a circle drawn in an X-Y plane with centre A as the origin. If a line with equation $\boldsymbol{x + y = k}$ intersects the circle at two points, which of the following is NOT a possible value of $\boldsymbol{k}$?

(A) 0

(B) $-2\sqrt{2}+1$

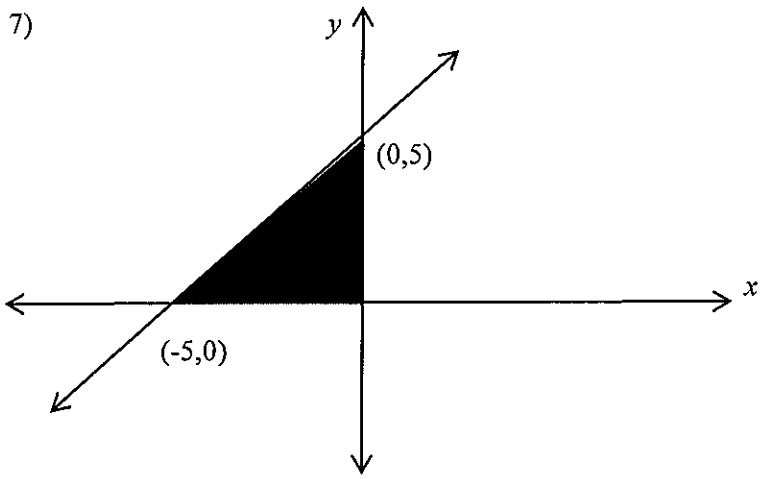
(C) 2

(D) $2\sqrt{2}+1$

6) Which of the following quadratic equations does **not** have real roots?

(A) $y = 3x^2 - 3x - 2$
(B) $y = 3x^2 + 3x - 2$
(C) $y = 3x^2 - 3x + 2$
(D) $y = 3x^2 + 3x - 2$

7)



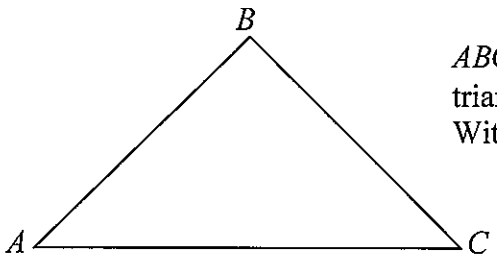
In the above diagram, the shaded region is given by:

(A) $x \leq 0,, y \leq 0$ and $x - y \leq -5$
(B) $x \geq 0, y \geq 0$ and $y \leq x - 5$
(C) $x \leq 0, y \geq 0$ and $x + y - 5 \leq 0$
(D) $x \leq 0, y \geq 0$ and $y \leq x + 5$

End of Multiple Choice

PART A **Start your answers on a new page.** 16 **Marks**

(a)



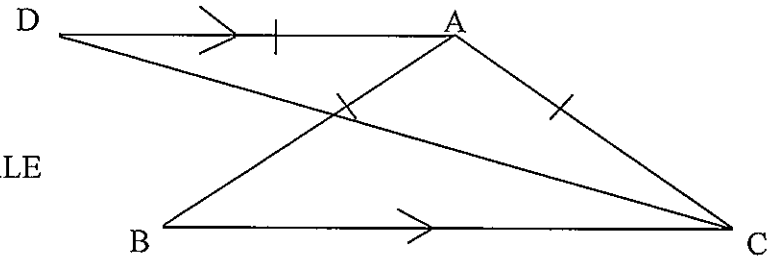
ABC is a right angled triangle with $\angle ABC = 90°$, With perpendicular bisector, *BD*.

(i) Copy diagram onto your paper indicating perpendicular bisector and showing all given information. (1)

(ii) Prove triangles *ABC* and *BDC* are similar (3)

(iii) Hence, show that BC^2 = ACxDC (1)

(b)



NOT TO SCALE

In the diagram, ΔABC is isosceles with *AB=AC*.
DA is parallel to *BC* and *DA=AC* with $\angle DAB = 55°$.

(i) Copy the diagram onto your worksheet clearly Showing all of the information. (1)

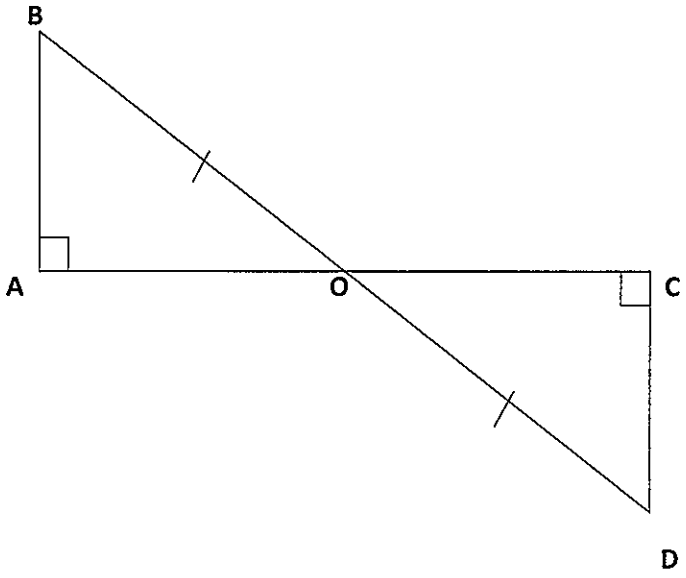
(ii) Show that $\angle ACB = 55°$ (2)

(iii) Find size of $\angle ADC$ (2)

(iv) Find the size of $\angle BCD$ (1)

PART A Contd.

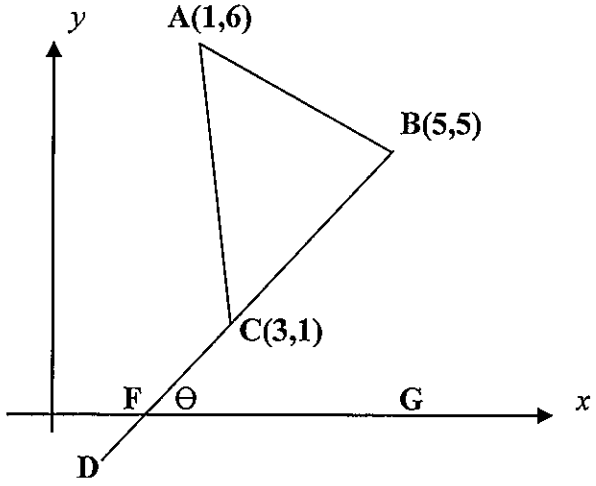
(c)



Prove that ΔAOB is congruent to ΔCOD. (3)

(d) Find the size of each exterior angle of a regular hexagon (2)

PART B **Start a new page.** 16 **Marks**



NOT TO SCALE

The points *A, B* and *C* have co-ordinates (1,6), (5,5) and (3,1). The points *B, C, F* and *D* are in a straight line, and angle BFG is Θ.

(a) Copy the diagram and answer the following.

(b) Find the gradient of the line *BC* (1)

(c) Calculate the size of angle θ to the nearest degree (1)

(d) Find the equation of line *BD* in general form (2)

(e) Find the co-ordinates of the point *D* if *C* is the midpoint of *BD* (2)

(f) Find the length of the interval joining *BC* (Leave answer in exact form) (2)

(g) i) Find the perpendicular distance of *A* from *BC* (Leave answer in exact form) (2)

ii) Hence, find the area of Δ *ABC* (2)

iii) Without calculating the area of Δ *ABD*, find the ratio of the areas of the triangles ABC and ABD (1)

(h) Find the co-ordinates of a point *E* such that *ABDE* forms a parallelogram (1)

(i) Find the area of the parallelogram *ABDE* . (2)

PART C **Start a new page.** **16 Marks**

(a) If α and β are the roots of the equation $2x^2 - 3x - 4 = 0$, find the value of

(i) $\alpha + \beta$ (1)

(ii) $\alpha\beta$ (1)

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$ (2)

(iv) $\alpha^2 + \beta^2$ (2)

(b) Solve the equation $4^x - 9\,(2^x) + 8 = 0$ (2)

(c) If $3x^2 - 5x + 6 = A(x-2)^2 + B(x-2) + C$ for all values of x, find values for A, B and C. (3)

(d) Find the values of m for which the equation $x^2 + (m-2)x + 4 = 0$ has equal roots; (2)

(e) The quadratic expression $Q(x)$ is given by:

$Q(x) = (1 + m)x^2 + 4x + m - 1$.

Find the range of values of m for which $Q(x) > 0$ for all x. (3)

End of Exam ☺

Name:_____ Teacher:_____

Multiple-choice answer sheet

Select the alternative A, B, C or D that best answers the question. Fill in the response oval completely, using a black pen.

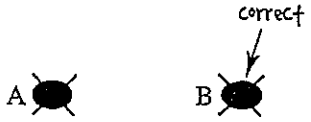
Sample: 2 + 4 = (A) 2 (B) 6 (C) 8 (D) 9

A○ B● C○ D○

If you think you have made a mistake, put a cross through the incorrect answer and fill in the new answer.

A● B⊗ C○ D○

If you change your mind and have crossed out what you consider to be the correct answer, then indicate the correct answer by writing the word correct and drawing an arrow as follows.



Start here →

1. A○ B○ C○ D○
2. A○ B○ C○ D○
3. A○ B○ C○ D○
4. A○ B○ C○ D○
5. A○ B○ C○ D○
6. A○ B○ C○ D○
7. A○ B○ C○ D○

Yr 11 Mathematics 2012

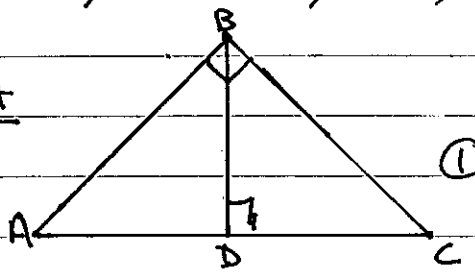
Task 2 June

Multiple Choice

1/ D 2/ A 3/ A
4/ D 5/ D 6/ C 7/ D

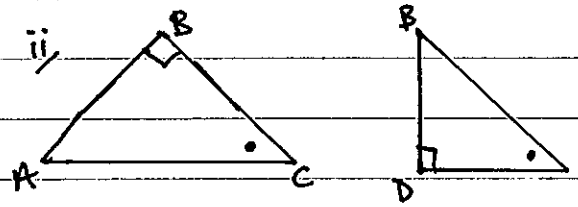
PART A

a) i/



(1)

ii/

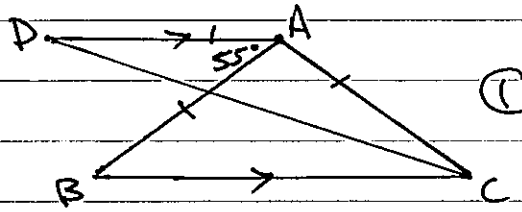


In ΔABC and ΔBDC
angle C is common (1)
angle ABC = angle BDC = 90° (given) (1)
$\therefore \Delta ABC \,|||\, \Delta BDC$
(equiangular) (1)

iii $\frac{BC}{AC} = \frac{DC}{BC}$ (corresponding sides in ratio) (1)

$BC^2 = AC \cdot DC$

b)



(1)

ii) $\angle ABC = 55°$ (alternate angle AD // BC) (1)

$\angle ABB = 55°$ (base angles equal isos triangle)

(1)

iii/ $\angle BAC = 180 - 2 \times 55$ (1)
$= 70$ (angle sum of triangle)

$\therefore \angle ADC = \frac{(80 - (70+55)}{2}$ (base angle of isos triangle ADC) (1)
$= 27.5$

iv/ $\angle BCD = 55 - 27.5$ (1)
$= 27.5°$ (adjacent angle)

c) ΔAOB and ΔCOD
$\angle AOB = \angle COD$ (vertically opposite) (1)
$\angle BAO = \angle DCO = 90°$ (given)
$BO = OD$ (given) (1)
$\therefore \Delta AOB \equiv \Delta COD$ (AAS) (1)

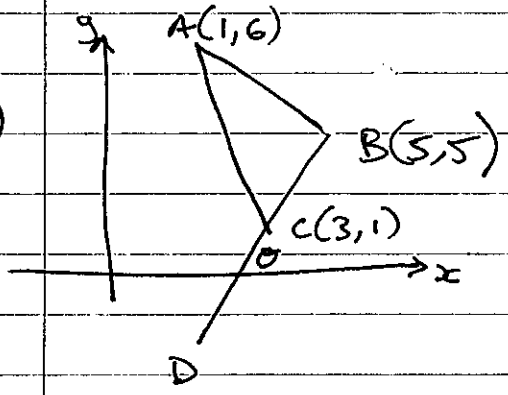
d) internal angle size $= \frac{(n-2) \times 180}{n}$

$= \frac{(6-2) \times 180}{6}$ (1)
$= 120°$

$\therefore$ External angle $180 - 120°$
$= 60°$ (1)

Part B.

a)



b) i) $m = \frac{5-1}{5-3} = \frac{4}{2} = 2$ (1)

c) $\tan\theta = 2$

$\theta = \tan^{-1} 2$

$\theta = 63°$ (1)

d) $y - 5 = 2(x-5)$ (1)

$y - 5 = 2x - 10$ (1)

Eqn BD: $2x - y - 5 = 0$

e) across 2 up 4 ∴ back 2 down 4 (1) various methods.

$D(3-2, 1-4)$

$D(1, -3)$ (1)

$d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

f) $d = \sqrt{2^2 + 4^2}$ (1)

$BC = \sqrt{20}$

$BC = 2\sqrt{5}$ (1)

$2x - y - 5 = 0$

g) i) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|2.1 - 6.1 - 5|}{\sqrt{2^2 + (1)^2}}$ (1)

$= \frac{|2 - 6 - 5|}{\sqrt{5}}$

$= \frac{9}{\sqrt{5}}$ or $\frac{9\sqrt{5}}{5}$ (1)

ii) $A_{\Delta ABC} = \frac{1}{2} \times 2\sqrt{5} \times \frac{9}{\sqrt{5}}$ (1)

$= 9$ units^2 (1)

iii) As ΔABC is half ΔABD ratio is 1 : 2 (1)

j) from A to B across 4 down 1

from D to E back 4 up 1 (1)

∴ $E(-3, -2)$

) Area of Parallelogram $= 2 \times$ Area Δ_{ABD} (1)

∴ $A = 2 \times 18$ units^2

$= 36$ units (1)

Part C

(a) for $2x^2 - 3x - 4$

i) $\alpha + \beta = \frac{-b}{a} = \frac{3}{2}$ (1)

ii) $\alpha\beta = \frac{c}{a} = -2$ (1)

iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

$= \frac{\alpha + \beta}{\alpha\beta}$ (1)

$= \frac{3/2}{-2}$

$= 3/-4$ (1)

iv) $\alpha^2 + \beta^2$

$= (\alpha+\beta)^2 - 2\alpha\beta$ (1)

$= \left(\frac{3}{2}\right)^2 + 4$

$= \frac{9}{4} + 4$

$= 6\frac{1}{4}$ (1)

b) $4^x - 9(2^x) + 8 = 0$

let $m = 2^x$

$\therefore 2^{2x} - 9(2^x) + 8 = 0$

$m^2 - 9m + 8 = 0$ (1)

$(m-8)(m-1) = 0$

$m = 8 \quad m = 1$

$\therefore 2^x = 8 \quad 2^x = 1$ (1)

$x = 3 \quad x = 0$

c) $3x^2 - 5x + 6 = A(x-2)^2 + B(x-2) + C$

$Ax^2 - 4Ax + 4A + Bx - 2B + C$

$Ax^2 - (4A - B)x + 4A - 2B + C$

equate co-efficients (1)

$A = 3$

$4A - B = 5$ (1)

$12 - B = 5$

$B = 7$

$4A - 2B + C = 6$

$12 - 14 + C = 6$

$C = 8$ (1)

$A = 3 \quad B = 7 \quad C = 8$

d) $x^2 + (m-2)x + 4 = 0$

$b^2 - 4ac = 0$ (1)

$(m-2)^2 - 4.1.4 = 0$

$m^2 - 4m + 4 - 16 = 0$

$m^2 - 4m - 12 = 0$

$(m-6)(m+2) = 0$

$m = 6$ or 2 (1)

e) $(1+m)x^2 + 4x + m - 1 > 0$

$\therefore b^2 - 4ac > 0$ (1)

$4^2 - 4.(1+m)(m-1)$

$16 - 4m^2 + 4$

$-4m^2 + 20 > 0$ (1)

$-4(m^2 - 5)$

$\therefore (m+\sqrt{5})(m-\sqrt{5}) < 0$

(1)