Name ____________________

Preliminary Mathematics

Assessment Task 2 - 2013

Time Allowed - 75 minutes

Instructions: Calculators may be used in any parts of the task. For 1 Mark Questions, the correct answer is sufficient to receive full marks. For Questions worth more than 1 Mark, necessary working MUST be shown to receive full marks.

Multiple Choice	/4
Question 5	/12
Question 6	/12
Question 7	/12
Total	/40

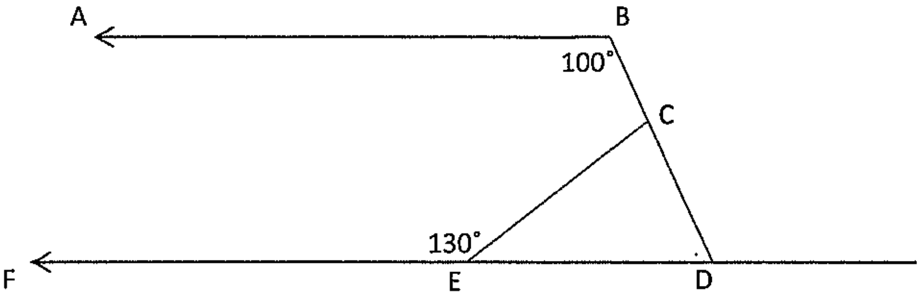
Fill in the correct answer on the answer sheet - Questions 1 - 4 are worth 1 mark each

1. If $f(x) = -x^3 - 2x^2 - 32$ find $f(-4)$

A 64 B 0

C -128 D -64

2.



In the diagram, AB is parallel to FD. The size of angle BCE is

A $100°$ B $110°$

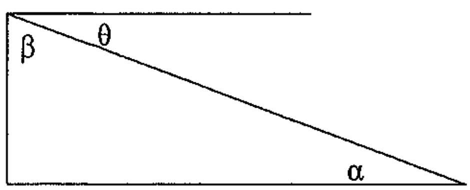
C $120°$ D $130°$

3. $\dfrac{\sin(360°-A)}{\sin(90°-A)} =$

A -1 B $tanA$

C 1 D $-tanA$

4.



A The angle of elevation is α and the angle of depression is β

B The angle of elevation is β and the angle of depression is θ

C The angle of elevation is θ and the angle of depression is β

D The angle of elevation is α and the angle of depression is θ

Question 5 12 Marks **(Begin a new sheet of paper)** **Marks**

a) (i) Sketch the graph of $y = \frac{1}{3-x}$ **2**

(ii) State its domain and range **2**

b) Sketch the region defined by

$y \geq x^2 - 4$ and $y < 2x - 4$ **4**

c) (i) Sketch the graph of $y = f(x)$ given

$$\begin{aligned} f(x) &= x^2 \quad for\ x > 2 \\ &= 2x - 1 \quad for -1 \leq x \leq 2 \\ &= -3 \quad for\ x < -1 \end{aligned}$$ **3**

1

(ii) Is this function even, odd or neither? Justify your answer.

Question 6 12 Marks **(Begin a new sheet of paper)** **Marks**

a) Find the exact value of

i) $\cos 330°$ **1**

ii) $\operatorname{cosec}(-120°)$ **1**

b) Solve i) $\sec\theta = 2$ for $0 \le \theta \le 360°$ **1**

ii) $\sin 2\theta = -\frac{\sqrt{3}}{2}$ for $0 \le \theta \le 360°$ **2**

c) Simplify $(1 + \tan^2\theta)(1 - \sin^2\theta)$ **1**

d) A ship sails on a bearing of 125° from a port P, to a port Q 20 km away.

At Q, it turns and sails a further 12 km on a bearing of 215° to R.

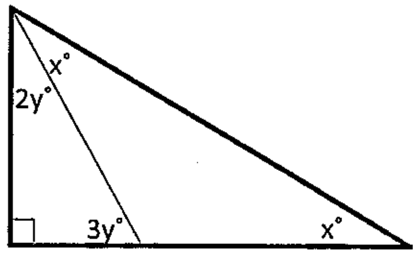
(i) Draw a diagram with this information **1**

(ii) On what bearing must it sail to return home? (nearest degree) **2**

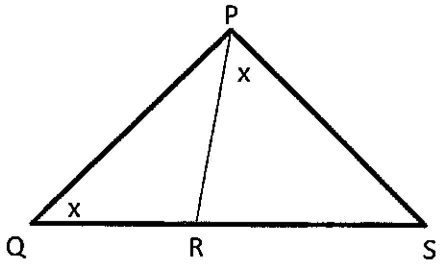
e) Prove $\dfrac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta} = \tan^2\theta + \cos^2\theta$ **3**

Question 7 12 Marks **(Begin a new sheet of paper)**

a) Find the values of x and y **2**



b) In the diagram, angle RPS =angle PQS
PS = 6cm, QS = 9cm and PQ = 7cm.



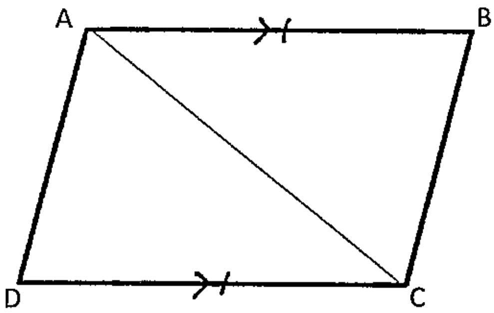
Copy the diagram

i) Prove angle PRS= angle QPS **1**

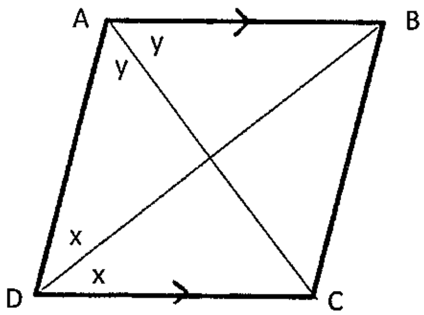
ii) Name a pair of similar triangles **1**

iii) Hence find the length of PR **2**

c) If a quadrilateral has one pair of opposite sides both equal and parallel, then it is a parallelogram. Copy the diagram and prove this test for parallelograms, by calling your quadrilateral ABCD and joining diagonal AC. (First prove $\Delta ABC \equiv \Delta CDA$) **2**



d) In a new quadrilateral ABCD, AB is parallel to DC and the bisectors of angles A and D pass through the points C and B respectively.



Copy the diagram

i) Prove that triangle ABD is isosceles **1**

ii) Prove that triangle ADC is isosceles **1**

iii) Prove that ABCD is a rhombus **2**

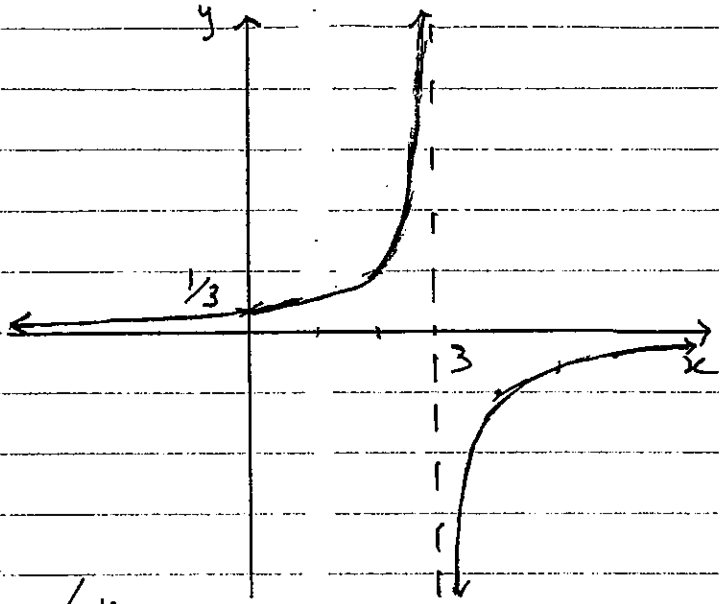
END OF TEST

Solutions to 2013 Prelim Task 2

1. $64 - 32 - 32 = 0$ B
2. D
3. $-\dfrac{\sin A}{\cos A} = -\tan A$ D
4. D

Question 5

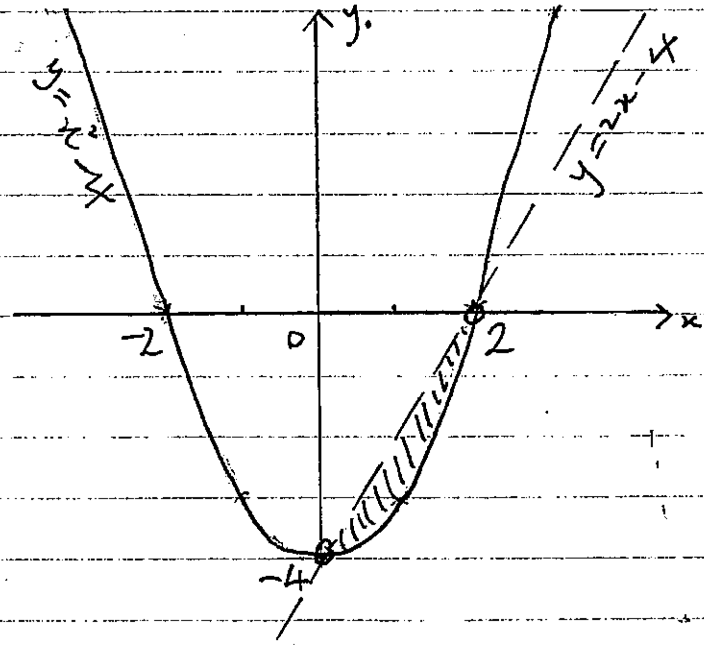
a) i) $y = \dfrac{1}{3-x} = \dfrac{-1}{x-3}$



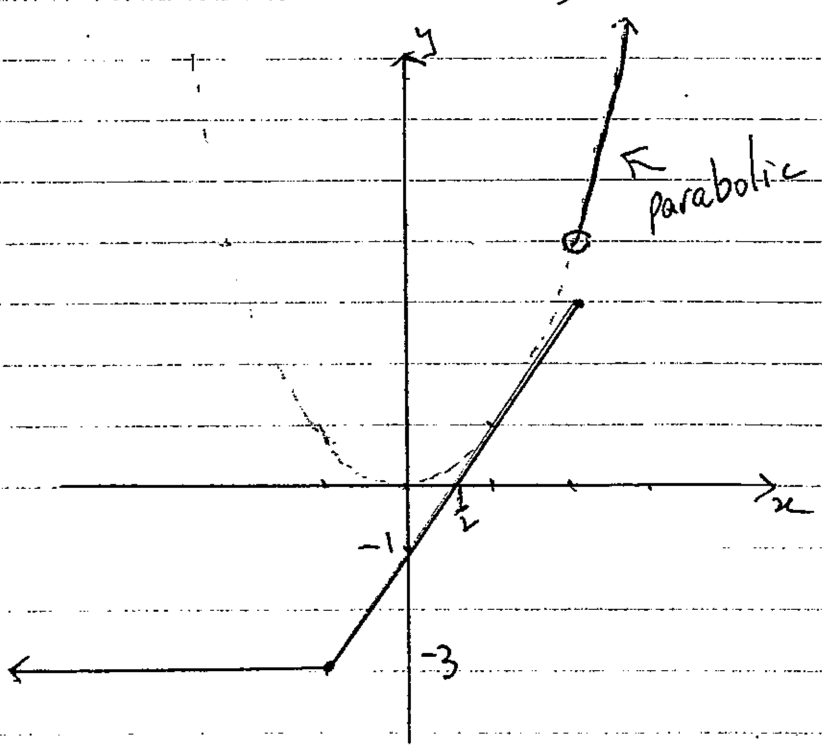
ii) Domain: All real x except $x=3$

Range: All real y except $y=0$

b)



c i)



c) ii) Neither even nor odd as there is no symmetry about y axis or point symmetry about origin.

Question 6

a) i) $\cos 330^\circ = +\cos 30^\circ$

$= \dfrac{\sqrt{3}}{2}$.

ii) $\operatorname{cosec}(-120^\circ) = \operatorname{cosec} 240^\circ$

$= -\operatorname{cosec} 60^\circ$

$= -\dfrac{2}{\sqrt{3}}$

b) i) $\sec\theta = 2$

$\cos\theta = \dfrac{1}{2}$

θ lies in 1st or 4th quad

$\theta = 60^\circ$ or 300°

ii) $\sin 2\theta = -\dfrac{\sqrt{3}}{2}$ $\qquad 0 \le 2\theta \le 720^\circ$

Acute $\angle$ is 60°. 2θ lies in 3rd or 4th quads.

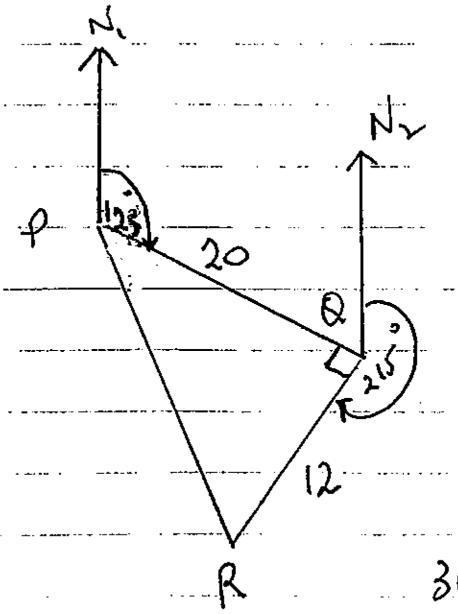
$2\theta = 240^\circ$ or 300° or 600° or 660°

$\theta = 120^\circ$ or 150° or 300° or 330°

c) $(1+\tan^2\theta)(1-\sin^2\theta) = \sec^2\theta \cdot \cos^2\theta$

$= 1$

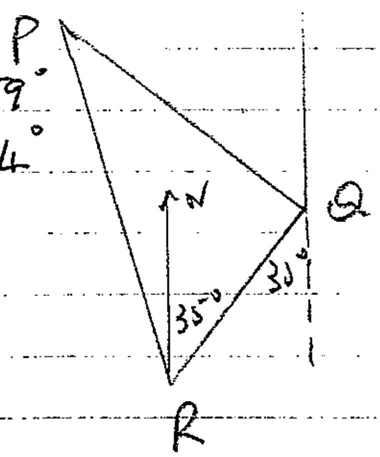
d)



$\tan R = \dfrac{20}{12}$

$= 1.666\ldots$

$\angle R = 59^\circ$



$\angle PRQ = 59^\circ$

$\angle PRN = 24^\circ$

$\therefore$ Bearing is

$360 - 24^\circ$

$= 336^\circ$

6e) $LHS = \dfrac{1 - \sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \dfrac{1}{\cos^2\theta} - \dfrac{\sin^2\theta\cos^2\theta}{\cos^2\theta}$

$= \sec^2\theta - \sin^2\theta$

$= (1+\tan^2\theta) - (1-\cos^2\theta)$

$= 1+\tan^2\theta - 1 + \cos^2\theta$

$= \tan^2\theta + \cos^2\theta$

<u>Question 7</u>

a) $2y + 3y = 90^\circ$ (angle sum of $\triangle$)

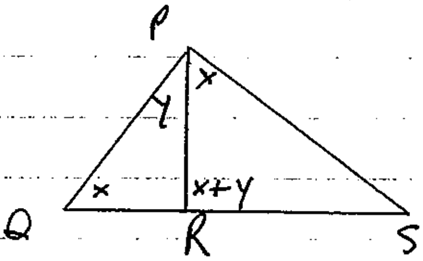
$y = 18$

$2x + 2y = 90$ (angle sum of $\triangle$)

$2x = 54$

$x = 27$

b)



let $\angle QPR = y$

$\therefore \angle PRS = x + y$ (exterior $\angle$ of $\triangle PQR$)

$\angle QPS = x + y$ (sum of $\angle QPR$ + $\angle RPS$)

$\therefore \angle PRS = \angle QPS$

ii) $\therefore \triangle QPS \,|||\, \triangle PRS$ (2 angles test)

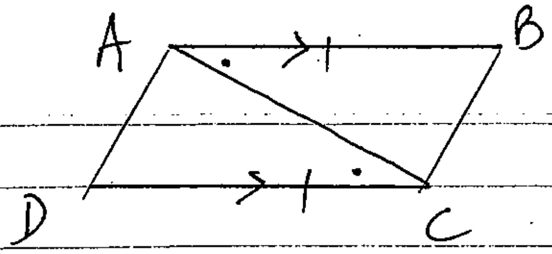
$\therefore \dfrac{QP}{PR} = \dfrac{PS}{RS} = \dfrac{SQ}{SP}$

$\dfrac{7}{x} = \dfrac{9}{6}$

$\dfrac{7}{x} = \dfrac{3}{2}$

$3x = 14$

$x = \dfrac{14}{3}$

7c) 

In $\triangle$'s ABC, CDA

1. ~~AB = DC (given)~~
2. AC is common
3. $\angle BAC = \angle ACD$ (alternate $\angle$'s $AB \parallel DC$)

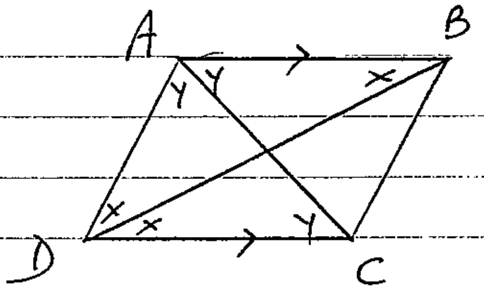
$\therefore \triangle ABC \equiv \triangle CDA$ (SAS test)

$\therefore \angle ACB = \angle CAD$ (matching $\angle$'s in congruent $\triangle$'s)

But these are alternate

$\therefore BC \parallel AD$

$\therefore$ ABCD is a parallelogram.
(both pairs of opposite sides parallel)

d) 

i) $\angle ABD = \angle BDC = \times$
Alternate $\angle$'s $AB \parallel DC$
$\therefore AD = AB$
(opposite equal $\angle$'s)
$\therefore \triangle ABD$ is isosceles

ii) $\angle BAC = \angle ACD = y$ (Alternate $\angle$'s $AB \parallel DC$)
$\therefore AD = DC$ (opposite equal $\angle$'s)
$\therefore \triangle ADC$ is isosceles.

iii) $\therefore AB = AD = DC$
$\therefore$ One pair of opposite sides, AB and DC are both equal + parallel.
$\therefore$ ABCD is a parallelogram from c)

But $AB = AD$
$\therefore$ One pair of adjacent sides are equal $\therefore$ ABCD is a rhombus.

Yr 11 Mathematics 2012

TASK 2 June

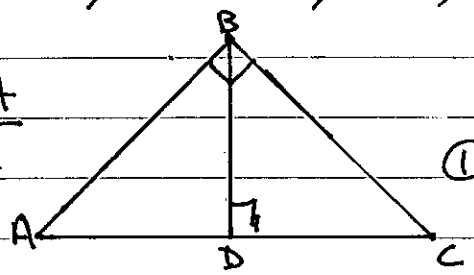
Multiple Choice

1/ D 2/ A 3/ A

4/ D 5/ D 6/ C 7/ D

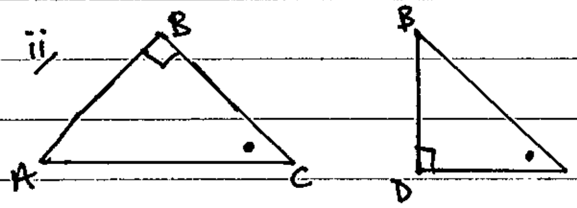
PART A

a) i/



1

ii/



In ΔABC and ΔBDC

angle C is common 1

angle ABC = angle BDC = 90° (given) 1

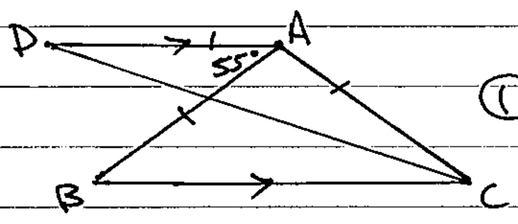
$\therefore \Delta ABC \ ||| \ \Delta BDC$

(equiangular) 1

iii $\frac{BC}{AC} = \frac{DC}{BC}$ (corresponding sides in ratio) 1

$BC^2 = AC \cdot DC$

b)



1

ii) $\angle ABC = 55°$ (alternate angle AD // BC) 1

$\angle ABB = 55°$ (base angles equal isos triangle)

1

iii/ $\angle BAC = 180 - 2 \times 55$ 1

$= 70$ (angle sum of triangle)

$\therefore \angle ADC = \frac{180 - (70 + 55)}{2}$ (base angle of isos triangle ADC) 1

$= 27.5$

iv/ $\angle BCD = 55 - 27.5$ 1

$= 27.5°$ (adjacent angle)

c) ΔAOB and ΔCOD

$\angle AOB = \angle COD$ (vertically opposite) 1

$\angle BAO = \angle DCO = 90°$ (given) 1

$BO = OD$ (given)

$\therefore \Delta AOB \equiv \Delta COD$ (AAS) 1

d) internal angle size $= \frac{(n-2) \times 180}{n}$

$= \frac{(6-2) \times 180}{6}$ 1

$= 120°$

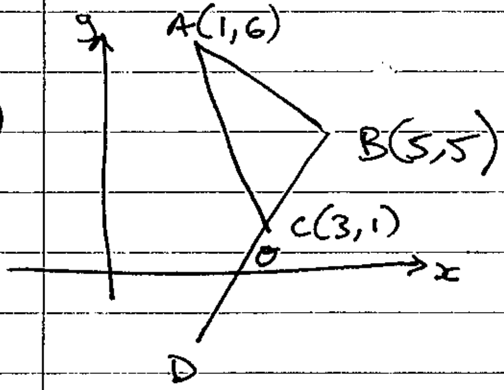
$\therefore$ External angle $180 - 120°$

$= 60°$ 1

2

Part B

a)



b) i) $M = \frac{5-1}{5-3} = \frac{4}{2} = 2$ (1)

c) $\tan\theta = 2$

$\theta = \tan^{-1} 2$

$\theta = 63°$ (1)

d) $y - 5 = 2(x-5)$ (1)

$y - 5 = 2x - 10$ (1)

Eqn_{BD}: $2x - y - 5 = 0$

e) across 2 up 4 ∴ back 2 down 4

(1) various methods.

$D(3-2,\ 1-4)$

$D(1,\ -3)$ (1)

f) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$d = \sqrt{2^2 + 4^2}$ (1)

$BC = \sqrt{20}$

$BC = 2\sqrt{5}$ (1)

$2x - y - 5 = 0$

g) i) $d = \frac{|ax+by+c|}{\sqrt{a^2+b^2}}$

$= \frac{|2.1 - 6.1 - 5|}{\sqrt{2^2 + (-1)^2}}$ (1)

$= \frac{|2 - 6 - 5|}{\sqrt{5}}$

$= \frac{9}{\sqrt{5}}$ or $\frac{9\sqrt{5}}{5}$ (1)

ii) $A_{\Delta ABC} = \frac{1}{2} \times 2\sqrt{5} \times \frac{9}{\sqrt{5}}$ (1)

$= 9$ units^2 (1)

iii) As ΔABC is half ΔABD ratio is 1:2 (1)

j) from A to B across 4 down 1

from D to E back 4 up 1

(1)

∴ $E(-3, -2)$

) Area of Parallelogram = 2 × Area Δ_{ABD} (1)

∴ $A = 2 \times 18$ units^2

$= 36$ units (1)

3/

Part C

(a) for $2x^2-3x-4$

i/ $\alpha+\beta = \frac{-b}{a} = \frac{3}{2}$ (1)

ii/ $\alpha\beta = \frac{c}{a} = -2$ (1)

iii $\frac{1}{\alpha}+\frac{1}{\beta}$

$= \frac{\alpha+\beta}{\alpha\beta}$ (1)

$= \frac{3/2}{-2}$

$= 3/_{-4}$ (1)

iv/ $\alpha^2+\beta^2$

$=(\alpha+\beta)^2-2\alpha\beta$ (1)

$= \left(\frac{3}{2}\right)^2+4$

$= \frac{9}{4}+4$

$= 6\frac{1}{4}$ (1)

b) $4^x - 9(2^x)+8=0$

let $m = 2^x$

$\therefore 2^{2x}-9(2^x)+8=0$

$m^2-9m+8=0$ (1)

$(m-8)(m-1)=0$

$m=8 \quad m=1$

$\therefore 2^x=8 \quad 2^x=1$ (1)

$x=3 \quad x=0$

c) $3x^2-5x+6 = A(x-2)^2+B(x-2)+c$

$Ax^2-4Ax+4A+Bx-2B+c$

$Ax^2-(4A-B)x+4A-2B+c$

equate co-efficients (1)

$A=3$

$4A-B=5$

$12-B=5$ (1)

$B=7$

$4A-2B+c=6$

$12-14+c=6$

$c=8$ (1)

$A=3 \quad B=7 \quad c=8$

d) $x^2+(m-2)x+4=0$

$b^2-4ac=0$ (1)

$(m-2)^2-4.1.4=0$

$m^2-4m+4-16=0$

$m^2-4m-12=0$

$(m-6)(m+2)=0$

$m=6$ or 2 (1)

e) $(1+m)x^2+4x+m-1>0$

$\therefore b^2-4ac>0$ (1)

$4^2-4.(1+m)(m-1)$

$16-4m^2+4$

$-4m^2+20>0$ (1)

$-4(m^2-5)$

$\therefore (m+\sqrt{5})(m-\sqrt{5})<0$

$m>\sqrt{5}$

$m<-\sqrt{5}$ (1)