HORNSBY GIRLS' HIGH SCHOOL

MATHEMATICS
YEAR 11
Preliminary Assessment Task 2
Half-Yearly Examination 2009

Student Number: ______________________

Time Allowed: ***90 minutes plus 5 minutes reading time***

Instructions:
- Attempt all questions
- Start a new page for each question
- The marks for each question are indicated
- Show all necessary working
- Marks may be deducted for untidy or badly arranged work
- Board approved calculators may be used

Outcomes Assessed:
P1 Demonstrates confidence in obtaining realistic solutions to problems
P2 Provides reasoning to support conclusions in the correct context
P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
P5 Understands the concept of a function and the relationship between a function and its graph

Marking Scheme:

	Total	
Algebra and Arithmetic	**Q1.**	**/14**
Functions and Graphs	**Q2.**	**/14**
Linear Functions	**Q3.**	**/14**
Trigonometric Functions	**Q4.** **Q5.**	**/14** **/14**
Total Marks:		**/70**

This assessment task constitutes 30% of the final Preliminary Course Assessment.

QUESTION 1 **(14 Marks)** **START A NEW PAGE** **Marks**

a) Evaluate: $\sqrt{\dfrac{1.25\times(3.6^4)}{0.075}}$ correct to 3 significant figures. 1

b) Factorise: $3y^3-81$ 2

c) Simplify: $(\sqrt{17}-2\sqrt{5})(\sqrt{17}+2\sqrt{5})$ 2

d) Show that the only real solution of $|4x-1|=3-5x$ is $x=\dfrac{4}{9}$ 3

e) If $f(x)=x^2-8x$, find $\dfrac{f(a+h)-f(a)}{h}$ 3

f) Solve for x: $|7-2x|>9$ and graph the solution on a number line. 3

QUESTION 2 **(14 Marks)** **START A NEW PAGE** **Marks**

a) State the domain and range for (i) $y=\dfrac{8}{x-1}$ 2

(ii) $y=\sqrt{16-x^2}$ 2

b) If $f(x)=2^{-x}+2^{x}$,

(i) find $f(a)$ and $f(-a)$. 1

(ii) hence, determine whether the function $f(x)$ is odd, even or neither. 1

c) Sketch the following functions on separate number planes showing important features and intercepts. 8

(i) $y=(x+2)^2$

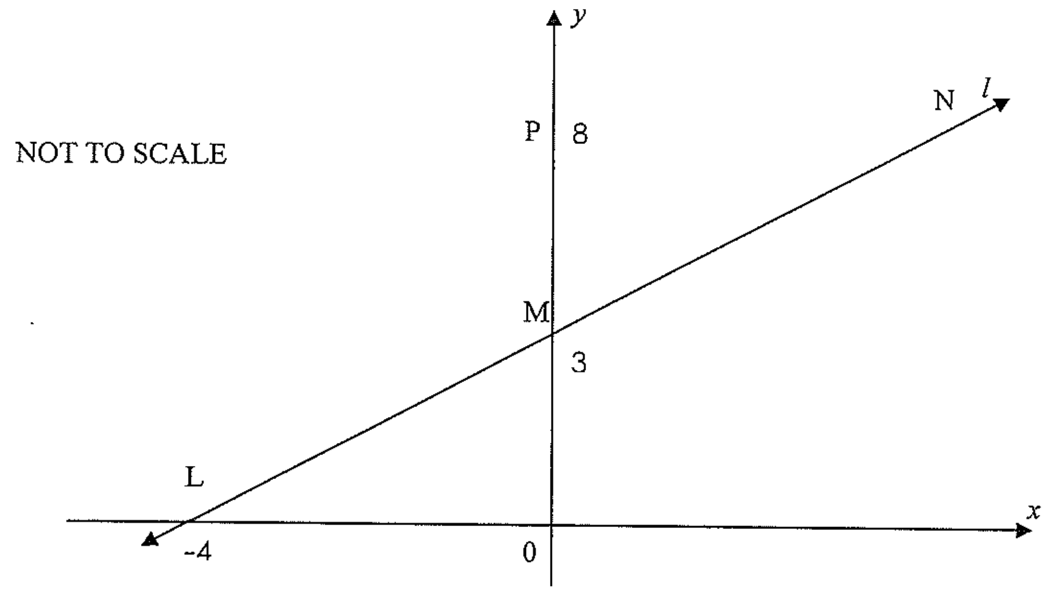
(ii) $y=|x+2|$

(iii) $y=\dfrac{1}{x+2}$

(iv) $y=2-x^3$

QUESTION 3 (14 Marks) START A NEW PAGE **Marks**

a)



The line l cuts the x axis at L (-4,0) and the y axis at M(0,3). N is a point on the line l and P is the point (0,8)

(i) Find the equation of the line l in general form. 2

(ii) Show that ΔLMP is isosceles. 2

(iii) M is the midpoint of LN find the coordinates of N. 2

(iv) Show that $\angle NPL$ is a right angle. 2

(v) Find the perpendicular distance from the point $Q(-4,5)$ to the line l. 2

(vi) Find the gradient of PQ and hence determine if PQ is parallel to line l, LN. 2

b) Find the equation of the straight line that makes an angle of 135° with the positive direction of x axis and passes through the point $(2,-3)$. 2

15

QUESTION 4 (14 Marks) **START A NEW PAGE** **Marks**

a) Simplify (i) $\frac{1}{\sec(-\theta)}$ $= \frac{1}{\sec\theta} = \cos\theta$ 1

(ii) $\text{cosec}\,(180^\circ - \theta).\ \sin\,(90^\circ - \theta)$ 2

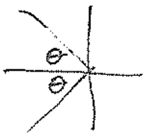
$\text{cosec}\,\theta \cdot \cos\theta = \cot\theta$

(iii) $2 - 2\sin^2\theta$ $= 2(1-\sin^2\theta)$ 1

$= 2\cos^2\theta$

4

b) Solve $\cos\theta = -0.6$ for $0^\circ \leq \theta \leq 360^\circ$. Give your answer to the nearest minute. 2



$\theta = 126^\circ 52'$

$233^\circ 8'$

c) Evaluate, giving the exact value as a single fraction for

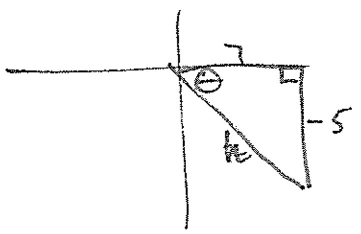
(i) $\tan 210^\circ$ $= \tan 30^\circ = \frac{1}{\sqrt{3}}$ or $\frac{\sqrt{3}}{3}$ 1

(ii) $\sin 60^\circ + \tan 210^\circ = \frac{\sqrt{3}}{2} + \frac{\sqrt{3}}{3}$ 2

$= \frac{5\sqrt{3}}{6}$

5

d) Given that $\tan\theta = \frac{-5}{7}$ and $\cos\theta > 0$, find the exact value of $\text{cosec}\,\theta$. Show necessary working. 3



$h^2 = 49 + 25$

$= 74$

$h = \sqrt{74}$

$\therefore \text{cosec}\,\theta = \frac{\sqrt{74}}{-5}$

e) Prove that $\frac{1-\tan^2\theta}{1+\tan^2\theta} = \cos^2\theta - \sin^2\theta$ 3

6

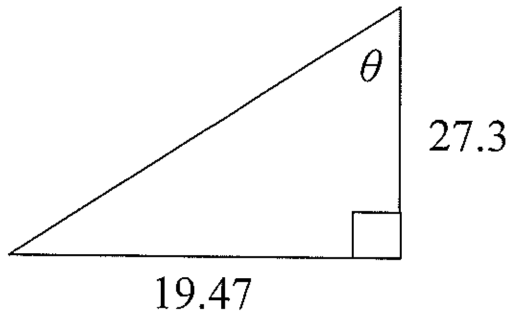
$\text{LHS} = \frac{1-\tan^2\theta}{\sec^2\theta}$

$= \frac{1 - \frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \frac{\frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}} = \cos^2\theta - \sin^2\theta$

$= \text{RHS}.$

QUESTION 5 (14 Marks) **START A NEW PAGE** **Marks**

a) Find θ to the nearest minute. 2



b) A ship leaves port and sails 58 kilometres on a bearing of 138°. How far due south of the port is the ship? Draw a diagram and give your answer correct to the nearest kilometre. 2

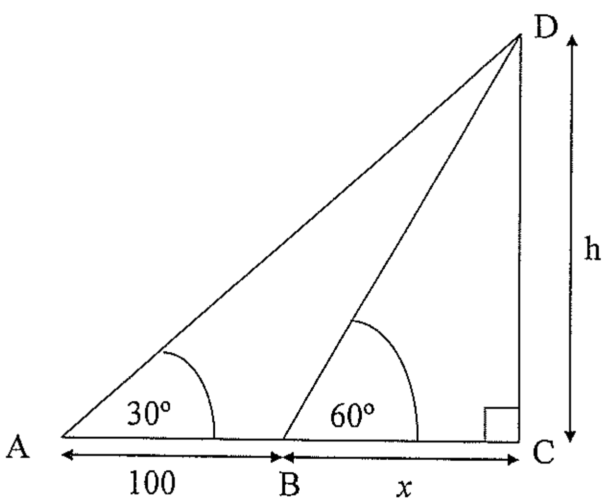
c) From an aircraft flying at 9 000m above sea level the angle of depression of a ship is $33^{\circ}25'$. How far is the ship from the aircraft in a straight line? (Give your answer correct to nearest metre). 2

d) Solve for $0^{\circ} \leq \theta \leq 360^{\circ}$

(i) $\operatorname{cosec}\theta = 2$ 2

(ii) $\cot\theta = 0$ 2

e) A child at A observes the angle of elevation of a kite caught in a tree to be 30 degrees. She then walks 100 metres towards the kite to a point B and finds the angle of elevation to be 60 degrees. If the kite has a height of h metres and the child has x metres still to walk before she is directly under the kite, find x. 4



END OF EXAMINATION

Q1 a) $\doteqdot 52.9089\ldots$ (from calculator)
$= 52.9$ (3 significant figures) 1

b) $3(y^3-27) = 3(y-3)(y^2+3y+9)$ 2

c) $17-20 = -3$ 2

d) either $4x-1 = 3-5x$ or $-(4x-1) = 3-5x$

$9x = 4$
$x = \frac{4}{9}$

Check: LHS $= \left|4\times\frac{4}{9}-1\right|$
$= \frac{7}{9}$
RHS $= 3-5\times\frac{4}{9}$
$= \frac{7}{9}$
LHS = RHS ✓
$\therefore x = \frac{4}{9}$ is the only solution.

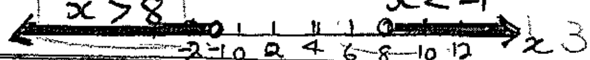
$-4x+1 = 3-5x$
$x = 2$

Check: LHS $= |4\times2-1|$
$= 7$
RHS $= 3-5\times2$
$= -7$
$\therefore$ Not a solution LHS ≠ RHS

3

e) $f(a+h) = (a+h)^2 - 8(a+h)$
$= a^2+2ah+h^2-8a-8h$
$f(a) = a^2-8a$
$\therefore \frac{f(a+h)-f(a)}{h} = \frac{a^2+2ah+h^2-8a-8h-(a^2-8a)}{h}$
$= \frac{2ah+h^2-8h}{h}$
$= \frac{h(2a+h-8)}{h}$
$= 2a+h-8$ 3

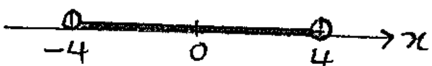
f) $|7-2x| > 9$
either $-(7-2x) > 9$
$7-2x < -9$ or $7-2x > 9$
$-2x < -16$ $\quad$ $-2x > 2$
$x > 8$ $\quad$ $x < -1$



3

Q2. a) (i) D: $x \neq 1$
R: $y \neq 0$

(ii) D: $16-x^2 \geq 0$
$(4-x)(4+x) \geq 0$

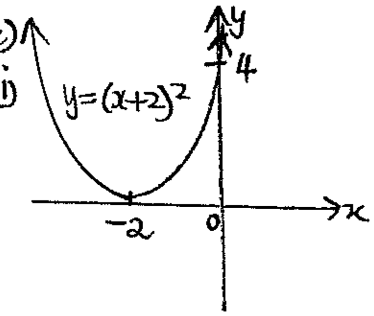


Test $x=0$ $16 \geq 0$ ✓
$\therefore -4 \leq x \leq 4$
R: $y \geq 0$

4

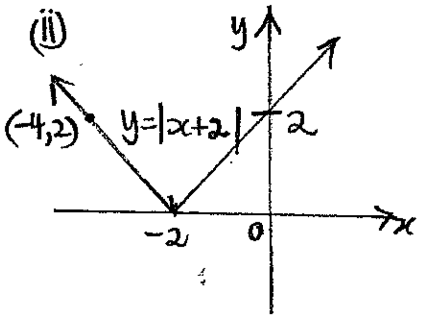
Q2 b) $f(a) = 2^{-a}+2^{a}$
$f(-a) = 2^{a}+2^{-a}$
$\therefore f(-a) = f(a)$ $\therefore$ Function is even. 2

c) (i)

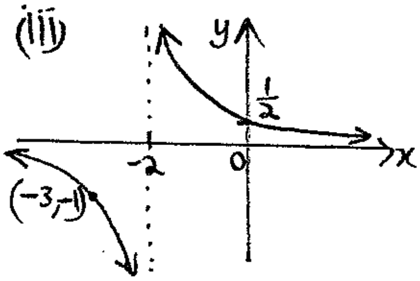


2 each {1 shape, 1 intercepts}

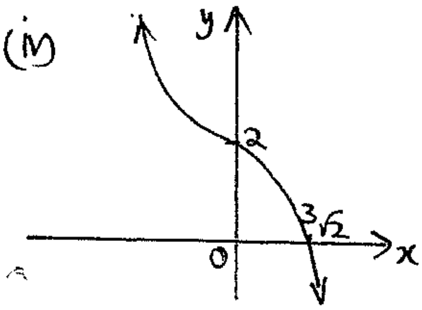
(ii)



(iii)



(iv)



Q3. a) (i) $\frac{x}{-4}+\frac{y}{3}=1$

$3x-4y=-12$

$\underline{3x-4y+12=0}$ 2

(ii) ∴ PM = 5 units

LM = 5 units {3,4,5} Pythagorean Triad)

or $d=\sqrt{3^2+4^2}$

$=\sqrt{25}$

$=5$

∴ $\triangle LMP$ is isosceles (2 equal sides) 2

(iii) $(0,3)=\left(\frac{-4+x_1}{2},\frac{0+y_1}{2}\right)$

$-4+x_1=0$, $\frac{0+y_1}{2}=3$

$x_1=4$, $y_1=6$

∴ $\underline{N \text{ is } (4,6)}$ 2

(iv) $m_{NP}=\frac{8-6}{0-4}$, $m_{PL}=\frac{8-0}{0+4}$

$=-\frac{1}{2}$, $=2$

∴ $m_{NP}\times m_{PL}=-\frac{1}{2}\times 2$

$=-1$ 2

∴ $NP \perp PL$

∴ $\underline{\angle NPL \text{ is a right angle}}$.

(v) $d=\frac{|3(-4)-4(5)+12|}{\sqrt{3^2+4^2}}$

$=\frac{|-20|}{5}$ units.

$=4$ 2

(vi) $m_{PQ}=\frac{8-5}{0+4}$, $m_{LN}=\frac{3}{4}$

$=\frac{3}{4}$ ∴ gradients are equal

∴ $PQ \parallel LN$ (line l) 2

b) $m=\tan 135°$

$=-1$

$y=-1x+b$

$-3=-1\times 2+b$

$b=-1$

∴ $\underline{y=-x-1}$ 2

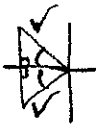
Q4. a) (i) $\cos(-\theta)=\cos\theta$ 1

(ii) $\frac{1}{\sin(180°-\theta)}\times \cos\theta=\frac{\cos\theta}{\sin\theta}$

$=\cot\theta$ 2

(iii) $2(1-\sin^2\theta)=2\cos^2\theta$ 1

b) Quadrants II, III $\cos\theta<0$



Acute angle α is $53°7'48''$ (from calculator)

∴ $\theta=180°-\alpha$ or $\theta=180°+\alpha$

$\doteqdot 126°52'12''$ $\doteqdot 233°7'48''$

$=\underline{126°52'}$ (correct to nearest minute) $=\underline{233°8'}$ (correct to nearest minute) 2

c) (i) $\tan 210°=\tan(180°+30°)$ 3rd quadrant $\tan\theta>0$

$=\tan 30°$

$=\frac{1}{\sqrt{3}}$ 1

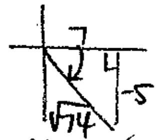
(ii) $\sin 60°+\tan 210°=\frac{\sqrt{3}}{2}+\frac{1}{\sqrt{3}}$

$=\frac{3+2}{2\sqrt{3}}$

$=\frac{5}{2\sqrt{3}}$ or $\frac{5\sqrt{3}}{6}$ 2

d) $\left.\begin{array}{l}\tan\theta<0\\ \cos\theta>0\end{array}\right\}$ ∴ 4th quadrant.

∴ $\sin\theta<0$



(By Pythagoras' Thm)

$h=\sqrt{5^2+7^2}$

$=\sqrt{74}$

$\text{cosec}\,\theta=\frac{1}{\sin\theta}$

$=\frac{1}{\frac{-5}{\sqrt{74}}}$

$=-\frac{\sqrt{74}}{5}$ 3

e) $LHS=\left(1-\frac{\sin^2\theta}{\cos^2\theta}\right)\div\sec^2\theta$

$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\frac{1}{\sec^2\theta}$

$=\frac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}\times\cos^2\theta$

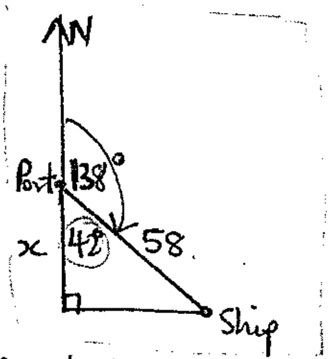
$=RHS.$ 3

Q5. a) $\tan\theta = \frac{19.47}{27.3}$

$\theta \doteqdot 35^\circ 29' 45''$ (from calc.)

$= 35^\circ 30'$ (to nearest minute) 2

b)



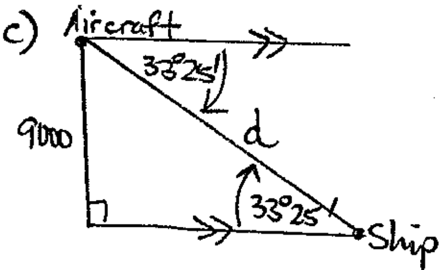
$\cos 42^\circ = \frac{x}{58}$

$x = 58 \cos 42^\circ$

$\doteqdot 43.10\ldots$ (from calculator)

$\therefore$ Ship is 43km south of the Port 2

c)



$\frac{9000}{d} = \sin 33^\circ 25'$

$d = \frac{9000}{\sin 33^\circ 25'}$

$\doteqdot 16342.13\ldots$ (from calculator) 2

$\therefore$ The distance between them is 16 342 m.

d) (i) $\operatorname{cosec}\theta = 2, \ 0^\circ \le \theta \le 360^\circ$

$\therefore \frac{1}{\sin\theta} = 2$

$\sin\theta = \frac{1}{2}$ $\quad \sin\theta > 0 \ \therefore$ I, II quads

$\theta = 30^\circ, 180^\circ - 30^\circ$

$= 30^\circ, 150^\circ$ 2

Q5 d) (ii) $\cot\theta = 0 \quad 0^\circ \le \theta \le 360^\circ$

$\frac{1}{\tan\theta} = 0$

$\tan\theta$ is undefined.

$\therefore \theta = 90^\circ, 270^\circ$ 2

e) $\frac{h}{x} = \tan 60^\circ$

$h = x\tan 60^\circ$ 1

$\frac{h}{(100+x)} = \tan 30^\circ$

$h = (100+x)\tan 30^\circ$ 1

$x\tan 60^\circ = 100\tan 30^\circ + x\tan 30^\circ$

$x\tan 60^\circ - x\tan 30^\circ = 100\tan 30^\circ$

$x(\tan 60^\circ - \tan 30^\circ) = 100\tan 30^\circ$ 1

$x = \frac{100\tan 30^\circ}{\tan 60^\circ - \tan 30^\circ}$

$= \frac{100 \times \frac{1}{\sqrt{3}}}{\sqrt{3} - \frac{1}{\sqrt{3}}}$

$= \frac{100}{\sqrt{3}} \div \left(\frac{3-1}{\sqrt{3}}\right)$

$= \frac{100}{\sqrt{3}} \times \frac{\sqrt{3}}{2}$

$= 50$ 1