HORNSBY GIRLS' HIGH SCHOOL

MATHEMATICS

YEAR 11

Preliminary Assessment Task 2

Half-Yearly Examination 2010

Student Number: _____

Time Allowed: ***90 minutes plus 5 minutes reading time***

Instructions:

- Attempt all questions
- Start a new page for each question
- The marks for each question are indicated
- Show all necessary working
- Marks may be deducted for untidy or badly arranged work
- Board approved calculators may be used

Outcomes Assessed:

P1 Demonstrates confidence in obtaining realistic solutions to problems
P2 Provides reasoning to support conclusions in the correct context
P3 Performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and trigonometric identities
P4 Chooses and applies appropriate arithmetic, algebraic, graphical, trigonometric and geometric techniques
P5 Understands the concept of a function and the relationship between a function and its graph

Outcomes	Total
Algebra and Arithmetic	**Q1. /13**
Functions and Graphs	**Q2. /13** **Q3. /10**
Linear Functions	**Q4. /12**
Trigonometric Functions	**Q5. /12** **Q6. /12**
Total Marks:	**/72**

This assessment task constitutes 30% of the final Preliminary Course Assessment.

QUESTION 1 **(START NEW PAGE)** **13 MARKS**

(A) Evaluate correct to 2 decimal places: 2

$$\frac{1}{(2.4)^2}+\frac{1}{\sqrt{3.7}}$$

(B) Rationalise the denominator of $\dfrac{1+\sqrt{xy}}{x\sqrt{x}}$ 2

where x and $y \geq 0$

(C) Factorise completely: $2p^3 + 54$ 2

(D) Expand $(h-2)^3$ 2

(E) Find all values of x which satisfy 2

$|3x-4|+2 \leq 7$

(F) Find rational numbers a and b such that 3

$$a+b\sqrt{3}=\frac{\sqrt{3}}{3+\sqrt{3}}$$

QUESTION 2 **(START NEW PAGE)** **13 MARKS**

(A) State the domain for the following

(i) $y = \dfrac{x^2}{x^2 - 4}$ 2

(ii) $f(x) = \dfrac{1}{\sqrt{3-x}}$ 1

(B) If $g(x) = \dfrac{x^2}{x^4 + 1}$

(i) find $g(-x)$ 1

(ii) state why $g(x)$ is an even function. 1

(C) If $h(x) = x^2 + 3x$ find:

(i) $h(-2)$ 1

(ii) $h(a)$ 1

(iii) $h(x-1)$ 2

(D) By sketching $q(x) = x(x-1)^2$ or otherwise, solve $q(x) \geq 0$. 2

(E) Sketch the function $f(x) = |x| + x$ for all x. 2

QUESTION 3 **(START NEW PAGE)** **10 MARKS**

(A) On separate diagrams sketch the following curves

(i) $y = x^2 + 2x - 3$ 2

(ii) $y = \sqrt{4 - x^2}$ 2

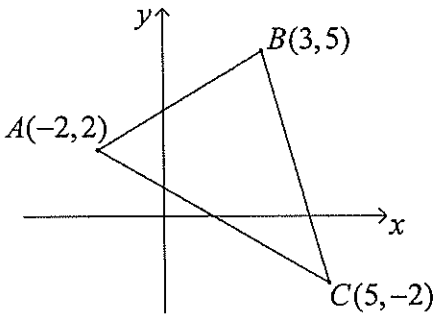
(B) Sketch the region defined by $y > x - 2$ and $y \leq 4 - x^2$ 3

(C) Sketch the graph of $f(x)$ if 3

$$f(x) = \begin{cases} 2x - 1 & for\ x \geq 0 \\ x^2 & for\ x < 0 \end{cases}$$

QUESTION 4 **(START NEW PAGE)** **12 MARKS**

(A) Consider the following ΔABC



(i) Find the exact distance AC. 1

(ii) Show that the equation of the line through A and C is $4x + 7y - 6 = 0$ 2

(iii) Find the altitude of ΔABC through vertex B. 2

(iv) Hence or otherwise, find the area of ΔABC. 1

(B) Find the equation of the straight line passing through the point $(1,2)$ and the point of intersection of the lines $3x - 5y - 3 = 0$ and $2x + 3y + 17 = 0$. 3

(C) If $2x - 3y - 3 = 0$ and $3x + ay + b = 0$ are perpendicular and intersect at $x = 3$, find the values of a and b. 3

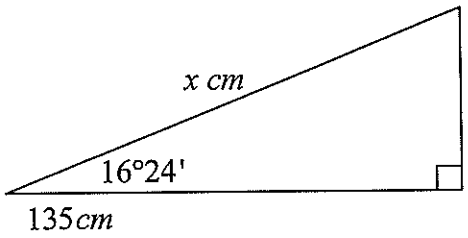
QUESTION 5 **(START NEW PAGE)** **12 MARKS**

(A) Find θ to the nearest minute if θ is an acute angle and

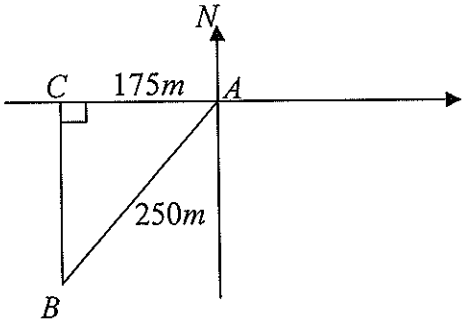
(i) $\sin\theta = \frac{3}{4}$ 1

(ii) $\sec\theta = 1.2230$ 1

(B) Find x correct to one decimal place. 2



(C) Find the bearing of B from A to the nearest degree. 2



(D) Find the exact value of

(i) $\tan 210°$ 1

(ii) $\sin 330°$ 1

(iii) $\sin 45° \cos 120°$ 2

(E) If $\sec\alpha = \frac{7}{3}$ and $\sin\alpha < 0$ find the exact value of $\tan\alpha$. 2

QUESTION 6 **(START NEW PAGE)** **12 MARKS**

(A) Draw sketches of the following for $0° \leq \theta \leq 360°$

(i) $y = \sin\theta$ 2

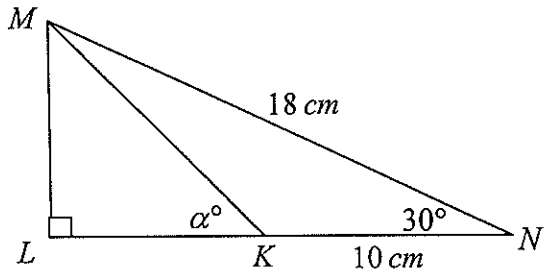
(ii) $y = \tan\theta$ 2

(B) Solve the following for $0° \leq \theta \leq 360°$:

(i) $\sqrt{2}\cos\theta = 1$ 2

(ii) $\tan(2\theta - 30°) = 1$ 3

(C) Find α in the following diagram to the nearest degree. 3



END OF EXAMINATION

Yr 11 2010

2u Trem. Half Yearly Solutions

* Please note: Marking scheme is a guideline only. Each students answers reconsidered on their merits. Alternative responses may also be acceptable.

2 – correct answer with rounding

1 – correct answer incorrect rounding

Question 1

(a) 0·69

(b) $\left(\frac{k+\sqrt{xy}}{x\sqrt{x}}\right) \times \frac{\sqrt{x}}{\sqrt{x}} = \frac{\sqrt{x}+x\sqrt{y}}{x^2}$ — 1 – multiplying; 1 – correct ans.

(c) $2p^3+54 = 2(p^3+27)$ — 1

$= 2(p+3)(p^2-3p+9)$ — 1

(E) $|3x-4|+2 \leq 7$

$|3x-4| \leq 5$

$-5 \leq 3x-4 \leq 5$ — 1 or equivalent

$-1 \leq 3x \leq 9$

$-\frac{1}{3} \leq x \leq 3$ — 1 correct answer

(F) $a+b\sqrt{3} = \frac{\sqrt{3}}{3+\sqrt{3}} \times \frac{(3-\sqrt{3})}{(3-\sqrt{3})}$ — 1

$= \frac{3\sqrt{3}-3}{9-3}$

$= \frac{-3+3\sqrt{3}}{6}$

$= \frac{-1+\sqrt{3}}{2}$ — 1

$\therefore a = -\frac{1}{2} \quad b = \frac{1}{2}$ — 1

(D) $(h-2)^3 = h^3 + 3(h)^2(-2) + 3(h)(-2)^2 + (-2)^3$

$= h^3 - 6h^2 + 12h - 8$

1 – correct terms

2 – correct terms & signs.

Solutions

Question 2

(A) (i) $D: x^2 - 4 \neq 0$

$(x-2)(x+2) \neq 0$

$\therefore x \neq \pm 2$ —— 2 (1 each)

(ii) $D: 3 - x > 0$

$-x > -3$

$x < 3$ —— 1

(B) $g(x) = \dfrac{x^2}{x^4+1}$

(i) $g(-x) = \dfrac{(-x)^2}{(-x)^4+1}$

$= \dfrac{x^2}{x^4+1}$ —— 1

(ii) $g(x) = g(-x)$ —— 1

$\therefore g(x)$ is even

(C) $h(x) = x^2 + 3x$

(i) $h(-2) = (-2)^2 + 3(-2)$

$= -2$ —— 1

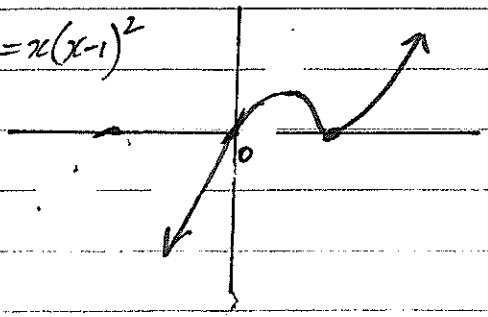
(ii) $h(a) = a^2 + 3a$ —— 1

(iii) $h(x-1) = (x-1)^2 + 3(x-1)$ —— 1

$= x^2 - 2x + 1 + 3x - 3$

$= x^2 + x - 2$ —— 1

(D) $q(x) = x(x-1)^2$



—— 1 or working

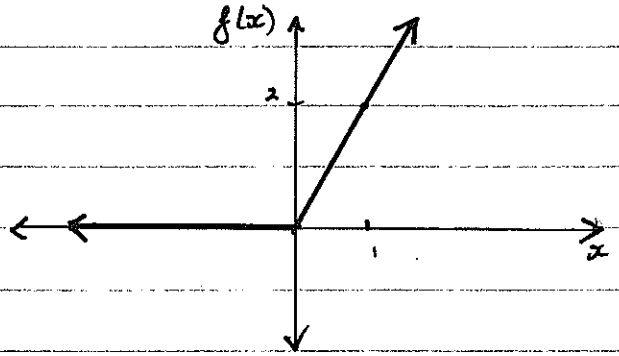
$\therefore q(x) \geq 0$ for $x \geq 0$ —— 1

(E) $|x| + x = x + x \quad x \geq 0$

$= 2x$

$= -x + x \quad x < 0$

$= 0$



1 for each branch.

Question 4

(A) (i) $AC = \sqrt{(5-2)^2+(-2-2)^2}$

$= \sqrt{49+16}$

$= \sqrt{65}$ units — 1

(ii) $A(-2,2)$ $C(5,-2)$

$\frac{y-2}{x+2} = \frac{-2-2}{5-2}$ or other correct method

$\frac{y-2}{x+2} = \frac{-4}{7}$

$-4x-8 = 7y-14$

$4x+7y-6=0$ as required — 2

(iii) $B(3,5)$ with $4x+7y-6=0$

$\therefore d = \left|\frac{4\times3+7\times5-6}{\sqrt{4^2+7^2}}\right|$

$= \frac{41}{\sqrt{65}}$ or $\frac{41\sqrt{65}}{65}$ — 2

(iv) Area $\Delta ABC = \frac{1}{2} \times \sqrt{65} \times \frac{41}{\sqrt{65}}$

$= \frac{41}{2}$ units^2 or other correct method

$= 20\frac{1}{2}$ units^2 — 1

(B) $3x-5y-3+k(2x+3y+17)=0$

sub $(1,2)$ $3-10-3+k(2+6+17)=0$

$-10+25k=0$

$k = \frac{10}{25}$

$= \frac{2}{5}$ — 1

$\therefore 3x-5y-3+\frac{2}{5}(2x+3y+17)=0$ or other correct method

$15x-25y-15+4x+6y+34=0$ — 1

$19x-19y+19=0$

$\therefore x-y+1=0$ // — 1

(C) If $2x-3y-3=0$ is perp to $3x+ay+b=0$

then $\boxed{a=2}$ — 1

$\therefore 3x+2y+b=0$

now $x=3$ is the point of intersection of both lines

$\therefore 2\times3-3y-3=0$

$6-3y-3=0$

$-3y=-3$

$y=1$ $\therefore (3,1)$ must fit or other correct method

— 1

$3\times3+2\times1+b=0$

$\therefore 11+b=0$

$\boxed{b=-11}$

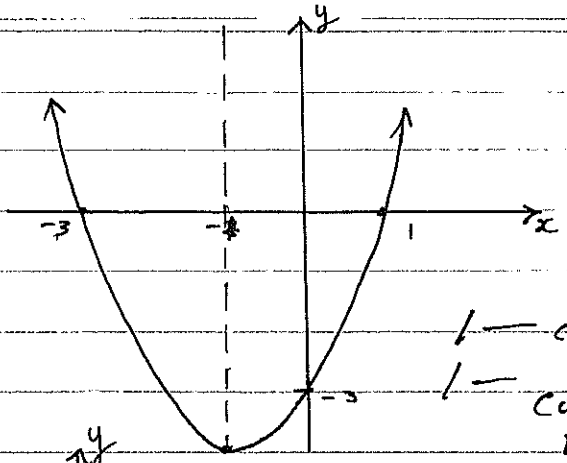
$\therefore \boxed{a=2, b=-11}$ // — 1

✓ ✓ ✓ ✓ ✓ ✓
1, 2, 3, 4, 5, 6

Question 3

(A) (i) $y = x^2 + 2x - 3$

$y = (x+3)(x-1)$

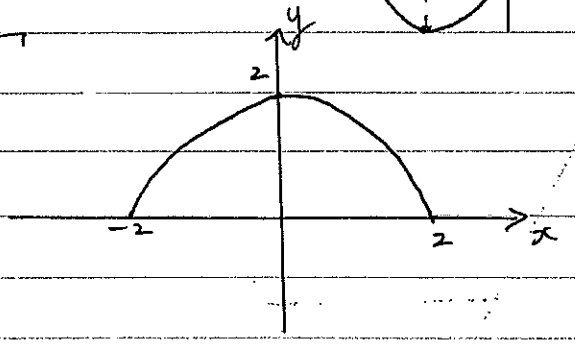


1 — correct shape
1 — correct values

(ii) $y = \sqrt{4 - x^2}$

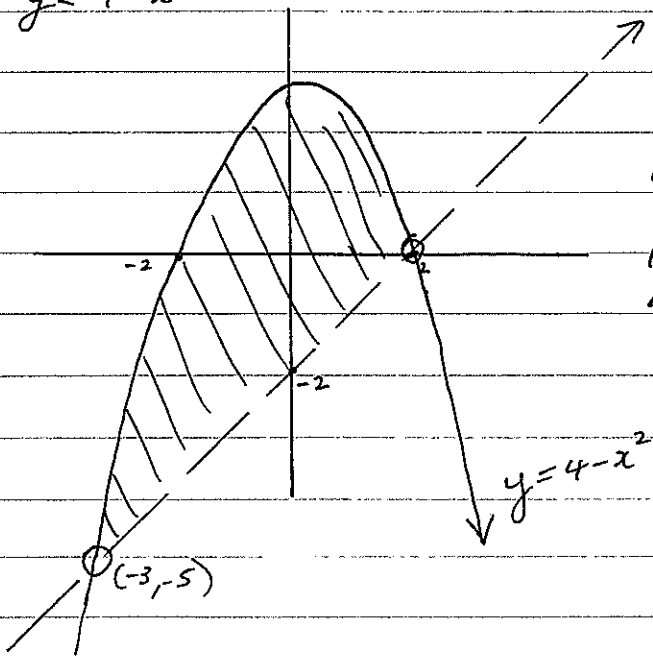
upper semi circle

centre (0,0) radius 2 units



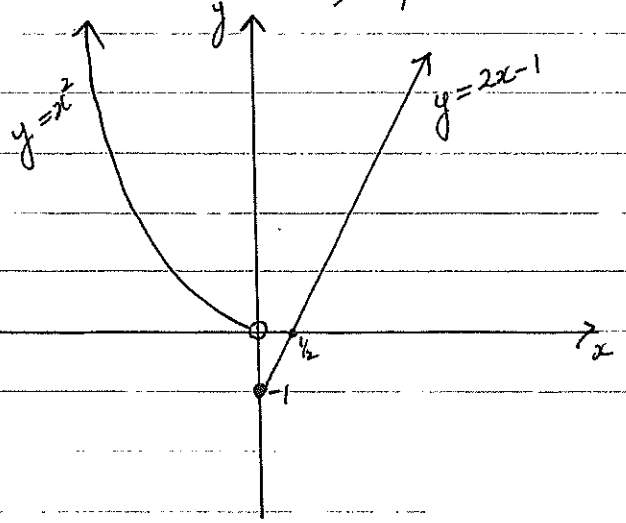
1 — correct shape
1 — correct values

(B) $y > x - 2$ and $y \leq 4 - x^2$



1 each correct lines
1 correct region including circles

(C)



$$f(x) = \begin{cases} 2x - 1 & \text{for } x \geq 0 \\ x^2 & \text{for } x < 0 \end{cases}$$

1 each correct lines
1 correct end pts.

Question 5

(A) (i) $\sin\theta = \frac{3}{4}$

$\theta = 48°35'$ ——— 1

(ii) $\sec\theta = 1.2230$ disregard minute rounding

$\cos\theta = \frac{1}{1.2230}$

$=$

$\theta = 35°9'$ ——— 1

(B) $\frac{135}{x} = \cos 16°24'$

$x = \frac{135}{\cos 16°24'}$ OR $135\sec 16°24'$ ——— 1

$= 140.7$ cm (1 d.p.) ——— 1

(C) Let $\angle WAB = \theta$

$\therefore \cos\theta = \frac{175}{250}$

$\therefore \theta = 46°$ (nearest deg) ——— 1

$\therefore$ Bearing of B from A is $270° - \theta = 270° - 46°$

$= 224°$

OR S 46° W ——— 1

(D) (i) $\tan 210° = -\tan 30°$

$= -\frac{1}{\sqrt{3}}$ ——— 1

(ii) $\sin 330° = -\sin 30°$

$= -\frac{1}{2}$ ——— 1

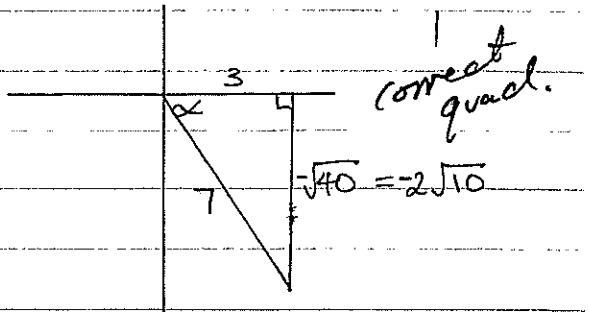
(iii) $\sin 45°\cos 120° = \frac{1}{\sqrt{2}} \times -\frac{1}{2}$ ——— 1

$= -\frac{1}{2\sqrt{2}}$

——— 1

(E) $\sec\alpha = \frac{7}{3}$ $\sin\alpha < 0$

$\therefore \cos\alpha = \frac{3}{7}$ $\therefore$ 4th Quad

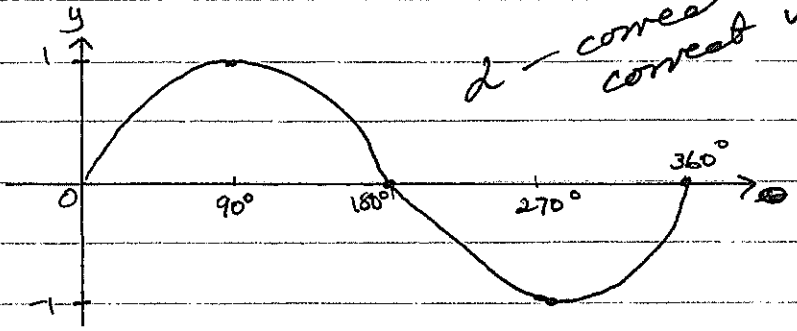


1 correct quad.

$\therefore \tan\alpha = \frac{-2\sqrt{10}}{3}$ ——— 1

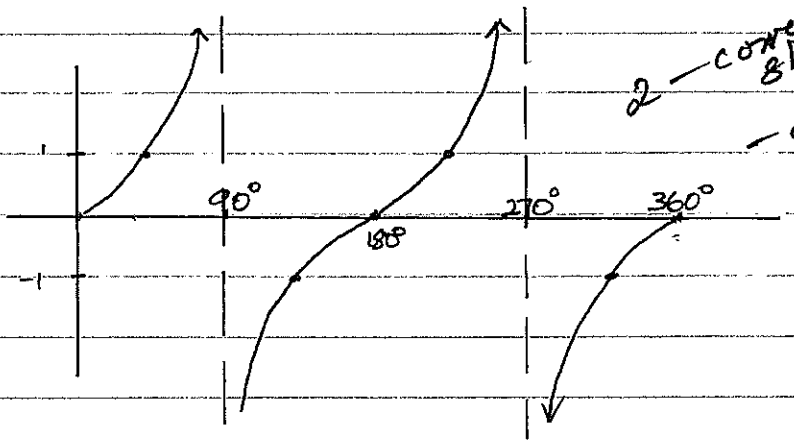
Question 6

(A) (i) $y = \sin\theta$



2 – correct shape
correct values

(ii) $y = \tan\theta$



2 – correct shape
– correct values

(B) (i) $\sqrt{2}\cos\theta = 1$ $\quad 0° \le \theta° \le 360°$

$\cos\theta = \frac{1}{\sqrt{2}}$ 1st and 4th Quad. — 1

$\theta = 45°, 315°$ — 1

(ii) $\tan(2\theta - 30°) = 1$ $\quad 0° \le \theta° \le 360°$

$2\theta - 30° = 45°, 225°, 405°, 495°$ $\quad 0° \le 2\theta° \le 720°$

$2\theta = 75°, 255°, 435°, 525°$ $\quad -30° \le 2\theta - 30 \le 690°$

$\theta = 37\frac{1}{2}°, 127\frac{1}{2}°, 217\frac{1}{2}°, 262\frac{1}{2}°$ — 3 for all 4 correct values
— 2 for 2,3 correct values
— 1 for 1 correct value

(C) $\frac{LN}{18} = \cos 30°$

$\therefore LN = 18\cos 30°$

$= 18 \times \frac{\sqrt{3}}{2}$

$= 9\sqrt{3}$ — 1

$\therefore \frac{LM}{9\sqrt{3}} = \tan 30°$

$\therefore LM = 9\sqrt{3} \times \frac{1}{\sqrt{3}}$

$= 9$ — 1

So $\frac{LM}{LK} = \frac{9}{9\sqrt{3} - 10} = \tan\alpha$

$\therefore \tan\alpha = \frac{9}{9\sqrt{3} - 10}$

$\therefore \alpha = 58°$ (nearest degree)

— 1