## QUESTION 1: 15 MARKS (START A NEW PAGE)

a) Write in simplest form:

$$\frac{x+8}{x^{-1}+8^{-1}}$$ 2

b) Simplify, leaving your answer in index form:

$$\frac{25^{-1}}{\left(5^{n}\right)^{3}\times(125)^{1-n}}$$ 3

c) Solve the following inequation and graph the solution on a number line:

$$|4-5x|<11$$ 3

d) Find the exact length of the diagonal of a rectangle if it makes an angle of 30° with the longer side, which is 48 *cm* in length. 2

e) A, B, C are the vertices of a triangle with coordinates (-2, -3), (4, 3) and ($k$, 5) respectively. Find:

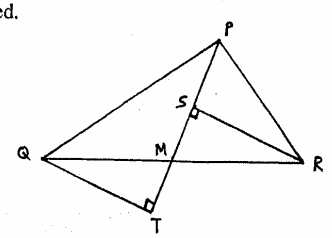
(i) the coordinates of the midpoint of AB, 1
(ii) the gradient of the interval AB, 1
(iii) the equation of the perpendicular bisector of AB, 1
(iv) hence, or otherwise, find $k$ if ΔABC is isosceles having base AB. 2

## QUESTION 2: 15 MARKS (START A NEW PAGE)

a) Differentiate with respect to $x$:

(i) $4\sin(3-x)$ 1
(ii) $\sqrt{1+x^{2}}$ 2
(iii) $3xe^{2x}$ 2
(iv) $\log_{e}(\log_{e}x)$ 1

b) In ΔPQR, M is the midpoint of QR, QT and RS are perpendicular lines drawn from Q and R to the line PM produced.




(i) Prove ΔQMT ≡ ΔRMS, giving reasons. 3
(ii) What type of quadrilateral is QSRT, giving reasons. 2

c) For what value(s) of $k$ will the following equation have equal roots? 2

$$x^{2}-(k+4)x+7+k=0$$

d) Evaluate the following limit: $\lim_{x\to 1}\frac{x^{2}-x}{1-x}$ 2

## QUESTION 3: 15 MARKS (START A NEW PAGE)

a) Determine the set of values of $x$ for which $\frac{dy}{dx} > 0$ in the function: $y = -x^3 + 13x^2 - 35x + 5$ — 3

b) (i) Solve for $x$ : $1 - 2\cos x = 0, \quad 0 \le x \le 360°$. — 2

(ii) Sketch: $y = 1 - 2\cos x, \quad 0 \le x \le 360°$. — 2

c) (i) Find the equation of the tangent to the curve $y = (2x-1)^2$ at the point A where $x = -1$. — 3

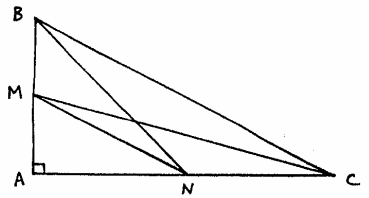
(ii) The tangent intersects the y axis at B. Find the area of the triangle OAB, where O is the origin. — 2

d) The minute and hour hands of a clock are respectively 9cm and 6cm long. Find the distance between the ends of the hands when the time is twenty past two. (Give your answer to 1 decimal place). — 3

## QUESTION 4: 15 MARKS (START A NEW PAGE)

a) In this figure, $\angle BAC = 90°$, M and N are the midpoints of BA and AC respectively.
Let AM = $a$ units and AN = $b$ units.

Prove: $BN^2 + CM^2 = 5 \times MN^2$ — 4




b) A and B are the points (-3, -1) and (7, 3) respectively. The point P $(x, y)$ moves so that $\angle APB = 90°$.

(i) Derive the equation of the locus of P. — 2

(ii) Find the centre and radius of the above circle. — 3

c) (i) A wheel has a radius of 15 *cm*. Through what angle (in radians) does a point on the wheel rotate if the wheel rolls 150 *cm* along a horizontal path? — 1

(ii) A point on the wheel was initially in contact with the ground. In rolling 150 *cm* show that the point on the wheel has completed more than $1\frac{1}{2}$ revolutions. — 2

(iii) What is the height above the ground of the point described in part (ii) after the wheel rolls 150 *cm*. (Give answer to 3 decimal places). — 3

**End of Exam Paper**

# Solutions to Yr 11 – HALF YEARLY 2004

## (24) MATHEMATICS LEVEL

4 x 15 marks each = 60 marks total

**Question 1 : 15 marks** — Marks

a) $\dfrac{x+8}{x^{-1}+8^{-1}} = \dfrac{x+8}{\frac{1}{x}+\frac{1}{8}} = \dfrac{x+8}{\frac{8+1}{8x}} = 8x$ — ① ① — 2

b) $\dfrac{25^{-1}}{(5^n)^3 \times (125)^{1-n}} = \dfrac{5^{-2}}{5^{3n} \times 5^{3-3n}} = 5^{-2-3n-3+3n}$ ① $= 5^{-5}$ ① — 3

c) $|4-5x| < 11$

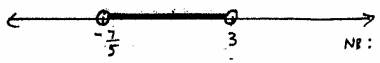
$4-5x < 11$ OR $4-5x > -11$

$-5x < 7$ $-5x > -15$

$x > -\frac{7}{5}$ $x < 3$

① ① — 3

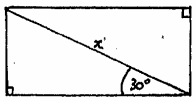
OR $-\frac{7}{5} < x < 3$




①

NB: $-\frac{1}{2}$ if closed circles
1 if graphed solution is correct from incorrect answers above
0 if arrows are pointing wrong way.

d)




$\frac{x}{48} = \sec 30°$ ① or equivalent first step.

$x = 48 \times \sec 30°$

$= 48 \times \frac{2}{\sqrt{3}}$

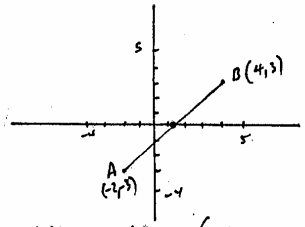
$= \frac{96}{\sqrt{3}}$ *

$= \frac{96\sqrt{3}}{3}$

$= 32\sqrt{3}$ ① — 2

* $-\frac{1}{2}$ if not rationalised

e)




(i) Midpoint AB $= \left(\frac{-2+4}{2}, \frac{-3+3}{2}\right) = (1, 0)$ — 1

$\frac{1}{2}$ for x coord. $\frac{1}{2}$ for y coord.

(ii) $m_{AB} = \frac{3+3}{4+2} = \frac{6}{6} = 1$ — 1

(iii) $\perp$ bis $m_{AB} = -1$ ½

$\therefore y - 0 = -1(x-1)$

$y = -x + 1$ } ½ for any equivalent equation — 1

(iv) The $\perp$ bisector will pass through C(k, 5), so satisfies the equation $y = -x+1$. ① — 2

$\therefore 5 = -k + 1$

$k = -4$ ①

## Question 2 (15 marks)

a) (i) $\frac{d}{dx}\left(4\sin(3-x)\right) = -1 \times 4\cos(3-x)$
$= -4\cos(3-x)$ — 1

$-\frac{1}{2}$ if no negative

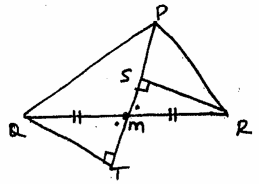
(ii) $\frac{d}{dx}\sqrt{1+x^2} = \frac{d}{dx}(1+x^2)^{\frac{1}{2}}$ ①
$= \frac{1}{2}(1+x^2)^{-\frac{1}{2}} \times 2x$ ①
$= x(1+x^2)^{-\frac{1}{2}}$ — 2

(iii) $\frac{d}{dx}\left(3x\,e^{2x}\right)$ ①
$= 3x \cdot 2e^{2x} + e^{2x} \cdot 3$ ①
$= 6x\,e^{2x} + 3e^{2x}$
OR
$= 3e^{2x}(2x+1)$ — 2

(iv) $\frac{d}{dx}\left(\log_e(\log_e x)\right) = \frac{\frac{1}{x}}{\log_e x} = \frac{1}{x\log_e x}$ ① — 1

$-\frac{1}{2}$ if left as this

b)




(i) In $\Delta$'s QMT & SRM:

| | | |
|---|---|---|
| RM = QM (given) | ½ | |
| $\angle RSM = \angle QTM = 90°$ (given) | ½ | |
| $\angle SMR = \angle QMT$ (vertically opposite) | 1 | * |
| $\therefore \Delta QMT \equiv \Delta SRM$ (AAS) | 1 | |

3.

* Alternatively:
SR ∥ QT (since the alternate angles RSM and MTQ are equal then the lines are parallel)
$\therefore \angle SRM = \angle TQM$ (alternate angles equal; SR ∥ QT)

(ii) There are several Alternatives possible.

Solution 1:
① SR ∥ QT (since the alternate angles $\angle RSM$ and $\angle QTM$ are equal, then the lines SR and QT are parallel)
½ SR = QT (corresponding sides of congruent $\Delta$'s) — 2
$\therefore$ QSRT is a parm (one pair of opposite sides equal and parallel) ½

Solution 2:
SM = MT (corresponding sides of congruent $\Delta$'s)
QM = MT (given)
$\therefore$ QSRT is a parm (diagonals ST and QR of a parm bisect each other) — 2

c) For equal roots, $\Delta = 0$
ie $(k+4)^2 - 4(7+k) = 0$ ①
$k^2 + 8k + 16 - 28 - 4k = 0$
$k^2 + 4k - 12 = 0$ ½
$(k+6)(k-2) = 0$
$k = -6$ or $2$ ½ — 2

d) $\lim_{x\to 1} \frac{x(x-1)}{1-x}$
$= \lim_{x\to 1} -x$ ① — 2
$= -1$ ①

## Question 3 (15 marks)

a) $y = -x^3 + 13x^2 - 35x + 5$

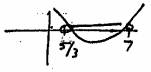
$\frac{dy}{dx} = -3x^2 + 26x - 35$ ①

$-3x^2 + 26x - 35 > 0$

$3x^2 - 26x + 35 < 0$

$(3x - 5)(x - 7) < 0$ ①

$\frac{5}{3} < x < 7$ ①




OR $x > \frac{5}{3}$ and $x < 7$ 3.

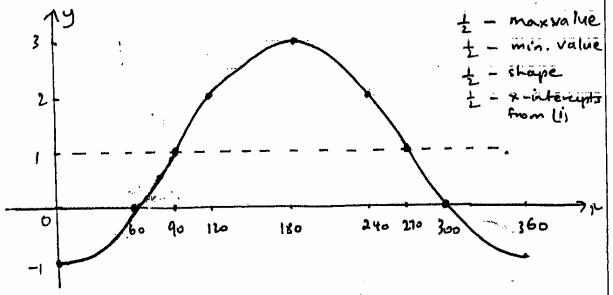
b) (i) $1 - 2\cos x = 0$, $0 \le x \le 360°$

$\cos x = \frac{1}{2}$ ①

$x = 60°, 300°$

$(\frac{1}{2})$ $(\frac{1}{2})$ 2

(ii) $y = 1 - 2\cos x$, $0 \le x \le 360°$




$\frac{1}{2}$ – max value
$\frac{1}{2}$ – min. value
$\frac{1}{2}$ – shape
$\frac{1}{2}$ – x-intercepts from (i) 2

c)(i) $y = (2x - 1)^2$

$\frac{dy}{dx} = 2(2x - 1) \cdot 2 = 4(2x - 1)$ (½)

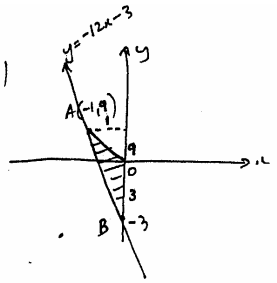
at $x = -1$, $\frac{dy}{dx} = 4(2x - 1 - 1) = -12$ (½)

at $x = -1$, $y = (2x - 1 - 1)^2 = 9$ ①

∴ Eq$^n$ of tangent: $y - 9 = -12(x + 1)$ ①

$y = -12x - 3$ 3.

(ii)

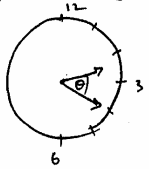



①

Area $\Delta OAB = \frac{1}{2} \times 12 \times 1 =$

$= \underline{6 \text{ units}^2}$ ① 2

d) Need angle between the hands at 2:20

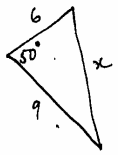



$\frac{360}{12} = 30°$ for each 5 minutes.

$\frac{4}{12} \times 30° = 10°$

∴ $\theta = 30 + 30 - 10$

$\underline{\theta = 50°}$ ①




$x^2 = 6^2 + 9^2 - 2.6.9.\cos 50°$ * ①

$= 47.578938 15$

$x = \pm 6.897748774$ ①

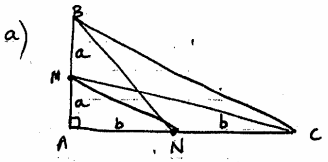
$x = \underline{6.9 \text{ (1dp)}}$ $-\frac{1}{2}$ if not correct to 1 dp

3.

* If angle of 50° is incorrect, max marks possible is ②/3.

## Question 4 (15 marks)

a)




$MN^2 = a^2 + b^2$

In $\Delta AMN$: $a^2 + b^2 = MN^2$ ①

In $\Delta BAN$: $(2a)^2 + b^2 = BN^2$

$4a^2 + b^2 = BN^2$ ①

In $\Delta CAM$: $a^2 + (2b)^2 = CM^2$

$a^2 + 4b^2 = CM^2$ ①

$\therefore BN^2 + CM^2 = 4a^2 + b^2 + 4b^2 + a^2$ ①½

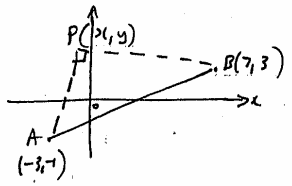
$= 5a^2 + 5b^2$

$= 5(a^2 + b^2)$ ½

$= 5 \times MN^2$

4

b)




(i) Condition required: $m_{PA} \times m_{PB} = -1$

$\frac{y+1}{x+3} \times \frac{y-3}{x-7} = -1$ ①

* Any of these are possible for ① mark.

$\frac{y^2-2y-3}{x^2-4x-21} = -1$

$y^2 - 2y - 3 = -x^2 + 4x + 21$

$y^2 - 2y + 1 + x^2 - 4x + 4 = 21 + 3 + 1 + 4$

$(y-1)^2 + (x-2)^2 = 29$

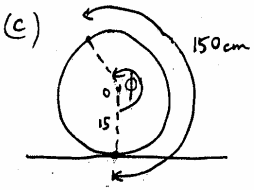
2

(ii) $(x-2)^2 + (y-1)^2 = 29$ ①

Centre of circle is $(2,1)$ ① , radius is $\sqrt{29}$ ①

3

(c)




(i) $\ell = r\theta$

$150 = 15 \times \theta$

$\theta = 10$ rads

(½) if wrong units used.

$\therefore$ Wheel rotates angle of 10 rads

1

(ii) $2\pi$ rads = 1 revolution

1 rad = $\frac{1}{2\pi}$ revolutions

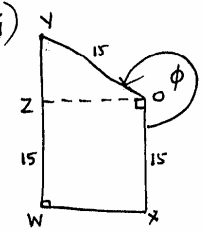
10 rads = $\frac{10}{2\pi}$ revolution ①

$\therefore$ 10 rads = 1.59 revolutions

Thus, the point has made 1.59 revolutions which is more than 1.5 revolutions. ①

2

(iii)




$\phi = (10 - 2\pi)$ rads ①

$(\doteq 3.716814693)$

$\angle YOZ = 2\pi - \phi - \frac{\pi}{2}$

$= 2\pi - (10 - 2\pi) - \frac{\pi}{2}$

$= 4\pi - 10 - \frac{\pi}{2}$

$\therefore \angle YOZ = (\frac{7\pi}{2} - 10)$ rads ①

$\frac{YZ}{15} = \sin \angle YOZ$

$\therefore YZ = 15 \sin(\frac{7\pi}{2} - 10)$

$= 12.58607294$ (½)

$\therefore YW = 15 + 12.58607294$

$= \underline{27.586}$ (3dp) (½)

$\therefore$ The height of point above ground is 27.586 cm

3