JAMES RUSE AGRICULTURAL HIGH SCHOOL
YEAR 11 MATHEMATICS
HALF YEARLY EXAM 2005

QUESTION 1 **Marks**

(a) Graph on the number line $\{ x: -1 \le x \le 3 \}$ **1**

(b) Simplify $\left(2^x\right)^x$ **1**

(c) Solve $|5x-6| \le 12$ **2**

(d) Shade the region $(x-1)^2 + (y-1)^2 \le 1$ **1**

(e) Find to 2 decimal places the solution of $3^x = 7$ **2**

(f) Rationalize the denominator : $\dfrac{3\sqrt{2}-5}{4+\sqrt{2}}$ **2**

(g) Solve $x^2 \ge 121$ **2**

(h) Solve $2\sin x + 1 = 0$ for $\{-90^0 \le x \le 90^0\}$ **1**

(i) Graph $y = x(x+1)(x+3)^2$, **3**

hence solve $x(x+1)(x+3)^2 \le 0$

QUESTION 2 (START A NEW PAGE)

(a) Triangle ABC is represented by the points $A(3, -2)$, $B(4, 5)$ and $C(-6, 1)$.

Find **:**

(i) the equation of line AB in general form. **2**

(ii) the equation of the line perpendicular to AB passing through point C. **2**

(iii) the length of the altitude from A to BC if the equation of BC is given by $2x-5y+17=0$. **2**

(iv) the co-ordinates M of the midpoint AB. **1**

(v) the length of the median from C to AB . **2**

(vi) the co-ordinates of point D if $ABCD$ is a parallelogram. **2**

(b) The roots of $2x^2 - 3x - 5 = 0$ are α and β .
Find the value of **:**

(i) $\alpha + \beta$ **1**

(ii) $\alpha\beta$ **1**

(iii) $\alpha^2 + \beta^2$ **2**

QUESTION 3 (START A NEW PAGE)

(a) Differentiate with respect to x **:**

(i) $\dfrac{9x^2 - 11}{x}$ **2**

(ii) $\sqrt{x}\,(3x - 5)$ **2**

(b) Find the equation of the tangent to the curve $y = 4x^3 - 8x + 7$ at $x = -1$. **3**

(c) Graph $y = \tan x^0$ in the domain $\{0^0 \le x^0 \le 180^0\}$ **2**

(d) Find the value of x giving reasons $(6 - x)(x + 2) = 3x$ **2**

(e) A plane travels 250 km on a bearing $N57^0 E$, then 380 km on a bearing of 193^0T.

(i) How far (to nearest km) is the plane from the initial position? **2**

(ii) What is the final bearing of the plane from the initial position ? **2**

QUESTION 4 (START A NEW PAGE)

(a) Graph $y = \sqrt{16 - x^2}$. **1**

State the domain and range. **2**

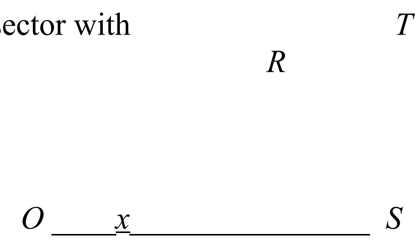
(b) Find the Cartesian equation of the locus of point $P(x, y)$ if point P is equidistant from $A(3, -2)$ and $B(-1, 4)$. **2**

(c) Find the values of k such that $(2k + 1)x^2 - 5x + 3$ is positive definite. **2**

(d)(i) Triangle OTS is right angled at S and ORS is a sector with OS and OR equal to one unit. **2**

Prove the inequality **:**

$\sin x < x < \tan x$ for $\{\ 0 < x < \dfrac{\pi}{2}\ \}$



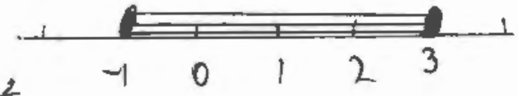
(ii) Hence deduce the value of $\lim\limits_{x \to 0} \dfrac{x}{\sin x}$. **1**

(e) A circular disc 8cm in diameter touches a flat surface at the point P. **2**

If the disc rolls 35 cm how high is the point P above the flat surface ?

(f) Solve for n **:** $2^{2n+1} - 9(2^n) + 4 = 0$ **3**

1(a)



(b) 2^{x^2}

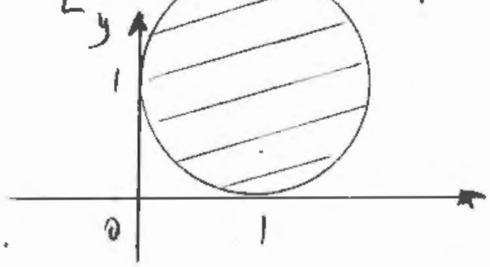
(c) $5x-6=\pm 12$

$5x=-6$ or 18

$x=-1\frac{1}{5}$ or $3\frac{3}{5}$

Soln $\left\{-1\frac{1}{5}\le x\le 3\frac{3}{5}\right\}$

(d)



(e) $3^x=7$

$x=\log_3 7$

$=\dfrac{\log 7}{\log 3}$

$=1.77$

(f) $\dfrac{(3\sqrt{2}-5)}{(4+\sqrt{2})}\cdot\dfrac{(4-\sqrt{2})}{(4-\sqrt{2})}$

$=\dfrac{12\sqrt{2}-6-20+5\sqrt{2}}{16-2}$

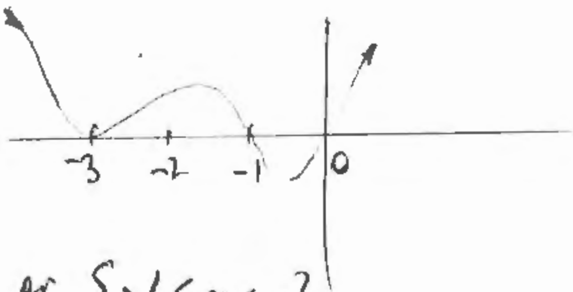
$=\dfrac{17\sqrt{2}-26}{18}$

(g) $\{x\ge 11\}$ or $\{x\le -11\}$

(h) $\sin x=-\frac{1}{2}$

$x=-30^\circ$

i



$x=3$ or $\{-1\le x\le 0\}$

2a(i) $m_{AB}=7$

$y+2=7(x-3)$

$7x-y-23=0$

(ii) $x+7y=-6+7$

$x+7y-1=0$

(iii) $D=\left|\dfrac{6+10+17}{\sqrt{2^2+5^2}}\right|$

$=\dfrac{33}{\sqrt{29}}$ units.

(iv) $M\left(\frac{7}{2},\frac{3}{2}\right)$

(v) $D=\sqrt{\left(\frac{-21}{2}\right)^2+\left(\frac{-1}{2}\right)^2}$

$=\dfrac{\sqrt{442}}{2}$ units

(vi) $M_{AC}=\left(\frac{-3}{2},\frac{-1}{2}\right)$

$M_{BD}\left(\frac{x+4}{2},\frac{y+5}{2}\right)$

$D\equiv(-7,-6)$

(b) (i) $\alpha+\beta=\frac{3}{2}$

(ii) $\alpha\beta=\frac{-5}{2}$

(iii) $\alpha^2+\beta^2=(\alpha+\beta)^2-2\alpha\beta$

$=\left(\frac{3}{2}\right)^2-2\cdot\frac{-5}{2}$

$=7\frac{1}{4}$

3(a) (i) $9 + \frac{11}{x^2}$

(ii) $\frac{d}{dx}\left[3x^{\frac{3}{2}} - 5x^{\frac{1}{2}}\right] = \frac{9}{2}x^{\frac{1}{2}} - \frac{5}{2}x^{-\frac{1}{2}}$

b) $y = 4x^3 - 8x + 7$

$\frac{dy}{dx} = 12x^2 - 8$

$x = -1$

$y = 11$

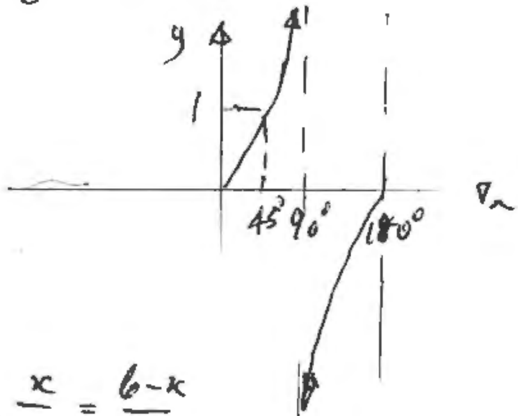
$m = 12 - 8$
$= 4$

Eqn Tangent

$y - 11 = 4(x+1)$

$4x - y + 15 = 0$

c)



$\frac{x}{x+2} = \frac{6-x}{3}$

d) $(6-x)(x+2) = 3x$ (A line parallel to one side of a triangle divides the other two in the same ratio)

$12 + 4x - x^2 = 3x$

$x^2 - x - 12 = 0$

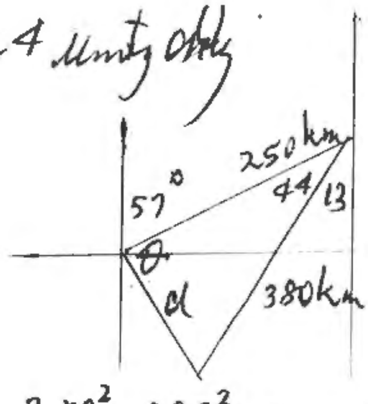
$(x-4)(x+3) = 0$

$x = 4$ or $x = -3$

But $\{0 < x < 6\}$

∴ $x = 4$ units only

(e)



$d^2 = 250^2 + 380^2 - 2 \times 250 \times 380 \cos 44^\circ$

$= 70225$

$d = 265$ km.

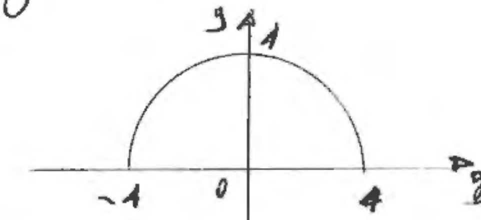
(ii) $380^2 = 70225 + 250^2 - 2 \times 250 \times 265 \cos\theta$

$\cos\theta = -0.09$

$\theta = 95^\circ 3'$

Brg $152^\circ T$

4



Domain $\{-4 \le x \le 4\}$

Range $\{0 \le y \le 4\}$

b) $(x-3)^2 + (y+2)^2 = (x+1)^2 + (y-4)^2$

$x^2 - 6x + 9 + y^2 + 4y + 4 = x^2 + 2x + 1 + y^2 - 8y + 16$

$-8x + 12y - 4 = 0$

$2x - 3y + 1 = 0$

(c) $2k + 1 > 0$

$k > -\frac{1}{2}$

And $\Delta < 0$

$25 - 4 \times 3 \times (2k+1) < 0$

$25 - 24k - 12 < 0$

$-24k < -13$

$k > \frac{13}{24}$

∴ Soln $k > \frac{13}{24}$

d(i) Area ΔORS < Area Sector < Area ΔOR

$\frac{1}{2} \cdot 1 \cdot 1 \sin x < \frac{1}{2} \cdot 1^2 \cdot x < \frac{1}{2} \cdot 1 \cdot \tan x$

$\sin x < x < \tan x$

(ii) $1 < \frac{x}{\sin x} < \frac{\sin x}{\cos x} \cdot \frac{1}{\sin x}$

$\lim_{x \to 0}(1) < \lim_{x \to 0}\left(\frac{x}{\sin x}\right) < \lim_{x \to 0} \frac{1}{\cos x}$

$1 < \lim_{x \to 0} \frac{x}{\sin x} < 1$

∴ $\lim_{x \to 0} \frac{x}{\sin x} = 1$

(e) $l = r\theta$

$35 = \cancel{4\theta}$

$\theta = 8.75$ rad.

height $= 4 + 4\sin\left(8.75 - \frac{\pi}{2}\right)$

$= 7.12$ cm.

(f) $2^{2n+1} - 9(2^n) + 4 = 0$

$2 \cdot (2^n)^2 - 9(2^n) + 4 = 0$

Let $u = 2^n$

$2u^2 - 9u + 4 = 0$

$(2u - 1)(u - 4) = 0$

$u = \frac{1}{2}$ or $u = 4$

$2^n = 2^{-1}$ or $2^n = 2^2$

$n = -1$ or $n = 2$.