Year 11 Mathematics (2u) – Half Yearly Examination 2006

Question 1 **Marks**

a. Solve $|2x+6| = 3x-1$. **3**

b. Find the domain and range for $y = \dfrac{1}{\sqrt{9-x^2}}$. **2**

c. Find the value of n for $\dfrac{9^n.2^{n-1}}{36^n} = 1$. **3**

d. Find the derivative of $\dfrac{4x+\sqrt{x}}{x}$. **2**

e. Evaluate $\lim\limits_{x \to -1} \dfrac{x+1}{3x^2+5x+2}$. **2**

f. Simplify and express with a rational denominator in its simplest form : **3**

$$\frac{\sqrt{5}(3-\sqrt{20})}{\sqrt{5}-2(\sqrt{5}-10)}$$

Question 2 (Start a new page)

a. Is the line $x + y = 3$ tangent to the circle $2x^2 + 2y^2 = 9$? Give reasons. **2**

b. i. Find, by first principles, the derivatives of $f(x) = x^2$. **2**
ii. Find the equation of tangent to $f(x) = x^2$ at $x = 3$. **2**

c. Let A and B be the fixed points (-1,0) and (2,0) and let P be the variable point (x, y),
i. Suppose that P moves so that $PA=2PB$. Deduce that P moves on a circle. **2**
ii. Find the centre and radius of the circle. **2**

d. i. If $\tan\theta = c$, for $90° < \theta < 180°$, find the value of $\sin\theta$ in terms of c. **2**
ii. Solve for x : $\cos^2 x = \dfrac{3}{4}$, $0^\circ \le x \le 360^\circ$ **3**

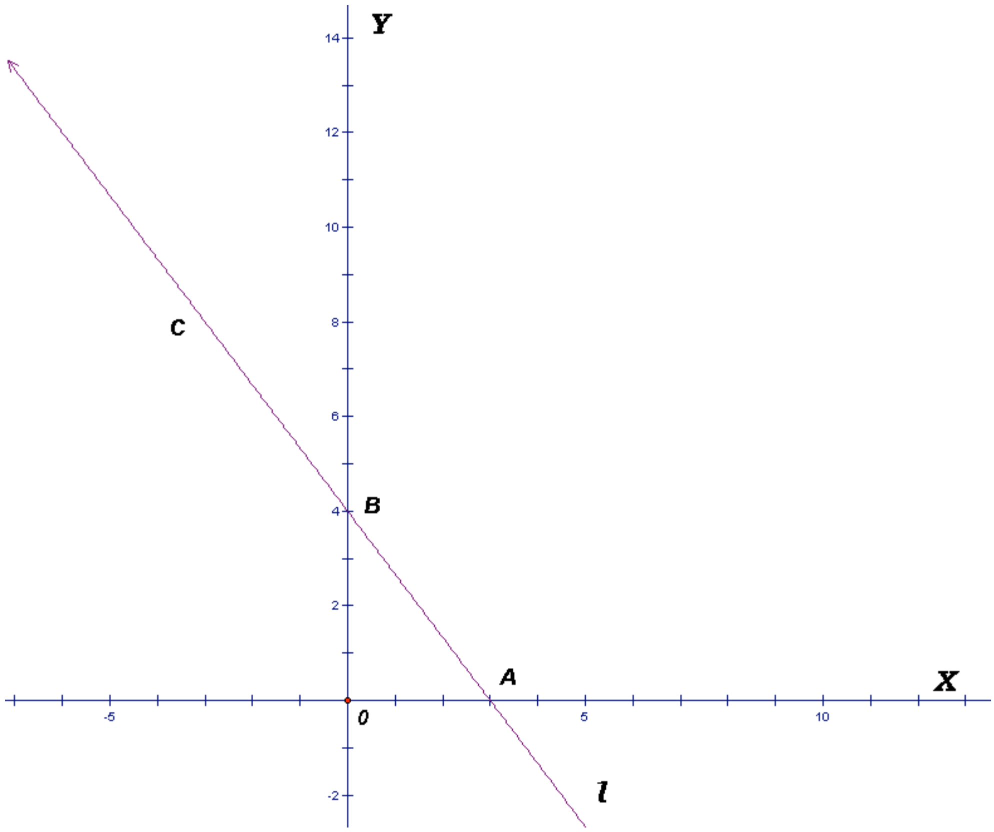
Question 3 (Start a new page)

a. From a point A on a straight road running north and south, the bearing of a point C is N 38° E. From B, 9570 metres north of A, the bearing of C is S 41° E. Calculate

i. the distance from A to C (to the nearest metre). **3**
ii. the shortest distance from C to the road (to the nearest metre). **2**

Question 3 (continued)

b. **Marks**



In the diagram above, the line l cuts the x axis at $A(3,0)$ and y axis at $B(0,4)$.
C is a point on l such that $AB=BC$.
$D(3,8)$ is on the line k where the line k (not shown on diagram) has equation :
$4x + 3y - 36 = 0$.

Copy or trace the diagram into your writing booklet.

i. Find the equation of l. **2**

ii. Draw the graph of k in your diagram indicating where it cuts the axes. **1**

iii. Show that the coordinates of C is (-3, 8). **1**

iv. Find the perpendicular distance of the point C from the line k: $4x + 3y - 36 = 0$. **2**

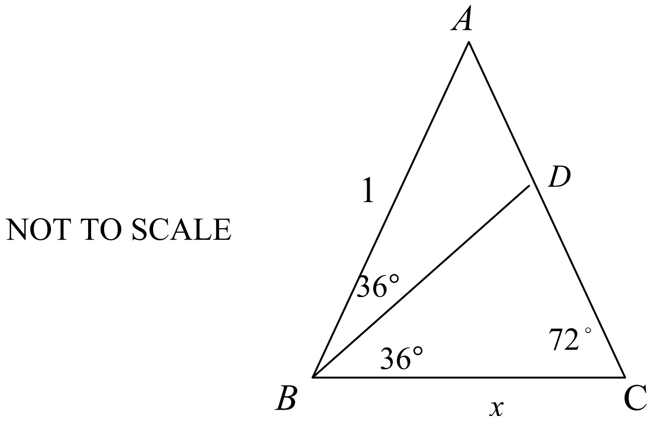
v. M is the midpoint of CD. Show the equation of a circle centre D and radius DM has the equation $x^2 - 6x + y^2 - 16y + 64 = 0$. **2**

vi. In your diagram shade the region that satisfies both $x^2 - 6x + y^2 - 16y + 64 \leq 0$ and $4x + 3y - 36 \geq 0$ simultaneously. **2**

Question 4 (Start a new page) **Marks**

a. Graph $y = \dfrac{x}{x+1}$, showing all the essential features. **3**

b.



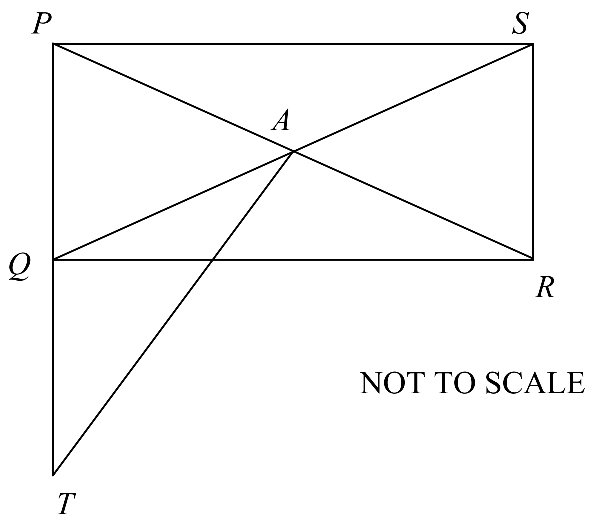
In the diagram above, ABC is an isosceles triangle where $\angle ABC = \angle BCA = 72^\circ$ and $AB = AC = 1$. Angle ABC is bisected by BD and $BC = x$.

Copy the diagram into your answer booklet.

i. Show that triangles ABC and BCD are similar. **2**

ii. By using part i, find the exact value of x. **4**

c.



In the diagram above, $PQRS$ is a rectangle. Diagonals PR and SQ meet at A. Q is the midpoint of PT and $PT = PR$.

Copy the diagram into your answer booklet.

i. Prove $\Delta PRQ \equiv \Delta TRQ$. **2**

ii. Hence, prove $AR \perp AT$. **4**

END of PAPER

Solutions to 2006 2u T2

Q1a) $2x+6 = 3x-1$ or $2x+6 = 1-3x$

$x = 7$ 1m or $x = -1$ 1m

check $|2x+6| = 3x-1$

If $x=7$ LHS $= |2\times7+6| = 20$

RHS $= 3(7)-1 = 20$

$\therefore x = 7$

If $x=-1$

LHS $= |2(-1)+6| = 4$

RHS $= 3(-1)-1 = -4$

$\therefore x \neq -1$

$\therefore x = 7$ only 1m

b) D: $-3 < x < 3$ 1m

R: $\frac{1}{3} \leq y$ 1m

c) $\dfrac{\cancel{3^{2n}} \cdot 2^{n-1}}{2^{2n} \cdot \cancel{3^{2n}}} = 1$ 1m

$2^{n-1} = 2^{2n}$ 1m

$\therefore -1 = n$ 1m

d) $\dfrac{4x + \sqrt{x}}{x} = 4 + \dfrac{1}{\sqrt{x}}$ 1m

$\therefore \dfrac{d}{dx}\left(4 + \dfrac{1}{\sqrt{x}}\right) = -\dfrac{1}{2\sqrt{x^3}}$ 1m

e) $\lim\limits_{x \to -1} \dfrac{\cancel{x+1}}{(3x+2)(\cancel{x+1})} = \lim\limits_{x \to -1} \dfrac{1}{3(-1)+2} = -1$ 1m 1m

f) $\dfrac{3\sqrt{5} - \sqrt{100}}{\sqrt{5} - 2\sqrt{5} + 20} = \dfrac{3\sqrt{5} - 10}{20 - \sqrt{5}} \times \dfrac{20 + \sqrt{5}}{20 + \sqrt{5}} = \dfrac{60\sqrt{5} - 200 + 15 - 10\sqrt{5}}{400 - 5}$ 1m 1m

$= \dfrac{50\sqrt{5} - 185}{395} = \dfrac{10\sqrt{5} - 37}{79}$ 1m 1m

Q2a) $y = 3 - x$

$2x^2 + 2(3-x)^2 = 9$

$2x^2 + 2(9 - 6x + x^2) = 9$

$4x^2 - 12x + 18 - 9 = 0$

$4x^2 - 12x + 9 = 0$

$(2x-3)^2 = 0$ 1m

$x = 3/2$ (double root)

yes, $x + y = 3$ is tangent to $2x^2 + 2y^2 = 9$ because it touches the circle at only 1 point. 1.

b) (i) $f(x) = x^2$

$\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h} = \lim\limits_{h \to 0} \dfrac{(x+h)^2 - x^2}{h} = \lim\limits_{h \to 0} \dfrac{\cancel{x^2} + 2xh + h^2 - \cancel{x^2}}{h}$ 1m

$= \lim\limits_{h \to 0} \dfrac{\cancel{h}(2x + h)}{\cancel{h}} = 2x$ 1m

(ii) Eq of tangent to $f(x) = x^2$ at $x = 3$, $y = 9$

$y - 9 = 2(3)[x - 3]$ 1m

$y - 9 = 6x - 18$

$y = 6x - 9$ 1m

c) $(PA)^2 = (x+1)^2 + y^2$ $\quad (PB)^2 = (x-2)^2 + y^2$

$PA = 2PB$ $\therefore (PA)^2 = 4(PB)^2$

$(x+1)^2 + y^2 = 4[(x-2)^2 + y^2]$ 1m

$x^2 + 2x + 1 + y^2 = 4(x^2 - 4x + 4 + y^2)$

$x^2 + 2x + 1 + y^2 = 4x^2 - 16x + 16 + 4y^2$

$-15 = 3x^2 - 18x + 3y^2$ 1m

c ii) $-15+27 = 3(x^2-6x+9)+3y^2$

$12 = 3(x^2-6x+9)+3y^2$

$4 = (x-3)^2+y^2$

$\therefore$ centre of circle is $(3,0)$, 1m radius $= 2$ unit 1m

d i) $\tan\theta$ and $\sin\theta$ have different signs for $90<\theta<180$

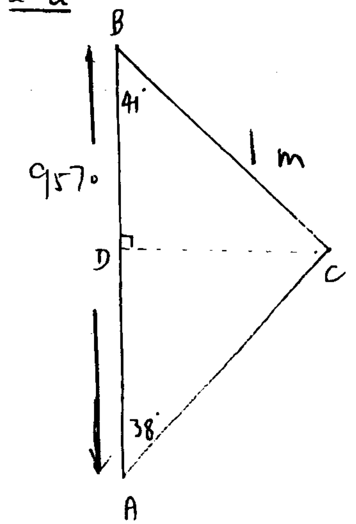
$\therefore \sin\theta = \dfrac{-c}{\sqrt{1+c^2}}$ #

1m for "-" sign

1m for correct answer

ii) $\cos x = \dfrac{\pm\sqrt{3}}{2}$ 1m $\therefore x = \cancel{60^\circ}\ 30,\ \cancel{120^\circ}\ 150,\ \cancel{240}\ 210,\ \cancel{300}\ 330$ 2m

23a



$\angle BCA = 180 - 41 - 38 = 101$

$\dfrac{9570}{\sin 101} = \dfrac{AC}{\sin 41}$ 1m

$AC = 9570\dfrac{\sin 41}{\sin 101} = 6396$ (nearest metre) 1m

$CD \perp AB$

$CD = AC \sin 38 = 3938$ (nearest metre)

1 m 1m

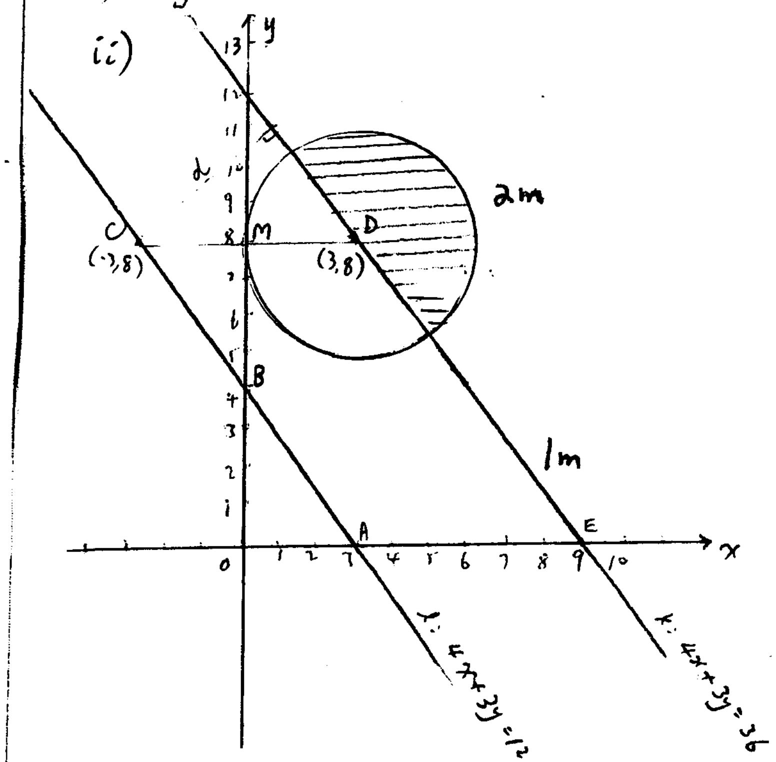
Q 3b

i) Eq of line l:

$y-0 = (x-3)\left[\dfrac{4}{-3}\right]$ 1m $\therefore$ $-3y = 4x-12$

$12 = 4x+3y$ 1m

ii)



iii) Let $C = (x,y)$

$\dfrac{x+3}{2} = 0 \quad \therefore x = -3$

$\dfrac{y+0}{2} = 4 \quad \therefore y = 8$ 1m

$\therefore C = (-3,8)$

iv) $d = \dfrac{|4(-3)+3(8)-36|}{\sqrt{4^2+3^2}}$ 1m $= \dfrac{|-12+24-36|}{5} = \dfrac{24}{5}$ 1m

v) D centre of circle $=(3,8)$ DM $= 3$ (radius)

$(x-3)^2+(y-8)^2 = 3^2$ 1m

$x^2-6x+\cancel{9}+y^2-16y+64 = \cancel{9}$

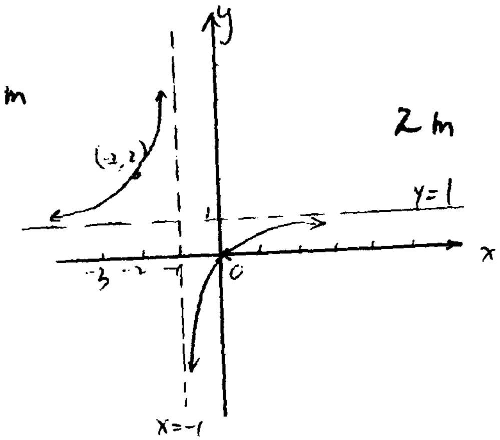
$x^2-6x+y^2-16y+64 = 0$ 1m

5

Asy : $x = -1$
$y = 1$ } 1m

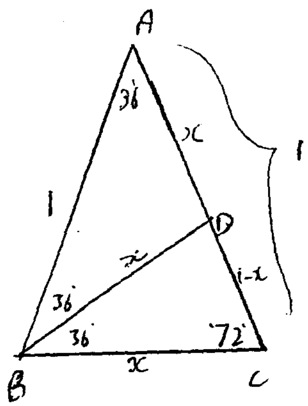
Intercept $x=0,\ y=0$

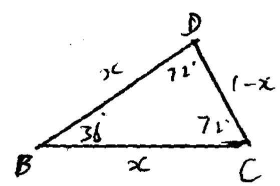
* any other pt on the other branch $-\frac{1}{2}$



2m

b)





m i) In ΔABC, $\angle ACB = \angle ABC = 72°$ (given)

m { In ΔBCD, $\angle DCB = 72°$ (given)
$\angle DBC = 36°$ (given)

m $\therefore \angle BDC = 180° - 36° - 72° = 72°$ (angle sum of triangle BDC)

$\therefore \angle DCB = \angle BDC = 72°$

m $\therefore \Delta ABC \ ||| \ \Delta BCD$ (equiangular)

ii) $\angle DBA = 36$ (given)

$\angle CAB = 180 - 72 - 72 = 36$ (angle sum of triangle ABC)

m $\therefore AD = BD$ (sides opposite equal angles are equal)

m Similarly $BC = BD$

1m $\frac{AB}{BC} = \frac{BC}{DC}$ (corresponding sides of similar triangles ABC, BCD)

Let $AD = BD = BC = x$, $DC = 1 - x$

1m $\frac{1}{x} = \frac{x}{1-x}$

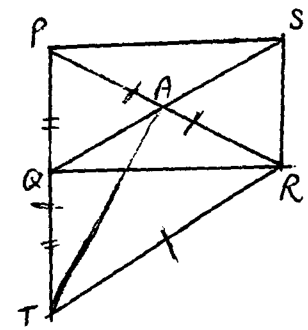
$1 - x = x^2$

$0 = x^2 + x - 1$

½m $x = \frac{-1 \pm \sqrt{1 - 4(-1)}}{2} = \frac{-1 \pm \sqrt{5}}{2}$

½m But $x > 0$ $\therefore x = \frac{\sqrt{5} - 1}{2}$ #

c)



To prove : i) $\Delta PRQ \equiv \Delta TRQ$

Proof In ΔPRQ, ΔRTQ,

½ $PQ = TQ$ (Q is mid pt of PT)

½ RQ is common

½ { $\angle PQR = 90$ (angle of rectangle)
½ $\angle TQR = 180 - 90 = 90$ (angle sum on straight line PQT)

½m $\therefore \Delta PRQ \equiv \Delta TRQ$ (SAS)

ii) To prove $AR \perp AT$:

½ $PR = TR$ (corresponding sides of congruent triangles PRQ, TRQ)

In ΔPAT and ΔRAT

½ $PT = TR$ (given)

½ AT is common

½ $PA = RA$ (diagonal of rectangle PSRQ bisect each other)

½ $\Delta PAT \equiv \Delta RAT$ (SSS)

½ $\angle PAT = \angle RAT$ (corresponding angle of congruent triangles PAT, RAT)

½ $\angle PAT + \angle RAT = 180$ (angle sum on straight line PAR)

½ $\therefore \angle PAT = \angle RAT = 180 \div 2 = 90$

ie $AR \perp AT$.