**Question 1 (15 Marks)** **Marks**

(a) Factorise $8x^3 + 64y^3$. **2**

(b) Simplify fully $\dfrac{2^{-x}.4^{2x+1}}{(2^{-x})^3}$. **2**

(c) Find the equation of a line passing through the point of intersection of the straight lines $2x - 3y + 5 = 0$ and $3x + 4y + 8 = 0$ and the point (2, 1). **3**

(d) Differentiate with respect to $x$:

(i) $3\pi^2 - 2$ **1**

(ii) $\dfrac{2x^3 - 5x}{x}$ **2**

(iii) $\dfrac{2x}{3-4x}$ **2**

(e) (i) State the natural domain and range for the function $y = \sqrt{3-x}$. **2**

(ii) Neatly sketch the function $y = \sqrt{3-x}$. **1**

**Question 2 (15 Marks)** **START A NEW PAGE** **Marks**

(a) Find the gradient of the tangent to the curve $y = \dfrac{1}{5-2x}$ at the point (3, −1) **2**

(b) Solve for $x$:

(i) $\tan x = 1$, where $0^0 \le x \le 360^0$. **2**

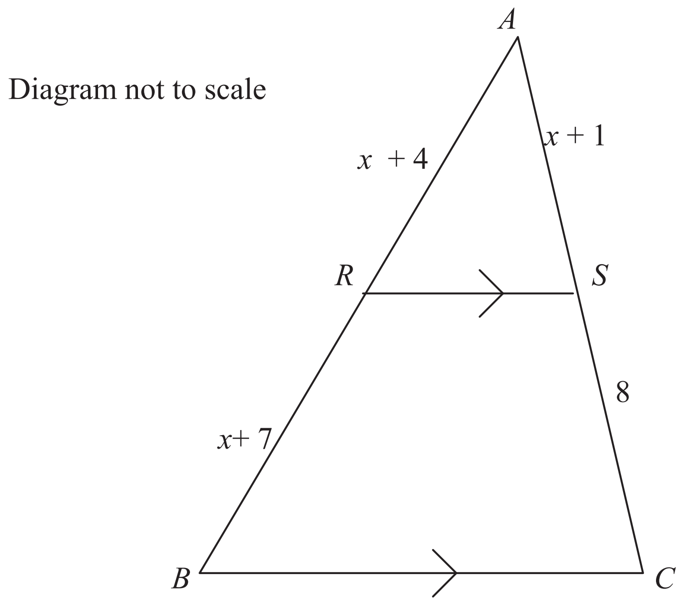
(ii) $\sqrt{3-2x} = x$ **3**

(iii) $\dfrac{18}{2x+1} - \dfrac{5}{x} = 1$ **2**

**Question 2 continues overleaf.**

**Question 2 continued** **Marks**

(c) In the diagram below $RS \parallel BC$. Find the value of $x$, giving reasons. **2**




(d) $ABCD$ is a quadrilateral such that $AB = 8$ units, $BC = 16$ units, $CD = 18$ units and $AD = 24$ units. Diagonal $AC = 12$ units. Draw a neat sketch on your answer page, clearly showing this information.

(i) Prove that $\Delta ABC \parallel\!\mid \Delta CAD$. **2**

(ii) Hence show that $AB \parallel DC$. **2**

**Question 3 (15 Marks)** **START A NEW PAGE** **Marks**

(a) Using a sketch, neatly indicate the region of the Cartesian plane for which the following inequalities hold true. **3**

$y \le x$ , $y \ge 0$ and $y \le \sqrt{9 - x^2}$

(b) Solve for $x$ and $y$, given that $4^x.8^{-y} = 1$ and $25^x = 125 \times 5^y$. **3**

(c) Derive the equation of the locus of $P(x, y)$ such that it's distance from $A(2, 4)$ is twice it's distance from $B(3, 5)$. **3**

**Question 3 continues overleaf.**

**Question 3 continued** **Marks**

(d) $HF$= 400cm and $GF$ = 200cm. $\angle EFH = 15^0$.

(i) Find the size of $\angle HFG$. **1**

(ii) Find the length of $EH$. **1**

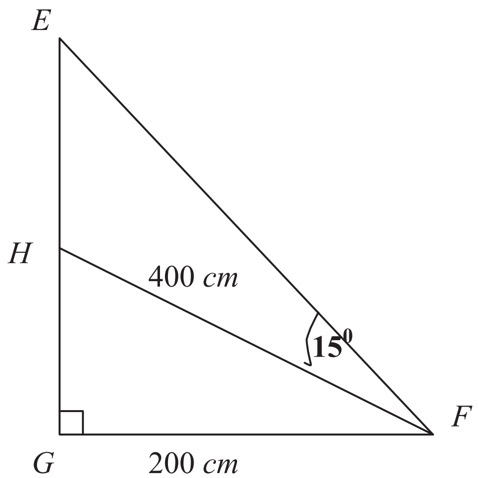



Diagram not to scale

(e) $A(1, 6)$ and $B(3, 2)$ are points on the curve $xy = 6$ and it is known that the tangent at $B$ has slope $-\frac{2}{3}$.

(i) Find the equation of the tangent at $B$. **1**

(ii) If this tangent meets the $x$, $y$ axes at $E$, $F$ respectively, prove that $B$ is the midpoint of $EF$. **3**

**Question 4 (15 Marks)** **START A NEW PAGE** **Marks**

(a) The line $x + y = 3\sqrt{2}$ is a tangent to a circle with centre (0, 0). Find the radius of this circle. **2**

(b) If $f(x)$ and $g(x)$ are odd functions and $h(x) = f(x).g(x)$, prove that $h(x)$ is an even function. **2**

(c) The lines $3x - y + 3 = 0$, $y = 3x - 6$, $y = x - 4$ and $5x + 3y = 15$ form a quadrilateral. Describe the type of quadrilateral formed, giving clear reasons. **3**

**Question 4 continued overleaf.**

**Question 4 continued.** **Marks**

(d) Solve for $x$:

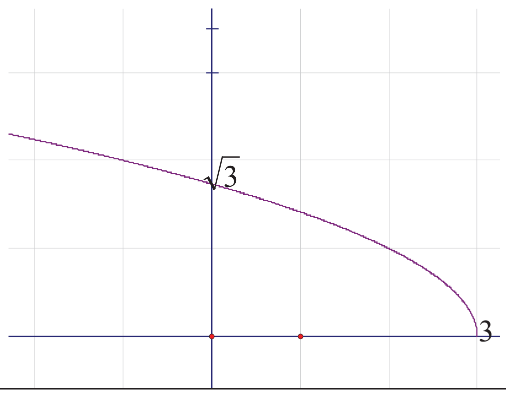
(i) $|3x - 2| < |3 + 2x|$ **3**

(ii) $\cos(180^0 - x) = \frac{1}{2}$; $0^0 \le x \le 360^0$ **2**

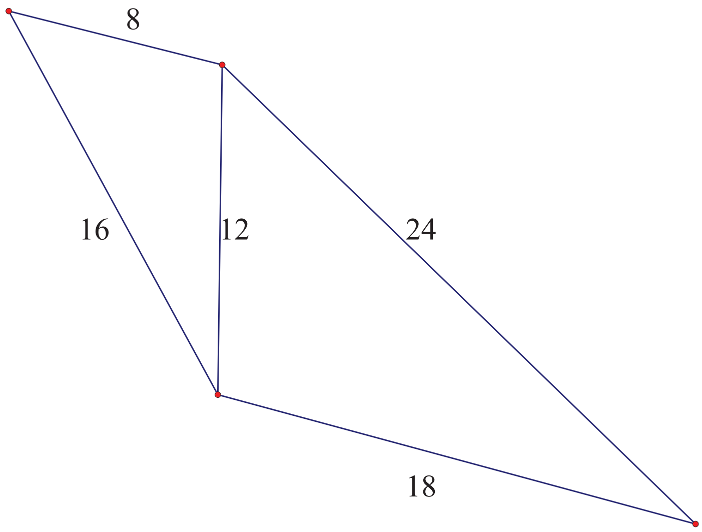
(e) Show, by the method of First principles, that the derivative of $y = \sqrt{2x}$ is given by $\frac{dy}{dx} = \frac{1}{\sqrt{2x}}$. **3**

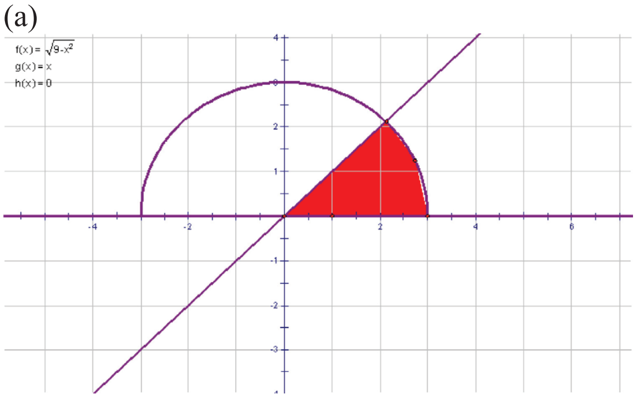
**~END OF TEST ~**

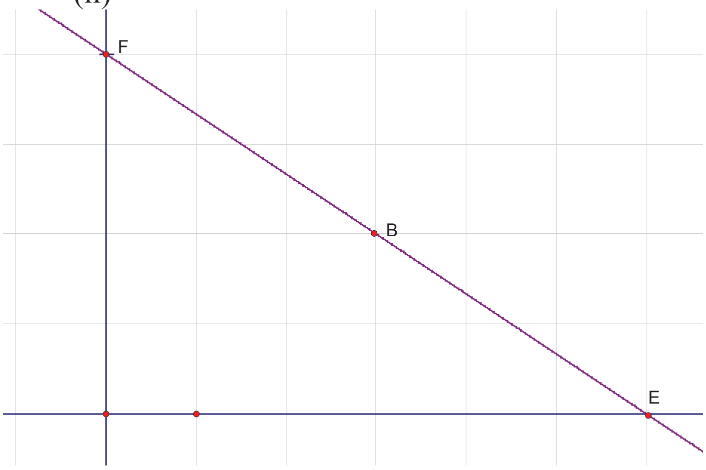
## Year 11 Preliminary Half Yearly Maths (2U) Examinations 2007 – SOLUTIONS

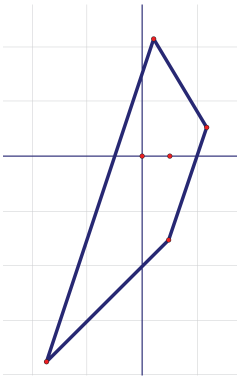
| | |
|---|---|
| **Question 1** | |
| (a) $8x^3 + 64y^3 = (2x + 4y)(4x^2 - 8xy + 16y^2)$ | ➔ ❷ Marks |
| (b) $\frac{2^{-x}.4^{2x+1}}{(2^{-x})^3} = \frac{2^{-x}\cdot 2^{4x+2}}{2^{-3x}}$ <br> $= 2^{6x+2}$ | ➔ ❷ Marks |
| (c) New line must satisfy $2x - 3y + 5 + k(3x + 4y + 8) = 0$ at (2, 1). | ➔ ❶ Mark |
| $\therefore\ 4 - 3 + 5 + k(6 + 4 + 8) = 0 \rightarrow k = -\frac{1}{3}$ | ➔ ❶ Mark for $k$ |
| $\therefore$ eqn of new line is: $3x - 13y + 7 = 0$ | ➔ ❶ Mark for equation. |
| (d) (i) $\frac{d}{dx}(3\pi^2 - 2) = 0$ | ➔ ❶ Mark |
| (ii) $\frac{d}{dx}\left(\frac{2x^3 - 5x}{x}\right) = \frac{d}{dx}\left(2x^2 - 5\right)$, $x \neq 0$ | ➔ ❶ Mark for simplification |
| $= 4x$ | ➔ ❶ Mark correct answer |
| (iii) $\frac{d}{dx}\left(\frac{2x}{3-4x}\right) = \frac{(3-4x)(2)-(2x)(-4)}{(3-4x)^2}$ | ➔ ❶ Mark correct quotient rule |
| $= \frac{6}{(3-4x)^2}$ | ➔ ❶ Mark correct answer |
| **Alternative solution** <br> $\frac{d}{dx}(2x)(3-4x)^{-1} = (2x)\cdot -1(3-4x)^{-2}\cdot -4 + (3-4x)^{-1}\cdot 2$ | ➔ correct use of rule ❶ Mark |
| $= \frac{8x}{(3-4x)^2} + \frac{2}{(3-4x)}$ <br> $= \frac{6}{(3-4x)^2}$ | ➔ ❶ Mark correct answer |
| (e) (i) Domain: $\{x : x \leq 3\}$; Range: $\{y : y \geq 0\}$ | ➔ ❶ Mark each |
| (ii)   | ➔ ❶ Mark |

| | |
|---|---|
| **Question 2** | |
| (a) $\frac{d}{dx}\left(\frac{1}{5-2x}\right)=\frac{2}{(5-2x)^2}$ | ➔ ❶ Mark correct differentiation |
| $\therefore$ gradient at (3, −1) is $m = 2$ | ➔ ❶ Mark |
| (b) (i) $\tan x = 1$<br>$x = 45^0$ or $225^0$ | ➔ ❶ Mark each |
| (ii) $\sqrt{3-2x}=x$ [Restriction : $0 \le x \le 1.5$] | ➔ ❶ Mark restriction |
| $\therefore 3 - 2x = x^2$ ➔ $x^2 + 2x - 3 = 0$ | ➔ ❶ Mark |
| $\therefore (x+3)(x-1)=0$ | |
| $\therefore x = -3$ or $x = 1$ $\therefore$ **only solution is $x = 1$** | ➔ ❶ Mark answer with justification |
| (iii) $\frac{18}{2x+1}-\frac{5}{x}=1$ [ $x \ne 0$ or $x \ne -\frac{1}{2}$ ] | |
| $\therefore 18x - 5(2x+1) = x(2x+1)$ | ➔ ❶ Mark |
| $\therefore 2x^2 - 7x + 5 = 0$ | |
| $\therefore (2x-5)(x-1)=0$ | |
| $\therefore x = 1$ or $x = \frac{5}{2}$ | ➔ ❶ Mark |
| (c) $\frac{x+4}{x+7}=\frac{x+1}{8}$<br>(A line parallel to one side of a triangle divides the other 2 sides in the same ratio) | ➔ ❶ Mark correct ratio with reason |
| $\therefore 8x + 32 = x^2 + 8x + 7$<br>$\therefore x^2 = 25$ ➔ $x = 5$ $(x > 0)$ | ➔ ❶ Mark answer with justification |
| **(d) Next page** | |

| Solution | Marking |
|---|---|
|  | |
| (d) (i) In $\Delta ABC$ and $\Delta ADC$<br>$\frac{AB}{AC}=\frac{8}{12}=\frac{2}{3},\ \frac{AC}{DC}=\frac{12}{18}=\frac{2}{3},\ \frac{BC}{AD}=\frac{16}{24}=\frac{2}{3}$<br>$\therefore\ \Delta ABC \parallel\!\mid \Delta ADC$ (3 corresponding sides in proportion) | ➔ ❶ Mark correct sides in ratio<br><br>➔ correct reasoning ❶ Mark |
| (ii) $\angle BAD = \angle ACD$ (Corresponding angles of similar triangles are equal)<br>$\therefore AB \parallel DC$ (Alternate angles equal only on parallel lines) | ➔ ❶ Mark<br><br>➔ ❶ Mark |

| Solution | Marking |
|---|---|
| **Question 3**<br>(a)<br> | ➔ ❶ Mark<br>$y=\sqrt{9-x^2}$<br>➔ ½ mark each for $y=x$ and $y=0$<br><br>➔ ❶ Mark correct region |
| (b) $4^x.8^{-y}=1$ ➔ $2x-3y=0$ ….(A)<br>$25^x=125\times 5^y$➔ $2x=3+y$ … (B)<br>Subst. (B) into (A)<br>$\therefore\ 3-2y=0\ \therefore\ y=\frac{3}{2}$<br>$\therefore\ 2x=4.5\quad \therefore\ x=\frac{9}{4}$ | ➔ ❶ Mark for (A)<br>➔ ❶ Mark for (B)<br><br>➔ ❶ Mark for answer $x$ and $y$ |

| **Question 3** | |
|---|---|
| (c) $PA = 2PB \rightarrow PA^2 = 4PB^2$ | |
| $\therefore (x-2)^2 + (y-4)^2 = 4[(x-3)^2 + (y-5)^2]$ | → ❶ Mark |
| $\therefore x^2 - 4x + 4 + y^2 - 8y + 16 = 4x^2 - 24x + 36 + 4y^2 - 40y + 100$ | → ❶ Mark |
| $\therefore 3x^2 + 3y^2 - 20x - 32y + 116 = 0 \rightarrow$ eqn of circle. | → ❶ Mark |
| (d) (i) $\cos \angle HFG = \frac{200}{400} \rightarrow \angle HFG = 60^0$ | → ❶ Mark |
| (ii) $\angle HEF = 15^0$ (Angle sum of $\Delta EGF$) | → ½ Mark |
| $\therefore EH = HF$ (Equal sides opposite equal angles) | → ½ Mark |
| $\therefore EH = 400$ cm | |
| (e) (i) gradient = $-\frac{2}{3}$ (given) | |
| $\therefore$ eqn. of tangent is: $y - 2 = -\frac{2}{3}(x-3)$<br>i.e. $2x + 3y - 12 = 0$ | → ❶ Mark<br>(eqn. can be of any form.) |
| (ii)<br><br> | |
| $E(6, 0)$ and $F(0, 4)$ $\therefore$ midpoint = $\left(\frac{6+0}{2}, \frac{0+4}{2}\right) = (3, 2)$<br>$\therefore B$ is the midpoint of $EF$. | → Point $E$ ❶ Mark<br>→ point $F$ ❶ Mark<br>→ Showing $B$ is the midpoint ❶ Mark |

| | |
|---|---|
| **Question 4** | |
| (a) If line is a tangent then the radius = perp. Distance of the line from the centre of the circle. | ➔ ❶ Mark correct use perp. Distance equation |
| $\therefore Radius = \left\| \frac{0+0-3\sqrt{2}}{\sqrt{2}} \right\| = 3$ units | ➔ ❶ Mark correct answer |
| (b) If $f(x)$ and $g(x)$ are odd functions then by definition $f(-x) = -f(x)$ and $g(-x) = -g(x)$ | ➔ ❶ Mark |
| $\therefore h(-x) = f(-x) \times g(-x)$<br>$= -f(x) \times -g(x)$<br>$= f(x) \times g(x) = h(x)$ | |
| $\therefore$ since $h(-x) = h(x)$; **$h(x)$ is an EVEN function** | ➔ ❶ Mark |
| (c) $L_1 : 3x - y + 3 = 0$ → gradient $m_1 = 3$<br>$L_2 : y = 3x - 6$ → gradient $m_2 = 3$<br>$L_3 : y = x - 4$ → gradient $m_3 = 1$<br>$L_4 : 5x + 3y = 15$ → $m_4 = -\frac{5}{3}$ | ➔ ❶ Mark |
|  $\therefore$ since $m_1 = m_2$ we have $L_1 \parallel L_2$<br>$\therefore$ quadrilateral formed is a trapezium (one pair of opposite sides parallel) | ➔ ❶ Mark<br>➔ ❶ Mark |
| (d) (i) $\|3x - 2\| < \|3 + 2x\|$<br>$\therefore 9x^2 - 12x + 4 < 9 + 12x + 4x^2$ (by squaring both sides) | ➔ ❶ Mark |
| $\therefore 5x^2 - 24x - 5 < 0$<br>$\therefore (5x + 1)(x - 5) < 0$ → $-\frac{1}{5} < x < 5$ | ➔ ❶ Mark Correct solution |
| (ii) $\cos(180^0 - x) = \frac{1}{2}$ | |
| $\cos x = -\frac{1}{2}$ (by supplementary angles) | ➔ ❶ Mark |
| $\therefore x = 120^0$ or $x = 240^0$ | ➔ ❶ Mark |

| **Question 4** | |
|---|---|
| (d) $y = \sqrt{2x}$ <br> $\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{(y + \Delta y) - y}{\Delta x}$ by First principles of diff. <br> $\therefore \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\sqrt{2(x + \Delta x)} - \sqrt{2x}}{\Delta x}$ <br> $= \lim_{\Delta x \to 0} \frac{\sqrt{2(x + \Delta x)} - \sqrt{2x}}{\Delta x} \times \frac{\sqrt{2(x + \Delta x)} + \sqrt{2x}}{\sqrt{2(x + \Delta x)} + \sqrt{2x}}$ <br> $= \lim_{\Delta x \to 0} \frac{2x + 2\Delta x - 2x}{\Delta x\left(\sqrt{2x + 2\Delta x} + \sqrt{2x}\right)}$ <br> $= \lim_{\Delta x \to 0} \frac{2\Delta x}{\Delta x\left(\sqrt{2x + 2\Delta x} + \sqrt{2x}\right)}$ <br> $= \lim_{\Delta x \to 0} \frac{2}{\left(\sqrt{2x + 2\Delta x} + \sqrt{2x}\right)}$ <br> $= \frac{2}{\left(\sqrt{2x} + \sqrt{2x}\right)}$ as $\Delta x \to 0$ <br> $= \frac{2}{2\sqrt{2x}} = \frac{1}{\sqrt{2x}}$ as required. | ➔ correct use of first principles ❶ Mark <br><br> ➔ ❶ Mark multiply by the conjugate of numerator <br><br> ➔ ❶ Mark correct simplification to result to answer |