YEAR 11

HALF YEARLY EXAMINATION 2009

MATHEMATICS

Time Allowed *– 90 minutes*
(Plus 5 minutes Reading Time)

All questions may be attempted

All questions are of equal value

Department of Education approved calculators are permitted

In every question, show all necessary working

Marks may not be awarded for careless or badly arranged work

No grid paper is to be used unless provided with the examination paper

The answers to all questions are to be returned in separate bundles clearly labelled Question 1, Question 2, etc. Each question must show your Candidate's Number.

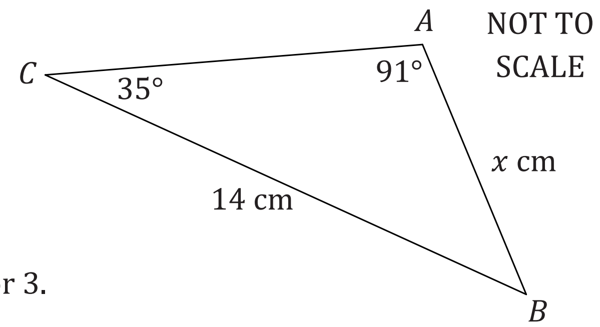
Question 1 (15 Marks) **Marks**

(a) The equation of a circle is given by $x^2 + 2x + y^2 - 6y = 0$, write down its centre and radius. **2**

(b) The tangent at the point $(-2, 1)$ on the curve $y = (x+3)^2$ cuts the x–axis at P and the y–axis at Q. If O is the origin, find the area of ΔPOQ. **3**

(c) (i) In ΔABC, find the value of x, correct to 2 decimal places. **2**

(ii) Hence calculate the area of ΔABC to 3 significant figures. **2**



(d) Simplify $\frac{5x+2}{x} + \frac{6}{x(x-3)}$ provided $x \neq 0$ or 3. **2**

(e) Factorise fully $x^4 - 16y^4$. **2**

(f) Sketch $y = 2\cos x$ for $-180° \leq x \leq 180°$. **2**

Question 2 (15 Marks) – START A NEW PAGE

(a) Find $\frac{dy}{dx}$ if:

(i) $y = \frac{x+1}{7}$ **1**

(ii) $y = \frac{6x^3 + \sqrt{x} - 1}{x^2}$ **3**

(b) (i) Sketch the graph of the line l whose equation is $y = x - 1$, clearly showing all intercepts with the co-ordinate axes. **1**

(ii) Find the co-ordinates of S and T, the points where the line intersects with the circle $x^2 + y^2 = 25$. **2**

(iii) Hence, shade the region that satisfies the union of the following inequalities, clearly labelling all boundaries and points of intersection: **2**
$y \leq x - 1$ and $x^2 + y^2 < 25$.

(c) Differentiate $y = x - 3x^2$ by First principles. **3**

(d) Find the domain and range of the function: $y = \sqrt{25 - (x-3)^2}$. **3**

Question 3 (15 Marks) – START A NEW PAGE **Marks**

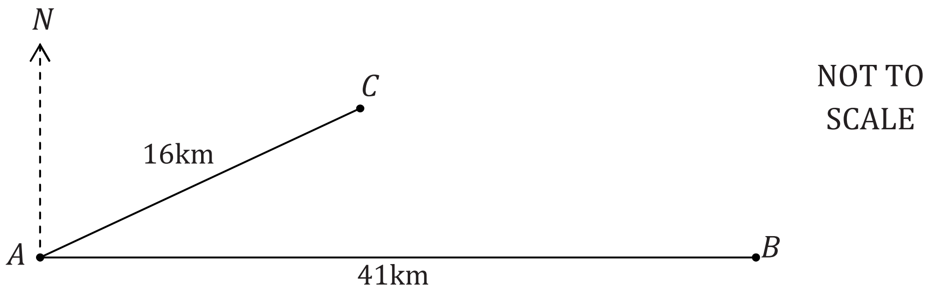
(a) (i) Find k if $5x - 4y + 2 = 0$ and $y = kx + 7$ are parallel. **2**

(ii) Calculate the perpendicular distance between these two lines. **2**

(b) In ΔPQR, X, Y and Z are the midpoints of QR, PR and PQ respectively. **4**
Draw a diagram and hence prove that $\Delta PQR \,|||\, \Delta XYZ$.

(c) Find the magnitude of the acute angle formed between the line $y = 3x - 5$ and the y–axis. **3**

(d) The diagram below shows a point A which is 41km due west of the point B. Point C is 16km from A and has a bearing from A of 068°T.



(i) Find the distance of B from C, correct to the nearest kilometre. **2**

(ii) Find the bearing of B from C, correct to the nearest minute. **2**

EXAM CONTINUES OVERLEAF

Question 4 (15 Marks) – START A NEW PAGE **Marks**

(a) Express $\frac{6+\sqrt{3}}{1-\sqrt{3}}$ in the form $a + b\sqrt{3}$, where a and b are rational. **2**

(b) (i) Graph $y = |x - 4|$, clearly labelling all features. **1**

(ii) Hence or otherwise solve $|x - 4| = 5 - 2x$. **2**

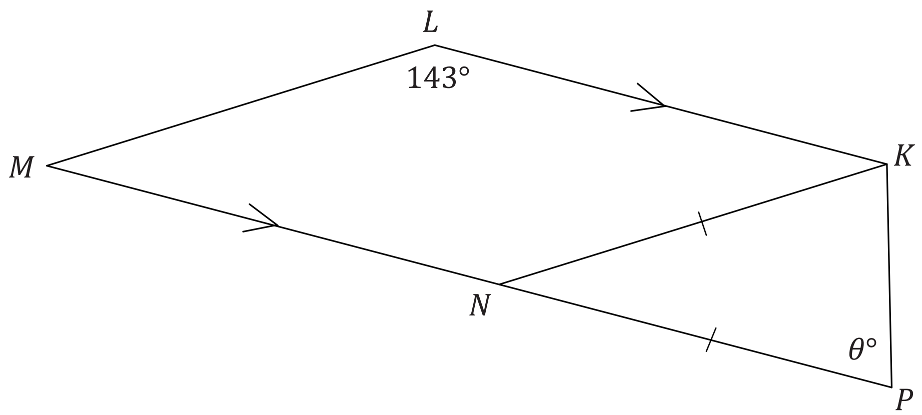
(c) For the function $f(x)$,

(i) Show that $g(x) = f(x) + f(-x)$ is even, and $h(x) = f(x) - f(-x)$ is odd. **2**

(ii) Deduce that any function can be written as the sum of even functions, odd functions, or even and odd functions. **2**

(d) Find the equation of the locus of all points which move so that its distance from the point $A(1, 1)$ is always twice their distance from the origin. **3**

(e) Given $KLMN$ is a parallelogram where MN is produced to P so that $NK = NP$, as shown in the diagram below. **3**



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If $\angle MLK = 143°$, copy the diagram and find the value of θ, giving all reasons.

END OF EXAM

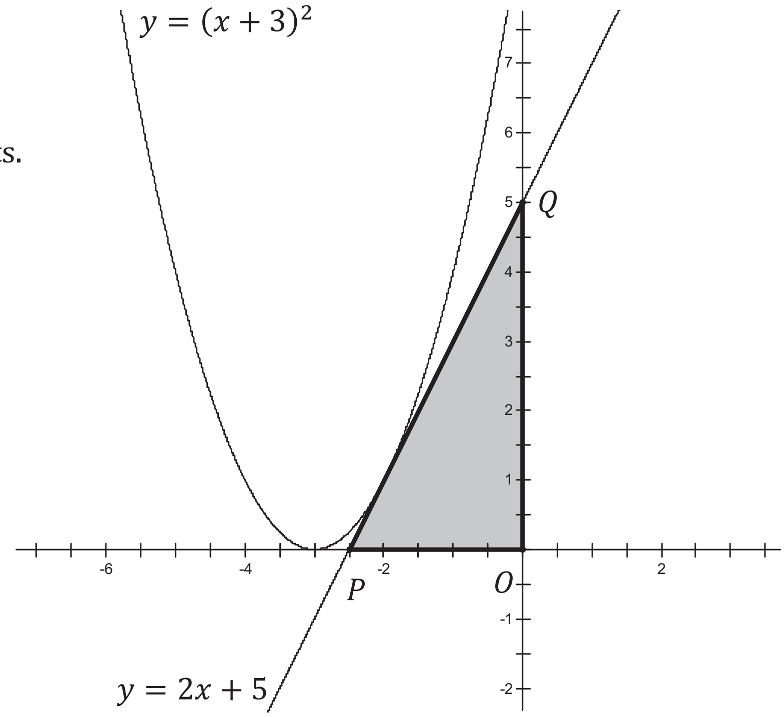
Question 1 (15 Marks)

(a) $x^2 + 2x + y^2 - 6y = 0$

$(x^2 + 2x + 1) + (y^2 - 6y + 9) = 10$

$(x + 1)^2 + (y - 3)^2 = 10$

$\therefore$ Centre is at $(-1, 3)$, radius is $\sqrt{10}$ units.



(b) $y = (x + 3)^2$

$\therefore\ y' = 2x + 6$

$y'(-2) = 2$

P is at $(-2\frac{1}{2}, 0)$, Q is at $(0, 5)$.

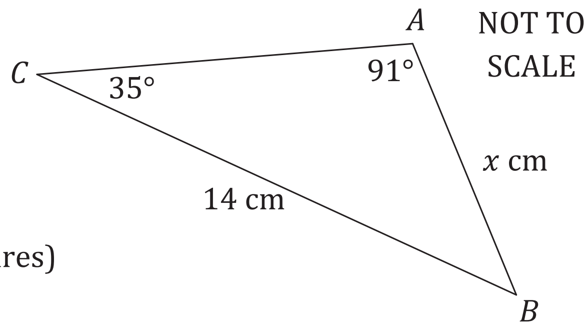
$\therefore\ A = \frac{1}{2} \times 2\frac{1}{2} \times 5$

$= 6\frac{1}{4}$ units^2

(c) (i) $\dfrac{x}{\sin 35°} = \dfrac{14}{\sin 91°}$

$x = \dfrac{14 \sin 35°}{\sin 91°}$

$x \approx 8.03$cm (2 decimal places)



(ii) $\angle ABC = 54°$

$\therefore \text{Area}_{\Delta ABC} = \frac{1}{2} \times 14 \times x \times \sin 54°$

$= \frac{1}{2} \times 14 \times x \times \sin 54°$

$\approx 45.5\text{cm}^2$ (3 significant figures)

(d) $\dfrac{5x+2}{x} + \dfrac{6}{x(x-3)} = \dfrac{(5x+2)(x-3)+6}{x(x-3)}$

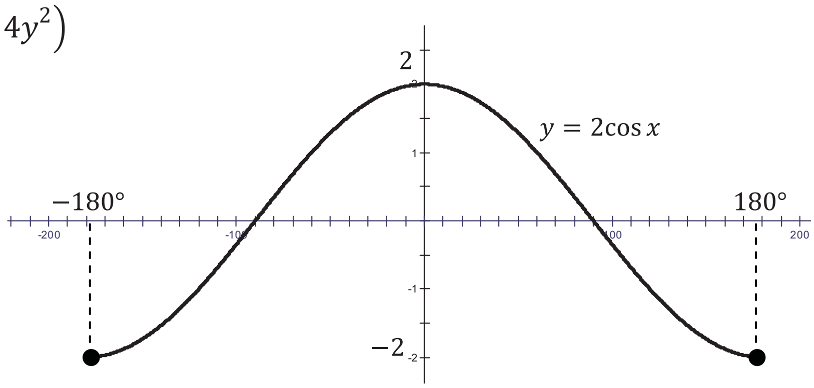
$= \dfrac{5x^2-15x+2x-6+6}{x(x-3)}$

$= \dfrac{5x^2-13x}{x(x-3)}$

$= \dfrac{5x-13}{(x-3)}$ $\quad (x \neq 0 \text{ or } 3)$

(e) $x^4 - 16y^4 = (x - 2y)(x + 2y)\left(x^2 + 4y^2\right)$

(f) $y = 2\cos x\ \{-180° \leq x \leq 180°\}$

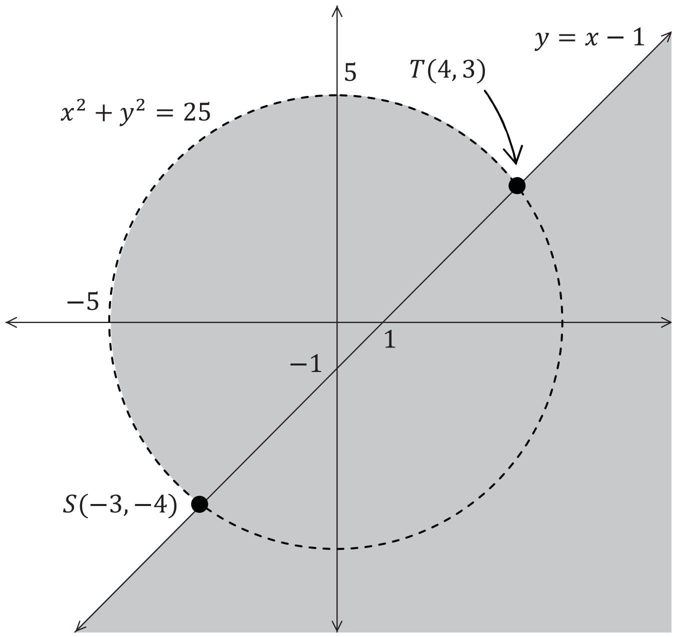


Question 2 (15 Marks)

(a) (i) $\frac{dy}{dx}=\frac{1}{7}$

(ii) $\frac{dy}{dx}=6-\frac{3}{2x^{\frac{5}{2}}}+\frac{2}{x^3}$

(b) (i) See diagram on right.



(ii) $y=x-1$

$x^2+y^2=25$

Solving simultaneously:

$x^2+(x-1)^2=25$

$x^2+x^2-2x+1=25$

$2x^2-2x-24=0$

$x^2-x-12=0$

$(x-4)(x+3)=0$

$\therefore x=-3$ or 4

$S(-3,-4)$

$T(4,3)$

(iii) See diagram above.

(c) $y=x-3x^2$

Let $y=f(x)$

$$f'(x)=\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

$$=\lim_{h\to 0}\frac{[(x+h)-3(x+h)^2]-[x-3x^2]}{h}$$

$$=\lim_{h\to 0}\frac{x+h-3(x^2+2xh+h^2)-x+3x^2}{h}$$

$$=\lim_{h\to 0}\frac{h-3x^2-6xh-3h^2+3x^2}{h}$$

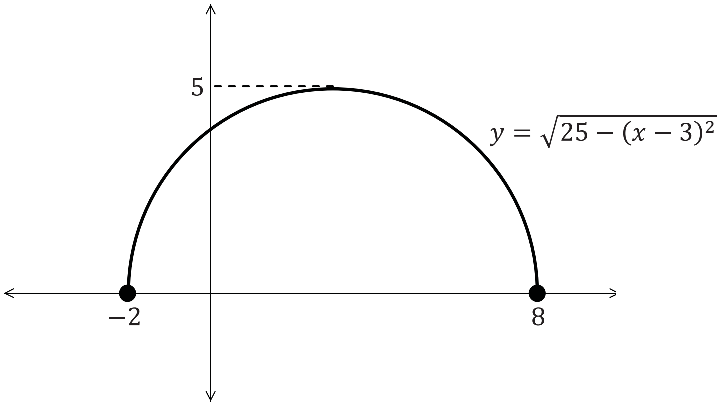
$$=\lim_{h\to 0}\frac{h-6xh-3h^2}{h}$$

$$=\lim_{h\to 0}(1-6x-3h)$$

$f'(x)=1-6x$

$\therefore y'=1-6x$

(d) According to diagram (see right), domain is $-2\le x\le 8$ and range is $0\le y\le 5$.



Question 3 (15 Marks)

(a) (i) $5x - 4y + 2 = 0$

$$-4y = -5x - 2$$

$$y = \frac{5}{4}x + \frac{1}{2}$$

$$\therefore k = \frac{5}{4}$$

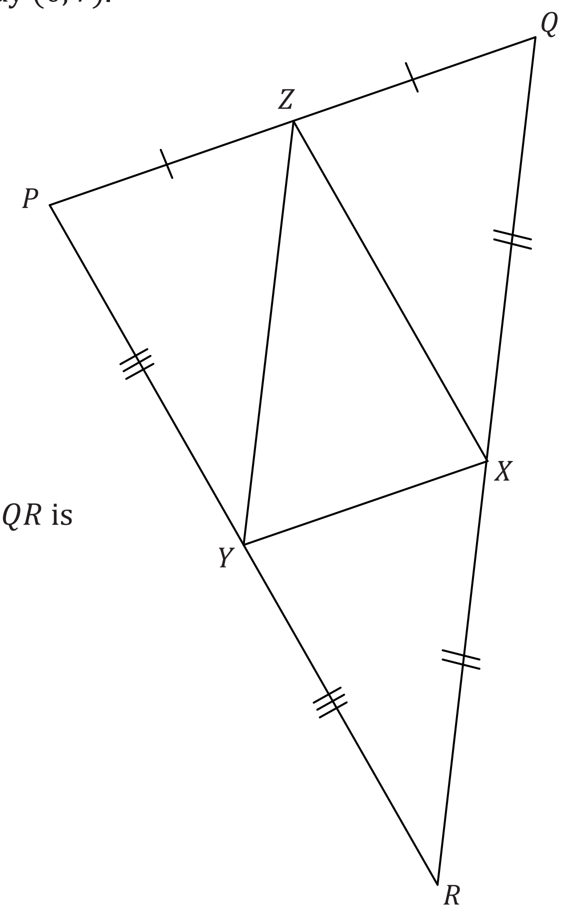
(ii) Consider a point on the line $y = kx + 7$, say $(0, 7)$.

$$d = \left|\frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}}\right|$$

$$= \left|\frac{5(0) - 4(7) + 2}{\sqrt{5^2 + 4^2}}\right|$$

$$= \left|\frac{-28 + 2}{\sqrt{41}}\right|$$

$$= \frac{26\sqrt{41}}{41} \text{ units}$$



(b) See diagram on right.

$XY = \frac{1}{2}PQ$ (interval joining midpoints of sides ΔPQR is half the length of the third side)

Similarly, $YZ = \frac{1}{2}QR$ and $XZ = \frac{1}{2}PR$.

In ΔPQR and ΔXYZ:

$$\frac{PQ}{XY} = 2$$

$$\frac{QR}{YZ} = 2$$

$$\frac{PR}{XZ} = 2$$

$\therefore \Delta PQR ||| \Delta XYZ$ (three pairs of sides in same ratio)

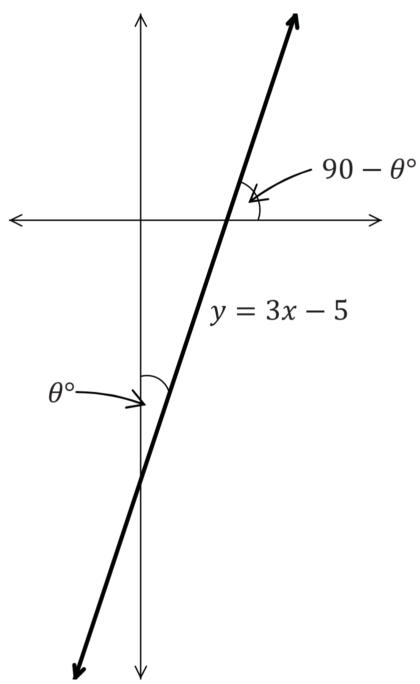
(c) $\tan(90 - \theta) = 3$

$$90 - \theta = \tan^{-1} 3$$

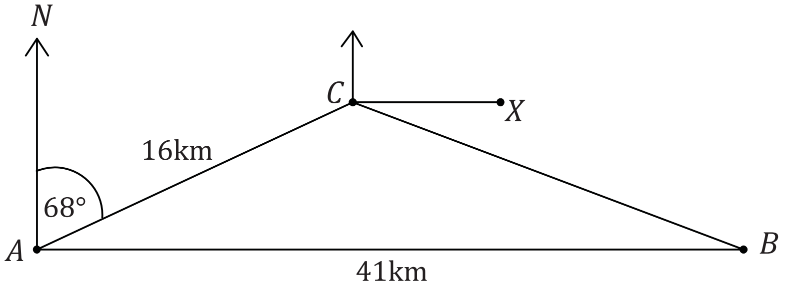
$$\theta = 90 - \tan^{-1} 3$$

$$\theta \approx 18°26' \text{ (nearest minute)}$$

or $18°$ (nearest degree)



(d)



(i) $BC^2 = 16^2 + 41^2 + 2(16 \times 41)\cos 22°$

$BC^2 \approx 720.5347828 \ldots$

$BC \approx 26.8427789 \ldots$

$\therefore$ The distance of B from C is 27km (nearest kilometre).

(ii) Construct X due east of C.

$\cos \angle ABC = \frac{AB^2 + BC^2 - AC^2}{2 \times AB \times BC}$

$\cos \angle ABC = \frac{41^2 + 27^2 - 16^2}{2 \times 27 \times 41}$

$\angle ABC \approx 12°54'$ (nearest minute)

$\therefore \angle BXC \approx 12°54'$ (alternate angles between parallel lines $AB || CX$ are equal).

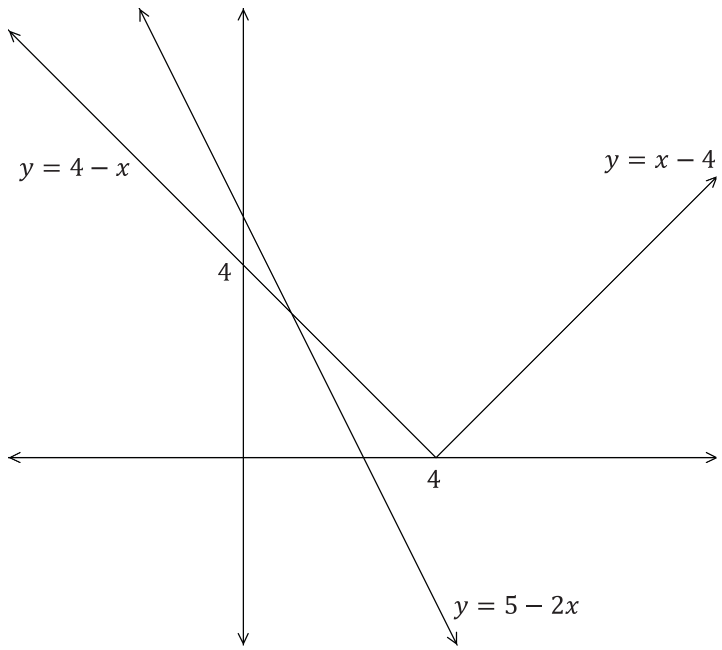
$\therefore$ The bearing of B from C is $102°54'T$ (nearest minute).

Question 4 (15 Marks)

(a) $\frac{6+\sqrt{3}}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{6+6\sqrt{3}+\sqrt{3}+3}{1-3}$

$= \frac{9+7\sqrt{3}}{-2}$

$= \frac{-9}{2} - \frac{7\sqrt{3}}{2}$



(b) (i) See graph on right.

(ii) Based on graph,

$4 - x = 5 - 2x$

$x = 1$

(c) (i) $g(x) = f(x) + f(-x)$

$\therefore g(-x) = f(-x) + f\big(-(-x)\big)$

$= f(-x) + f(x)$

$= g(x)$

$\therefore g(x)$ is even.

$h(x) = f(x) - f(-x)$

$\therefore h(-x) = f(-x) - f\big(-(-x)\big)$

$= f(-x) - f(x)$

$= -\big(f(x) - f(-x)\big)$

$= -h(x)$

$\therefore h(x)$ is odd.

(ii) $g(x) + h(x) = f(x) + f(-x) + f(x) - f(-x)$

$g(x) + h(x) = 2f(x)$

$f(x) = \frac{1}{2}\big(g(x) + h(x)\big)$

$\therefore$ Any function $f(x)$ can be expressed as the sum of an odd and even function.

(d) Consider the points $A(1,1), O(0,0)$ and $P(x,y)$.

The locus satisfies the equation $d_{AP} = 2d_{op}$.

$$\therefore \sqrt{(x-1)^2 + (y-1)^2} = 2\sqrt{x^2 + y^2}$$

$$(x-1)^2 + (y-1)^2 = 4(x^2 + y^2)$$

$$x^2 - 2x + 1 + y^2 - 2y + 1 = 4x^2 + 4y^2$$

$$3x^2 + 2x + 3y^2 + 2y = 2$$

$$x^2 + \tfrac{2}{3}x + y^2 + \tfrac{2}{3}y = \tfrac{2}{3}$$

$$x^2 + \tfrac{2}{3}x + \tfrac{1}{9} + y^2 + \tfrac{2}{3}y + \tfrac{1}{9} = \tfrac{6}{9} + \tfrac{2}{9}$$

$$\left(x + \tfrac{1}{3}\right)^2 + \left(y + \tfrac{1}{3}\right)^2 = \tfrac{8}{9}$$

(e) $\angle NKP = \angle KPN$ (equal angles opposite equal sides, $NK = NP$)

$= \theta°$

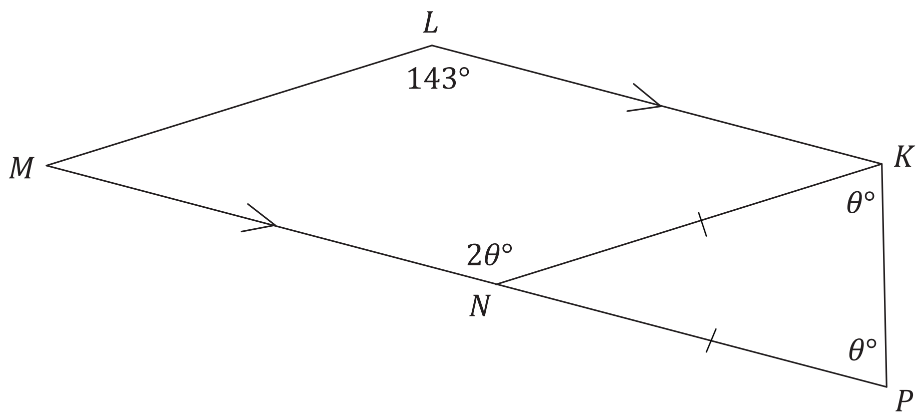
$\angle MNK = 2\theta°$ (external angle of ΔNKP is equal to the sum of the opposite two interior angles)

But $\angle MNK = \angle MLK$ (opposite angles in parallelogram $KLMN$ are equal)

$= 143°$

$\therefore 2\theta = 143$

$\theta = 71.5$



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