



THE KING'S SCHOOL

May 2011
Preliminary Certificate Course
Half Yearly Examination (PAT2)

Mathematics Year 11

General Instructions

- Time allowed – 50 minutes
- All questions are to be attempted.
- All working must be shown in every question.
- Marks may be deducted for careless or badly arranged work.
- Each Question is to be returned in a separate Writing Booklet, clearly labelled, indicating your Name and Class.

Examiners' Use Only			
Question	Functions	Trigonometry	Total
1	/13		/13
2		/13	/13
3		/12	/12
Total	/13	/25	/38

Question 1 (Functions) (13 marks)**Marks**

-
- (a) A function is defined as $y = x^2 - 6x + 5$
- (i) Sketch the function, clearly indicating x and y intercepts and the vertex. **3**
 - (ii) State the range of the function. **1**
 - (iii) For what values of x is the curve decreasing? **1**
- (b) Sketch $y = \frac{1}{x + 4}$ **2**
- (c) A function is defined as $f(x) = \sqrt{121 - x^2}$
- (i) Sketch the function $f(x)$. **2**
 - (ii) State the domain. **1**
- (d) Shade the region in the Cartesian plane for which the inequalities $y < x - 2$, $y \geq 0$ and $x > 3$ hold simultaneously. **3**

End of Question 1

Start a new Writing Booklet
Question 2 (Trigonometry) (13 marks)

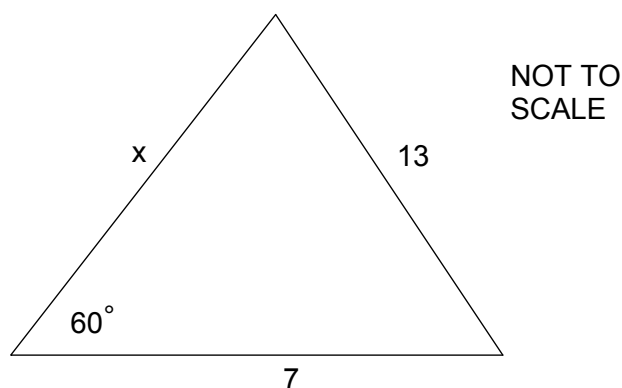
Marks

-
- (a) Find a value of x when $\sin x^\circ = \cos(x + 50^\circ)$ **1**
- (b) Given that A is an acute angle, find the size of A if $\tan A = 2.6314$. Give your answer to the nearest minute. **1**
- (c) A ship sails on a bearing of 106° from A to B . B is 76 nautical miles further east than A .
- (i) Draw a diagram, showing all the information. **1**
- (ii) Find, to the nearest nautical mile, how far the ship has sailed. **2**
- (d) If $\cos A = -\frac{3}{7}$ and $\tan A > 0$, find the exact value of $\sin A$ and $\cot A$. **3**
- (e) Find the value of $\sin 420^\circ$ in exact form. **2**
- (f) Find the value of $\frac{\cot 60^\circ + \sin 30^\circ}{\sin 60^\circ}$ in simplest exact form. **3**

End of Question 2

- (a) Find the area of triangle PQR correct to the nearest square cm if $\angle R = 50^{\circ}55'$, $QR = 12.1\text{cm}$ and $PR = 17.6$ 2
- (b) If $0^{\circ} \leq \theta \leq 360^{\circ}$, find all values of θ which satisfy the equation $2\cos^2\theta = 1$. 3

(c)



The diagram shows a triangle with sides 7cm, 13 cm and x cm, and an angle of 60° as marked.

- (i) Use the cosine rule to show that $x^2 - 7x - 120 = 0$. 2
- (ii) Hence find the exact value of x . 2
- (d) Prove $(2\cos\theta + 3\sin\theta)^2 + (3\cos\theta - 2\sin\theta)^2 = 13$ 3

End of Examination Paper

Question 1

a) i) $y = x^2 - 6x + 5$

X-axis $y = 0$

$0 = (x-5)(x-1)$

$x = 5$ or $x = 1$

Y-axis $x = 0$

$y = 5$

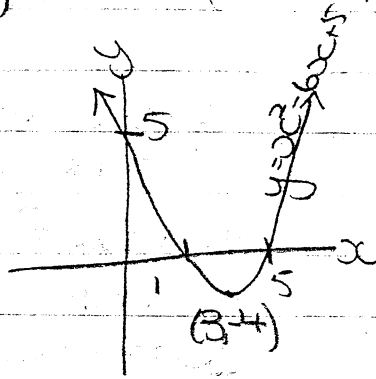
Vertex $x = \frac{-b}{2}$

$x = 3$

$y = 3^2 - 6(3) + 5$

$y = -4$

$(3, -4)$



Award 3 for correct answer showing x, y intercepts & vertex

Award 2 for one error or omission

Award 1 for two errors

NOTE CFPA on y co-ord of vertex from incorrect x value

ii) R: $y \geq -4$

OR CFPA

Award 1 for correct

answer OR from CFPA

iii) $x < 3$

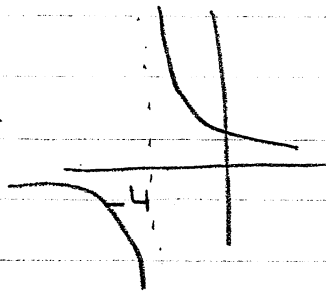
OR CFPA

Award 1 for correct

answer OR from CFPA

b) $x + 4 \neq 0$

$x \neq -4$

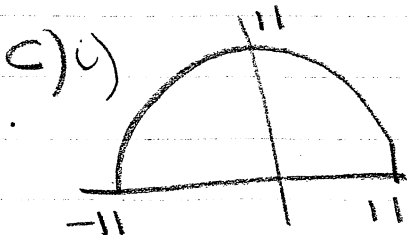


Award 2 for correct answer x asym must be labelled

Award 1 for correct sketch from incorrect domain

OR x asym correct

y asym incorrect



Award 2 for correct sketch, must include x & y intercepts

Award 1 for circle with correct

radius OR semicircle in with incorrect

radius OR semicircle with correct

radius in wrong quad

c) $D: -1 \leq x \leq 1$

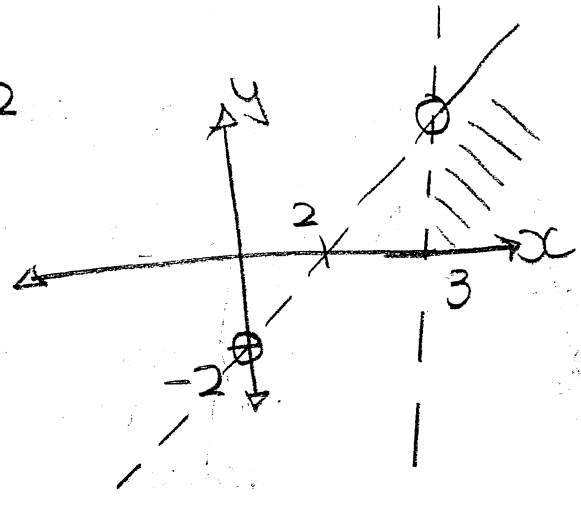
NOTE: CFPA

Award 1 for correct answer
OR CFPA

d) $y < x - 2$
x-axis $y = 0$

$0 = x - 2$
 $2 = x$
y-axis $x = 0$
 $y = -2$

Test $(0, 0)$
 $0 < 0 - 2$
 $0 < -2$ (F)



Award 3 for correct answer including broken lines & axes intercepts labelled. (no need for circles)

Award 2 for correct sketch incorrect shading from correct working

OR Correct graph full lines
OR Correct graph & lines no labels

OR Correct graph with one error

Award 1 two errors

Award 0 more than 2 errors

Correct graph full lines

Award 1

Correct graph & lines no labels

Correct graph with one error

Award 2

Award 3

Correct graph full lines

Correct graph & lines no labels

Correct graph with one error

Correct graph with two errors

Correct graph with more than 2 errors

Correct graph with more than 2 errors

Correct graph with more than 2 errors



Award 1

Award 2

Award 3

Award 0

Award 1

Award 2

Award 3

Award 0

Award 1

Award 2

Award 3

Award 0

Q2

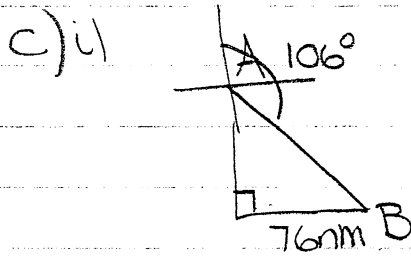
a) $x = 20^\circ$

Award 1 for correct answer

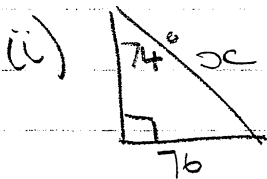
b) $A = 69^\circ 12'$

Award 1 must be correct to the nearest minute.

Award 0 for incorrect rounding.



Award 1 for correct diagram.



$$\sin 74 = \frac{76}{x}$$

$$x = \frac{76}{\sin 74}$$

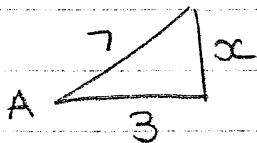
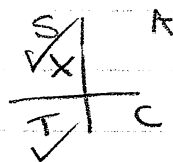
$x = 79$ nautical miles

Award 2 for correct answer on CFPA.

Award 1 for incorrect use of sin. ie wrong substitution

OR correct calculation from wrongly placed x .
OR incorrect angle ie 106° .

d) $\cos A = -\frac{3}{7}$



$$\sin A = \frac{-2\sqrt{10}}{7}$$

$$x^2 = 7^2 - 3^2$$

$$x = \sqrt{40}$$

$$x = 2\sqrt{10}$$

$$\tan A = \frac{2\sqrt{10}}{3}$$

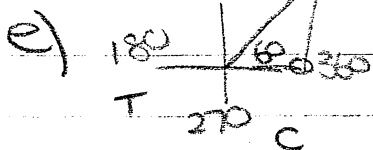
$$\cot A = \frac{3}{2\sqrt{10}}$$

Award 3 for correct calc of $\sin A$ & $\cot A$ (need not be simplified)

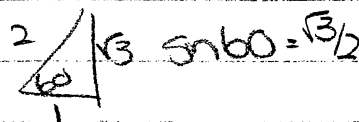
Award 2 for correct calc of $\cot A$ but wrong sign on $\sin A$.
OR correct calc of $\sin A$ but incorrect $\cot A$.

OR correct calc of $\sin A$ & $\cot A$ from wrong wrong quadrant.

Award 1 for correct calc of x only or correct identification of quad 3



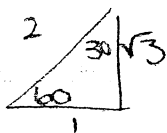
$$\theta = 60$$



Award 2 for correct answer.

Award 1 for $\theta = 60$

$$(f) \frac{\cot 60 + \sin 30}{\sin 60}$$



$$\frac{\cos 60 + \sin 30}{\sin 60}$$

sin 60.

$$\frac{\frac{1}{2} + \frac{\sqrt{3}}{2}}{\frac{\sqrt{3}}{2}}$$

$$\frac{\frac{1}{2} \times \frac{2}{\sqrt{3}} + \frac{1}{2}}{\sqrt{3}/2}$$

$$\frac{\frac{1}{\sqrt{3}} + \frac{1}{2}}{\sqrt{3}/2}$$

$$\left(\frac{2}{2\sqrt{3}} + \frac{\sqrt{3}}{2\sqrt{3}} \right) \times \frac{2}{\sqrt{3}}$$

$$\frac{(2 + \sqrt{3}) \times 2}{2\sqrt{3} \sqrt{3}}$$

$$\frac{4 + 2\sqrt{3}}{6}$$

$$= \frac{2(2 + \sqrt{3})}{6 \cdot 3}$$

$$= \frac{2 + \sqrt{3}}{3}$$

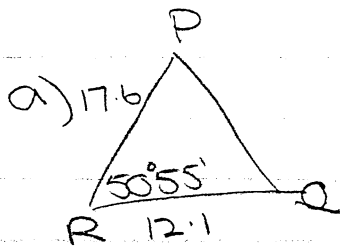
Award 3 for correct answer

Award 2 for correct substitution of exact values, with mistake in calculation.

Award 1 for 2 errors.

Award 0 for more than 2 errors.

Question 3.



$$A = \frac{1}{2} \times 12.1 \times 17.6 \sin 50^\circ 55'$$

$$A = 82.65$$

$$A = 83 \text{ cm}^2$$

Award 2 for correct answer
(ignore round)

Award 1 for correct answer for
incorrect diagram.

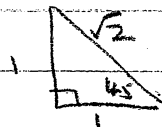
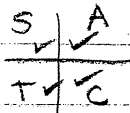
OR one error in working from
incorrect diagram

b) $2 \cos^2 \theta = 1$

$$\cos^2 \theta = \frac{1}{2}$$

$$\cos \theta = \pm \sqrt{\frac{1}{2}}$$

$$\cos \theta = \pm \frac{1}{\sqrt{2}}$$



$$\theta = 45^\circ, 180 - 45, 180 + 45, 360 - 45$$

$$\theta = 45, 135, 225, 315$$

Award 3 for correct answers.

Award 2 for correct answer without
 \pm ie only two values 45° & 135° .

Award 1 for one error in calc

or not $\sqrt{\quad}$ ie $\cos \theta = 1/2$

Award 0 for more than 2 errors
in working

c) i) $13^2 = 7^2 + x^2 - 2 \times 7 \times x \cos 60$

$$169 = 49 + x^2 - 14x \cos 60$$

$$169 = 49 + x^2 - 14x \times \frac{1}{2}$$

$$120 = x^2 - 7x$$

$$0 = x^2 - 7x - 120$$

Award 2 for correct answer.

Award 1 first line of substit
correct with one subsequent error

Award 0 for two or more error

ii) $(x-15)(x+8) = 0$

$$x = 15 \text{ or } x = -8$$

$x = -8$ can not be a

solⁿ, no negative length

$$\therefore x = 15$$

Award 2 for correct answer.

Award 1 for correctly solved
equation, but no or inappropri
conclusion. (including $x = -8$)
ie

Award 0 for 2 or more errors

$$d) (2\cos\theta + 3\sin\theta)^2 + (3\cos\theta - 2\sin\theta)^2 = 13.$$

$$(\cancel{4\cos^2\theta} + \cancel{12\sin\theta\cos\theta} + 9\sin^2\theta) + (\cancel{9\cos^2\theta} - \cancel{12\sin\theta\cos\theta} + 4\sin^2\theta)$$

$$4\cos^2\theta + 4\sin^2\theta + 9\sin^2\theta + 9\cos^2\theta$$

$$4(\cos^2\theta + \sin^2\theta) + 9(\sin^2\theta + \cos^2\theta)$$

$$4(1) + 9(1)$$

$$= 13$$

$$\text{LHS} = \text{RHS}$$

$$\therefore (2\cos\theta + 3\sin\theta)^2 + (3\cos\theta - 2\sin\theta)^2 = 13$$

must include full expansion. ← Award 3 for correct answer

Award 2 for ^{correct} expansion but 1 error in working.

Award 1 for incorrect expansion
ie no middle term

ie correct answer from incorrect working.