

2013

## YEAR 11 MATHEMATICS

## TERM 2 ASSESSMENT TASK

| Date: | $7^{\text {th }}$ June, Period 4 |
| :--- | :--- |
| Time allowed: | 45 minutes (plus 2 minutes reading time) |
| Total marks: | 35 marks |

## Directions to Candidates

- Attempt all questions.
- Marks are indicated next to each question.
- All necessary working should be shown.
- Board-approved calculators may be used.
- Begin each question on a new page with your student number clearly written at the top.
- Write in black pen and use black pen for all diagrams


## Outcomes

## A student:

P2 provides reasoning to support conclusions which are appropriate to the context.
P3 performs routine arithmetic and algebraic manipulation involving surds, simple rational expressions and identities.
P4 chooses and applies appropriate arithmetic, algebraic, graphical and geometrical techniques.
P5 understands the concept of a function and the relationship between a function and its graph.

Student Name: $\qquad$
Student Number: $\qquad$

## Section I

Total Marks (5)
Attempt Questions 1 - 5
Answer Section I on the Multiple Choice Answer Sheet provided.

1. What is the value of $x$ ?

(A) 18
(B) 27
(C) 36
(D) 45
2. What is the domain and range of the function $f(x)=\sqrt{1-x^{2}}$ ?
(A) Domain: $0 \leq x \leq 1$, Range: $-1 \leq y \leq 1$
(B) Domain: $-1 \leq x \leq 1$, Range: $-1 \leq y \leq 1$
(C) Domain: $-1 \leq x \leq 1$, Range: $0 \leq y \leq 1$
(D) Domain: $0 \leq x \leq 1$, Range: $0 \leq y \leq 1$
3. The diagram shows the graph of the function $y=5 x-x^{2}$.

Which pair of inequalities specifies the shaded region?

(A) $y \leq 5 x-x^{2}$ and $y \leq 0$.
(B) $y \leq 5 x-x^{2}$ and $y \geq 0$.
(C) $y \geq 5 x-x^{2}$ and $y \leq 0$.
(D) $y \geq 5 x-x^{2}$ and $y \geq 0$.
4. Consider $f(x)=\frac{6}{x}$ and $g(x)=2 x+4$.

What are the values for $x$ for which $f(x)=g(x)$ ?
(A) $x=-1$ or $x=3$
(B) $x=-3$ or $x=-1$
(C) $x=1$ or $x=3$
(D) $x=-3$ or $x=1$
5. In the diagram below, $A B C$ is a triangle and $D E$ is parallel to $B C$.


Given that $A D=2, B D=5$ and $D E=1.5$, what is the value of $B C$ ?
(A) 4.00
(B) 5.25
(C) 7.50
(D) 9.33

## Section II

Total marks (30)
Attempt questions 6-7
All questions are of equal value
a) For the function with equation $y=\frac{3}{x+2}$
i) Write down the equations of the asymptotes.
ii) Sketch the graph of this function showing all necessary features.
b) i) Find the centre and radius of the circle with the equation

$$
x^{2}+14 x+(y-1)^{2}-15=0
$$

ii) Does this circle cross the $y$-axis? Give reasons for your answer.
c) Sketch the graph of $y=|x-4|+3$
d) The function $f(x)$ is defined as $f(x)=\left\{\begin{array}{cc}x^{2}+5, & \text { if } \\ 5 \geq 0 \\ 5+x, & \text { if } \\ x<0\end{array}\right.$

Find:
i) $\quad f(-3)+f(2)$
ii) $\quad f\left(m^{2}\right)$
e) i) Sketch the graphs of $y=3^{x}-1$ and $y=3$ on one number plane.
ii) Explain why the equation $3^{x}-1=3$ has only one solution.
a) Find the number of sides of a regular polygon with each interior angle equal to $160^{\circ}$ ?
b) The parabola $y=x^{2}$ is shifted 2 units to the right and 5 units up.

For the shifted parabola:
i) state the coordinates of the vertex,
ii) write down the equation of this parabola,
iii) sketch the parabola showing all essential features.
c) For which values of $x$ is $x^{2}+4 x-21<0$ ?
d) Show that $f(x)=\frac{x^{3}}{x^{2}-4}$ is an odd function.
e) Explain how a function is different from a relation.

Give an example of a function and a relation, where the relation is not a function.
f) In the diagram below $P Q R S$ is parallelogram and $P A=B R$.

i) Show that $\triangle P A S \equiv \triangle R B Q \quad 2$
ii) Hence or otherwise show that $A Q B S$ is also a parallelogram.

## Multiple Choice Answer Sheet

Student Number:
1
A
$B \bigcirc$
$\mathrm{C} \bigcirc$
D
2
A
$B \bigcirc$
$\mathrm{C} \bigcirc$
D $\bigcirc$
3 A $\bigcirc$
$B \bigcirc$
$\mathrm{C} \bigcirc$
D
$A \bigcirc$
$B \bigcirc$
$\mathrm{C} \bigcirc$
D
A $\bigcirc$
$B \bigcirc$
$\mathrm{C} \bigcirc$
D

Year 11 Mathematics Assessment T2 2013 Solutions and Marking Criteria

| M/C | Solution | Marking Criteria |
| :---: | :---: | :---: |
| 1. | $\begin{aligned} & 5 y=90\left(\text { angle sum of triangle is } 180^{\circ}\right) \\ & y=18 \\ & 2 x=3 \times 18 \text { (exterior angle of triangle equals } \\ & \quad \quad \text { sum of } 2 \text { opposite interior angles) } \\ & 2 x=54 \\ & x=27 \end{aligned}$ | 1 - correct answer B |
| 2. | Upper half of semicircle, with radius one <br> Domain $-1 \leq x \leq 1$ <br> Range $0 \leq y \leq 1$ | 1 - correct answer C |
| 3. | $y \leq 5 x-x^{2}$ and $y \geq 0$ | 1 - correct answer B |
| 4. | $\begin{aligned} & \frac{6}{x}=2 x+4, x \neq 0 \\ & 2 x^{2}+4 x-6=0 \\ & 2\left(x^{2}+2 x-3\right)=0 \\ & 2(x+3)(x-1)=0 \\ & x=-3 \text { or } x=1 \end{aligned}$ | 1 - correct answer D |
| 5. | $\begin{aligned} & \triangle A E D\\|\\| \triangle A C B \text { (equiangular) } \\ & \frac{B C}{1.5}=\frac{7}{2} \text { (corresponding sides in } \\ & \quad \quad \quad \text { congruent triangles are in same ratio) } \\ & B C=\frac{7}{2} \times 1.5 \\ & B C=5.25 \end{aligned}$ | 1 - correct answer B |


| 6. <br> a) i) | $x=-2, y=0$ | 2 - correct answer <br> 1 - one asymptote correct |
| :---: | :---: | :---: |
| a) <br> ii) |  | 2 - correct graph, showing asymptotes and $y$-intercept 1 - correct graph with asymptotes |
| b) i) | $\begin{aligned} & x^{2}+14 x+(y-1)^{2}-15=0 \\ & x^{2}+14 x+49+(y-1)^{2}=15+49 \\ & (x+7)^{2}+(y-1)^{2}=64 \end{aligned}$ <br> Centre ( $-7,1$ ), Radius $r=8$ | 3 - correct solution <br> 2 - substantially correct solution <br> 1 - correctly completing square or correctly concluding centre or radius from incorrect calculations |
| $\begin{aligned} & \text { b) } \\ & \text { ii) } \end{aligned}$ | Yes, since the centre is 7 units from the $y$-axis and the radius of the circle is 8 units. <br> Can be shown algebraically by substituting $x=0$ and showing that $(y-1)^{2}-15=0$ has two solutions, ie $y$-intercepts. | 1- Correct explanation |
| c) |  | 2 - correct graph 1 - correct shape, not clearly showing intercept or vertex. |
| $\begin{aligned} & \text { d) } \\ & \text { i) } \end{aligned}$ | $\begin{aligned} f(-3)+f(2) & =5-3+2^{2}+5 \\ & =11 \end{aligned}$ | 1 - correct solution |
| d) ii) | $f\left(m^{2}\right)=m^{4}+5$, since $m^{2} \geq 0$ | 1 - correct answer |


| e) i) |  | 2 - both graphs correct <br> 1 - one correct graph |
| :---: | :---: | :---: |
| e) <br> ii) | Because the two graphs only intersect in one point. | 1- correct answer |
| 7. <br> a) | $\begin{aligned} & \text { Exterior angle }=180^{\circ}-160^{\circ} \text { (supplement of interior } \\ & \quad=20^{\circ} \quad \text { angle) } \\ & 360^{\circ} \div 20^{\circ}=18 \text { (exterior angle sum of polygon is } 360^{\circ} \text { ) } \\ & \therefore \text { polygon has } 18 \text { sides. } \\ & \text { OR } \\ & \text { Angle sum of interior angles in polygon }=(2 n-4) \\ & \text { right angles } \\ & \frac{(2 n-4) \times 90^{\circ}}{n}=160 \\ & 180 n-360^{\circ}=160 n \\ & 20 n=360 \\ & n=18 \end{aligned}$ | 2 - correct solution <br> 1 - correctly finding exterior angle <br> OR <br> correctly setting up equation $\frac{(2 n-4) \times 90^{\circ}}{n}=160$ |
| b) <br> i) | Vertex $=(2,5)$ | 1 - correct answer |
| b) <br> ii) | $y=(x-2)^{2}+5$ | 1 - correct equation |
| b) <br> iii) |  | 1 - correct graph, showing $y$-intercept |


| c) | $\begin{aligned} & x^{2}+4 x-21<0 \\ & (x-3)(x+7)<0 \end{aligned}$  $\therefore-7<x<3$ | 2 - correct solution 1 - correct factorisation, OR correct solution from their factors |
| :---: | :---: | :---: |
| d) | $\begin{aligned} f(-x) & =\frac{(-x)^{3}}{(-x)^{2}-4} \\ & =\frac{-x^{3}}{x^{2}-4} \\ & =-\left(\frac{x^{3}}{x^{2}-4}\right) \\ & =-f(x) \end{aligned}$ <br> $\therefore f(x)$ is an odd function | 2 - correct solution <br> 1 - correct substitution of ( $-x$ ) and attempt at simplification |
| e) | In a function every $x$-value has only one matching $y$-value, whereas in a relation an $x$ value can have more than one matching $y$-value. <br> Possible examples: <br> Function: $y=x$ <br> Relation: $x^{2}+y^{2}=4$ | 2 - correct explanation and examples 1 - correct explanation or correct examples for function and relation |
| f) i) | $\begin{aligned} & \text { In } \triangle P A S \text { and } \triangle R B Q \\ & P A=R B \text { (given) } \\ & P S=R Q \text { (opposite sides in parallelogram are equal) } \\ & \angle P=\angle R \text { (opposite angles in parallelogram are equal) } \\ & \therefore \triangle P A S \equiv \triangle R B Q \text { (SAS) } \end{aligned}$ | 2 - correct proof fully justified 1 - correct proof but not all reasons given or significant attempt at proof |
| f) ii) | Let $P A=R B=a$ (given) <br> $P Q=R S$ (opposite sides in parallelogram are equal) <br> hence $A Q=P Q-a$ $\begin{aligned} & =R S-a \\ & =B S \end{aligned}$ <br> $A S=B Q$ (corresponding sides in congruent triangles) <br> $\therefore A Q B S$ is a parallelogram (opposite sides are equal) | 2 - correct proof fully justified <br> 1 - correct attempt at proof indicated by a correct test for parallelogram OR correct proof not fully justified |

## Communication

6b)i) Clear reasoning based on distance/ graphical or AW 1 algebraic calculation of $y$-intercepts
6e)ii) Clear explanation with correct vocabulary. AW1
7b)iii) Axes labelled and scale accurate AW 1
7c) Shows graph of parabola AW 1
7d) Shows substitution of ( $-x$ ) AW 1
7e) Clear explanation of function AW 1

