Question 1 *(10 marks)* **Marks**

a) For the following functions, **8**

i) Sketch the following showing all the important features.

ii) Find the natural domain and range.

A) $y = |x+1| - 1$ B) $y = \dfrac{2}{x-3}$

b) A function is defined by *f(x)* = $\begin{cases} x^2 \; for, x \leq -1 \\ x+1 \; for, x > -1 \end{cases}$

Evaluate *f*(–2) +2*f*(0) – *f*(3). **2**

Question 2 *(10 marks)* **<u>Start this question in a new booklet.</u>**

a) Show by 'completing the square' that **4**

$x^2 + 4x + y^2 - 2y - 4 = 0$ is a circle, and state its centre and radius.

b) A function is defined as $f(x) = x^3 - 5x$ **2**

Show this function is either even, odd or neither.

c) Find the exact value of (i) $\tan 240^0$ **1**

(ii) $\sin(-45)^0$ **1**

d) Given that $\tan\theta = \dfrac{-5}{12}$ and $\sin\theta > 0$, find **2**

(i) $\sin\theta$

(ii) $\sec\theta$

continued on the next page ....

Question 3 *(10 marks)* **<u>Start this question in a new booklet.</u>** **Marks**

a) If $\sin\theta = \cos 35°$ find θ, if θ is acute. **1**

b) Simplify the following $\cot\theta - \cot\theta\cos^2\theta$ **2**

c) Prove $\left(1+\tan^2 A\right)\cos^2 A = 1$ **2**

d) Solve the following, for $0^\circ \le \theta \le 360^\circ$

(i) $\tan\theta = \dfrac{1}{\sqrt{3}}$ **2**

(iii) $2\cos 2\theta - 1 = 0$ **3**

Question 4 *(9 marks)* **<u>Start this question in a new booklet.</u>**

a) Prove $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = 2\sec^2 x$ **3**

b) From a harbour H, a boat B travelled 20km on a bearing of 125 degrees .
At the same time a yacht Y sailed 25km, on a bearing of 245 degrees.

(i) Draw a diagram, marking on it all the information supplied. **1**

(ii) Find the distance of the boat B, from the yacht Y.
Give your answer as an exact value. **2**

(ii) Find the bearing of the yacht Y from the boat B.
Give your answer to the nearest degree. **3**

continued on the next page ....

Question 5 *(10marks)* **<u>Start this question on a new page.</u>** **Marks**

The points A, B and C have the co-ordinates (–2,2) , (–1, -5) and (6,–6) respectively.

a) Sketch the triangle ABC, on a number plane using a ruler. Make the diagram about $\frac{1}{2}$ page in size. **1**

b) Show the midpoint P of AC has the co-ordinates (2,-2). **1**

c) Calculate the gradient of AC. **1**

d) Find the exact value of the length of side AC. **1**

e) Show AC and BP are perpendicular. **2**

f) Find the area of triangle ABC. **2**

g) Find the equation of the line AC. **2**

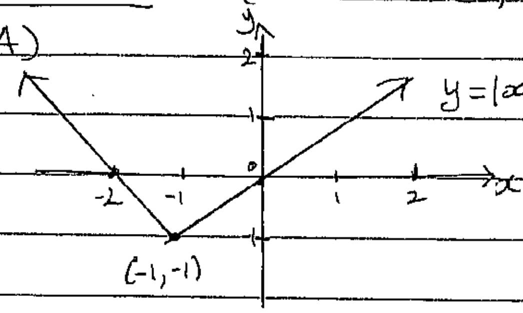
End of the Paper

Year 11 2013 Mathematics Assessment 2

Question 1 (10 marks)

a) (A)

(i)



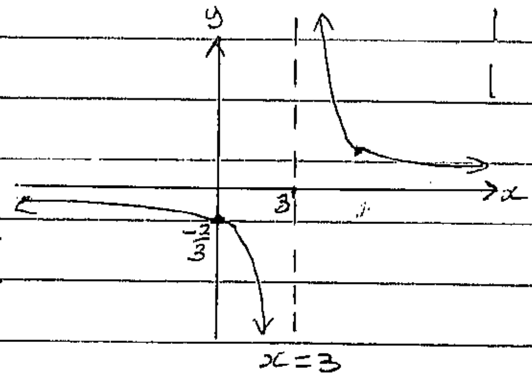
1 correct shape

1 all points correct.

(ii) Domain: all real x ✓ 1 mark each

Range: $y \geq -1$ ✓

(B) $y = \dfrac{2}{x-3}$



1 correct shape

1 correct y int and both asymptotes

(ii) Domain: all real x, $x \neq 3$ ✓

Range: all real y, $y \neq 0$ ✓

1 mark each.

b) $f(x) = \begin{cases} x^2, & \text{for } x \leq -1 \\ x+1, & \text{for } x > -1 \end{cases}$

$f(-2) = -2^2$
$= 4$

$f(0) = 0+1$
$= 1$

$f(3) = 3+1$
$= 4$

$\therefore f(-2) + 2f(0) - f(3) = 4 + 2 \times 1 - 4$ ✓ 2 marks.
$= 2$ ✓

2013 Year 11 Mathematics Assessment 2

<u>Question 2</u> (10 marks)

a) $x^2+4x+y^2-2y-4=0$ (4 marks)

$x^2+4x+\left(\frac{4}{2}\right)^2+y^2-2y+\left(\frac{-2}{2}\right)^2=4+5$ ✓

$(x+2)^2+(y-1)^2 \quad =9$ ✓

$\therefore$ its a circle centre $(-2,1)$ ✓

and radius 3 ✓

b) $f(x)=x^3-5x$

$f(-x)=(-x)^3-5(-x)$ ✓

$\quad =-x^3+5x$ 1 mark

$-f(x)=-(x^3-5x)$ = 2 marks.

$\quad =-x^2+5x$

$\therefore f(-x)=-f(x)$ 1 mark

$\therefore f(x)$ is an odd function ✓

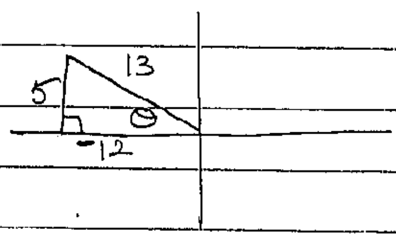
c) (i) $\tan 240^\circ = +\tan 60^\circ$

$\quad =+\sqrt{3}$ ✓ 1 mark.

(ii) $\sin(-45^\circ) = -\sin 45^\circ$

$\quad = -\frac{1}{\sqrt{2}}$ ✓ 1 mark

d) $\tan\theta = \frac{-5}{12}$, $\sin\theta > 0$



(i) $\sin\theta = \frac{5}{13}$ ✓ 1 mark

(ii) $\cos\theta = \frac{-12}{13}$

$\sec\theta = \frac{-13}{12}$ ✓ 1 mark.

2013 Year11 Mathematics Ass 2

Question 3 (10 marks)

a) $\sin\theta = \cos 35^\circ$

$\therefore \theta + 35 = 90^\circ$

$\theta = 90 - 35^\circ$

$\theta = 55^\circ$ ✓ 1mark

b) $\cot\theta - \cot\theta\cos^2\theta$

$= \cot\theta\,(1-\cos^2\theta)$

$= \cot\theta\,(\sin^2\theta)$ ✓

$= \dfrac{\cos\theta}{\sin\theta} \times \sin^2\theta$

$= \cos\theta\sin\theta$ ✓ 2 marks

c) Prove $(1+\tan^2 A)\cos^2 A = 1$

$L.H.S. = (1+\tan^2 A)\cos^2 A$

$= \sec^2 A \times \cos^2 A$ ✓ 2marks.

$= \dfrac{1}{\cos^2 A} \times \cos^2 A$ ✓

$= 1$

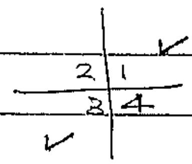
$= RHS.$

$\therefore LHS = RHS$

$\therefore (1+\tan^2 A)\cos^2 A = 1$

d) (1) $\tan\theta = \dfrac{1}{\sqrt{3}}$ $0^\circ \leq \theta \leq 360^\circ$

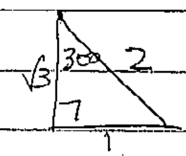
tan is positive in Q1 + Q3



related angle

$\tan\theta = \dfrac{1}{\sqrt{3}}$

$\theta = 30^\circ$ ✓



$\therefore$ Q1 $\theta = 30^\circ$

Q3 $\theta = 180 + 30^\circ = 210^\circ$

$\therefore \theta = 30^\circ, 210^\circ$ ✓ 2 marks.

(d) (ii) $2\cos 2\theta - 1 = 0 \qquad 0 \le \theta \le 360^\circ$

$\qquad\qquad\qquad\qquad\qquad\qquad \times 2 \quad \times 2 \quad \times 2$

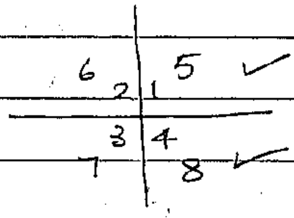
$\cos 2\theta = \frac{1}{2}$

Let $x = 2\theta \qquad 0 \le 2\theta \le 720^\circ$

$\cos x = \frac{1}{2} \qquad \therefore 0 \le x \le 720^\circ$

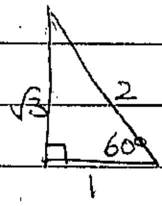
cos is pos in Q1 + Q4, Q5 + Q8

related angle



$\cos x = \frac{1}{2}$

$x = 60^\circ$ ✓



$\therefore$ Q1 $\quad x = 60^\circ$

Q4 $\quad x = 360 - 60$

$= 300^\circ$

Q5 $\quad x = Q1 + 360^\circ$

$= 360^\circ + 60^\circ$

$= 420^\circ$

Q8 $\quad x = Q4 + 360^\circ$

$= 300^\circ + 360^\circ$

$= 660^\circ$

$\therefore x = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$2\theta = 60^\circ, 300^\circ, 420^\circ, 660^\circ$

$\theta = \underbrace{30^\circ, 150^\circ}_{✓}, \underbrace{210^\circ, 330^\circ}_{✓}$ $\qquad$ 3 marks

2013 Year11 Mathematics Ass 2

Question 4 (9 marks

a) Prove $\dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = 2\sec^2 x$

$$LHS = \frac{1}{1+\sin x} + \frac{1}{1-\sin x}$$

$$= \frac{1(1-\sin x)}{(1+\sin x)(1-\sin x)} + \frac{1(1+\sin x)}{(1-\sin x)(1+\sin x)} \checkmark$$

$$= \frac{(1-\sin x)+(1+\sin x)}{(1+\sin x)(1-\sin x)}$$

$$= \frac{2}{1-\sin^2 x} \checkmark$$

$$= \frac{2}{\cos^2 x} \checkmark$$

3 marks.

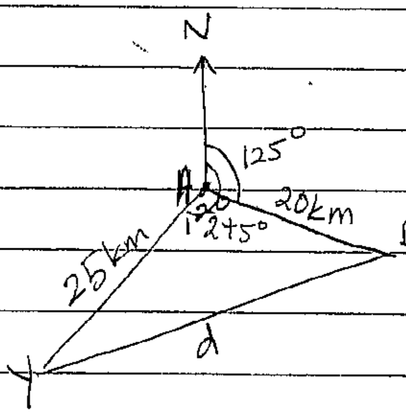
$$= 2\sec^2 x$$

$$= RHS.$$

$\therefore LHS = RHS.$

$\therefore \dfrac{1}{1+\sin x} + \dfrac{1}{1-\sin x} = 2\sec^2 x$

b)(i)



✓ 1mark all info

(ii) $d^2 = a^2 + b^2 - 2ab\cos D$

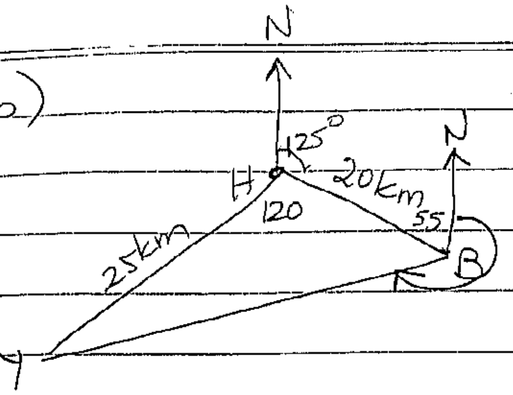
$= 20^2 + 25^2 - 2 \times 20 \times 25 \times \cos 120°$ ✓

$d^2 = 1525$

$d = \sqrt{1525}$ ✓ 2 marks

$d = 5\sqrt{61}$

Q4 (b)



(iii) $\dfrac{\sin A}{a} = \dfrac{\sin B}{b} = \dfrac{\sin C}{c}$

$\dfrac{\sin 120}{\sqrt{1525}} = \dfrac{\sin B}{25}$

$\sin B = \dfrac{25 \times \sin 120}{\sqrt{1525}}$

$= 0.55441 59532$

$B = \sin^{-1}(0.5544159532)$

$B = 33.67049651$

$B = 34^\circ$ (n. degree) ✓

$\angle NBH = 180^\circ - 125^\circ = 55^\circ$ ✓

∴ the bearing of Y from B is

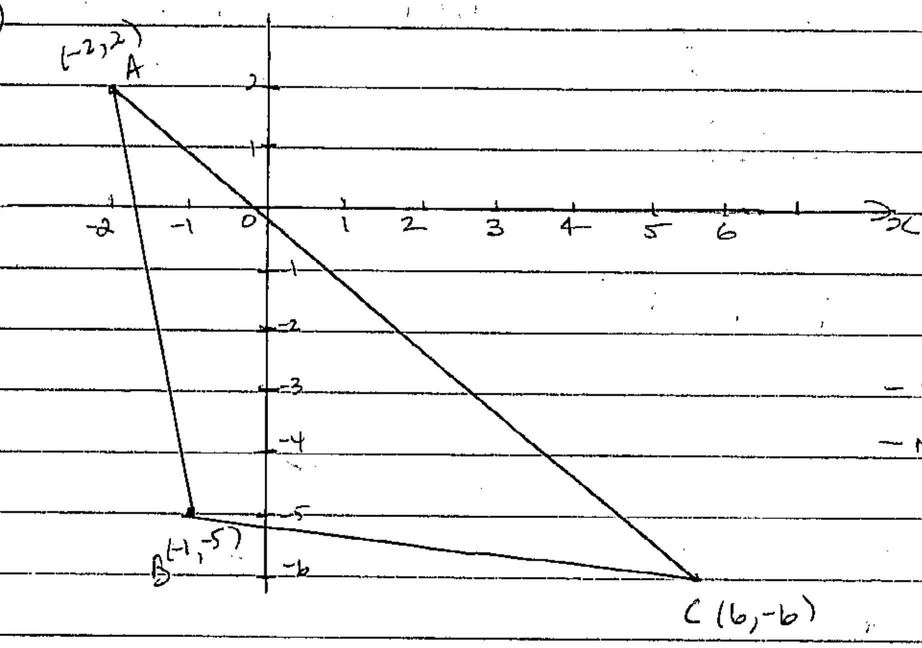
$= 360^\circ - 55^\circ - 34^\circ$ 3 marks.

$= 271^\circ$ ✓

2013 Year 11 Mathematics Ass 2

Question 5 (10 marks)

(a)



1mark.
- must use a ruler
- must be large.

(b) Midpoint of AC

$P = \left(\frac{x_2 + x_1}{2}, \frac{y_2 + y_1}{2}\right)$

$= \left(\frac{-2+6}{2}, \frac{2+-6}{2}\right)$ ✓ 1mark

$= \left(\frac{4}{2}, \frac{-4}{2}\right)$

$= (2, -2)$

(c) Gradient AC

$m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{-6-2}{6+2}$

$= \frac{-8}{8}$

$\therefore m = -1$ ✓ 1mark

(d) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(+6+2)^2 + (-6-2)^2}$

$= \sqrt{64+64}$

$\left.\begin{array}{l} = \sqrt{128} \\ = 8\sqrt{2} \end{array}\right\}$ ✓ 1mark

(e) $m_{AC} = -1$ (from c)

$m_{BP} = \frac{y_2 - y_1}{x_2 - x_1}$ $B(-1,-5)$ $P(2,-2)$

$= \frac{-2+5}{2+1}$

$= \frac{3}{3}$

$= 1$ ✓

Now $m_{AC} \times m_{BB} = -1 \times 1$ ✓

$= -1$

$\therefore AC \perp BP$ 2marks.

(f) $A = \frac{1}{2}bh$

$= \frac{1}{2} \times AC \times BP$ $BP = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \frac{1}{2} \times 8\sqrt{2} \times \sqrt{18}$ $= \sqrt{(2+1)^2 + (-2+5)^2}$

$= \sqrt{9+9}$

$= 4\sqrt{36}$ $= \sqrt{18}$ ✓

$= 24 \text{ units}^2$ ✓ 2marks

(g) line AC $A(-2,2)$ $m = -1$

$y - y_1 = m(x - x_1)$

$y - 2 = -1(x+2)$ ✓

$y - 2 = -x - 2$

$\left.\begin{array}{l} y = -x \\ x + y = 0 \end{array}\right\}$ ✓ 2marks.