NORTH SYDNEY BOYS' HIGH SCHOOL
2008 Preliminary Course Assessment Task 2

MATHEMATICS

General instructions

- Working time – 60 minutes.
- Write in the booklet provided.
- Each new question is to be started on a new page.
- Write using blue or black pen.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.
- Question are **not** of equal value.
- At the conclusion of the examination, bundle the booklet within this paper and hand to examination supervisors.

Total Marks: 64

NAME:

Marker's use only.

QUESTION	MARKS
1	/14
2	/28
3	/11
4	/11
Total	/64
Total (%)	/100

Question 1 (14 Marks) Commence a **new** page. **Marks**

(a) Express $0.6\dot{1}$ as a fraction in simplest form. **2**

(b) A metal cube has volume of 157.464 m^3. Find its surface area. **2**

(c) Factorise $125a^3 - 8b^3$. **2**

(d) Solve:

i. $\dfrac{x}{2} + 7 = 9$. **2**

ii. $y(2y + 5) = 12$. **3**

(e) Sketch the curve $y = (x + 5)(4x - 3)$. Hence or otherwise, solve **3**

$$(x + 5)(4x - 3) \geq 0$$

Question 2 (28 Marks) Commence a **new** page.

- Sketch the following on separate number planes, showing any intercepts with axes or asymptotes.
- State the domain & range for each function.

(a) $y = -2x + 5$. **4**

(b) $g(x) = |1 - x|$. **4**

(c) $f(x) = 16 - x^2$. **4**

(d) $f(x) = \sqrt{16 - x^2}$. **4**

(e) $y = 4^{-x}$. **4**

(f) $y = \sqrt{x - 1}$. **4**

(g) $y = -\dfrac{1}{x - 2}$. **4**

Question 3 (11 Marks) Commence a **new** page. **Marks**

(a) Show whether the following functions are odd, even or neither:

i. $y = x^2 - 1$. **2**

ii. $y = x^5 - 2x^3$. **2**

iii. $y = 2x + 1$. **2**

iv. $y = |2x| + 1$. **2**

(b) Solve the inequality $|2x - 1| > 6$ and graph the solution on the number line. **3**

Question 4 (11 Marks) Commence a **new** page. **Marks**

(a) Given $f(x) = x^2 - 3$, evaluate:

i. $f(2)$. **1**

ii. $f(3) - f(0)$. **1**

iii. $f(a + 1)$ **2**

(b) Sketch the following function: **3**

$$f(x) = \begin{cases} -x & x \geq 2 \\ -1 & x < 2 \end{cases}$$

(c) For the function $y = \dfrac{x^2 - 4}{x + 2}$

i. Find its domain. **1**

ii. By simplifying the function, find its range. **1**

iii. Sketch the function. **2**

End of paper.

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Solutions

Question 1

(a) (2 marks)

$$\begin{aligned} x &= 0.6\dot{1} \\ 10x &= 6.1\dot{1} \\ \hline \therefore 9x &= 5.5 \qquad [1] \\ x &= \frac{55}{90} = \frac{11}{18} \qquad [1] \end{aligned}$$

(b) (2 marks)

$$V = 157.464 \text{ m}^3$$

Letting x be the side length of the square,

$$\begin{aligned} x^3 &= 157.464 \\ x &= \frac{27}{5} \qquad [1] \\ SA &= 6x^2 \\ &= 6 \times \left(\frac{27}{5}\right)^2 \\ &= 174.96 \text{ m}^2 \qquad [1] \end{aligned}$$

(c) (2 marks)

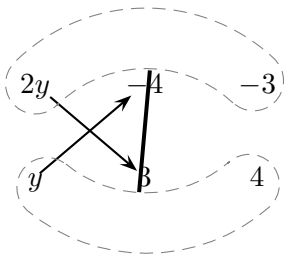
$$125a^3 - 8b^3 = \overbrace{(5a-2b)}^{[1]}\overbrace{(25a^2+10ab+4b^2)}^{[1]}$$

(d) i. (2 marks)

$$\begin{aligned} \frac{x}{2} + \underset{-7}{7} &= \underset{-7}{9} \\ \frac{x}{2} &= 2 \qquad [1] \\ x &= 4 \qquad [1] \end{aligned}$$

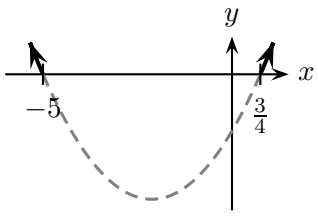
ii. (3 marks)

$$\begin{aligned} y(2y+5)_{-12} &= \underset{-12}{12} \\ 2y^2 + 5y - 12 &= 0 \qquad [1] \end{aligned}$$



$$\begin{aligned} (2y-3)(y+4) &= 0 \qquad [1] \\ \therefore y &= \frac{3}{2}, -4 \qquad [1] \end{aligned}$$

(e) (3 marks)



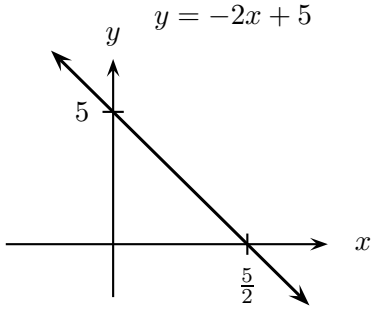
[2]

$$x < -5 \quad \text{or} \quad x > \frac{3}{4} \qquad [1]$$

Question 2

(a) (4 marks)

$y = -2x + 5$

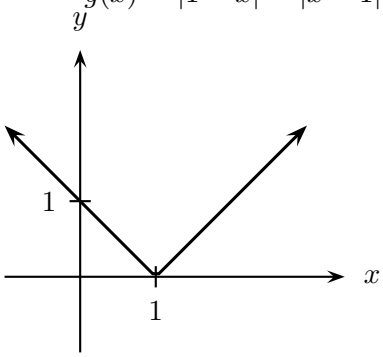


[2]

$$\begin{aligned} D &= \{x : x \in \mathbb{R}\} \qquad [1] \\ R &= \{y : y \in \mathbb{R}\} \qquad [1] \end{aligned}$$

(b) (4 marks)

$g(x) = |1 - x| = |x - 1|$

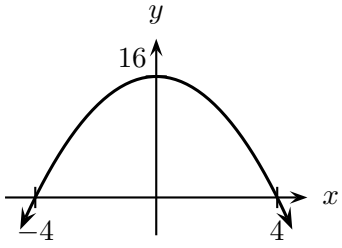


[2]

$$\begin{aligned} D &= \{x : x \in \mathbb{R}\} \qquad [1] \\ R &= \{y : y \geq 0\} \qquad [1] \end{aligned}$$

(c) (4 marks)

$f(x) = 16 - x^2$

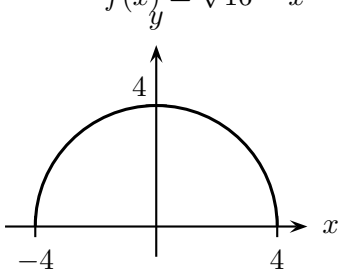


[2]

$D = \{x : x \in \mathbb{R}\}$ [1]
$R = \{y : y \leq 16\}$ [1]

(d) (4 marks)

$f(x) = \sqrt{16 - x^2}$

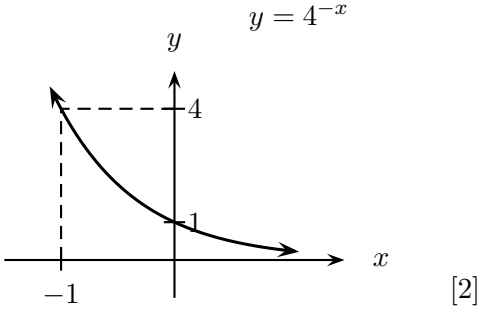


[2]

$D = \{x : -4 \leq x \leq 4\}$ [1]
$R = \{y : 0 \leq y \leq 4\}$ [1]

(e) (4 marks)

$y = 4^{-x}$

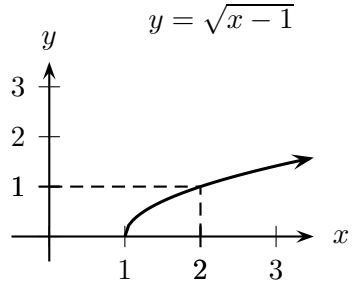


[2]

$D = \{x : x \in \mathbb{R}\}$ [1]
$R = \{y : y > 0\}$ [1]

(f) (4 marks)

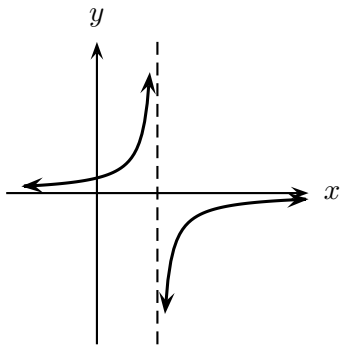
$y = \sqrt{x - 1}$



[2]

$D = \{x : x \geq 1\}$ [1]
$R = \{y : y > 0\}$ [1]

(g) (4 marks)



[2]

$D = \{x : x \neq 2\}$ [1]
$R = \{y : y \neq 0\}$ [1]

Question 3

(a) i. (2 marks)

$$f(x) = x^2 - 1$$
$$f(a) = a^2 - 1$$
$$f(-a) = (-a)^2 - 1 f(a) = f(-a)$$
$$= a^2 - 1$$
$$\therefore f(x) = x^2 - 1 \text{ is even.}$$

Alternatively, a sketch will suffice.

ii. (2 marks)

$$f(x) = x^5 - 2x^3$$
$$f(-a) = (-a)^5 - 2(-a)^3$$
$$= -a^5 + 2a^3$$
$$= -(a^5 - 2a^3) = -f(a)$$
$$f(-a) = -f(a)$$
$$\therefore f(x) = x^5 - 2x^3 \text{ is odd.}$$

iii. (2 marks)

$$\begin{aligned} f(x) &= 2x+1 \\ f(-a) &= 2(-a)+1 \\ &= -2a+1 \\ -f(a) &= -2a-1 \\ f(a) &= 2a+1 \\ f(a) &\neq f(-a) \neq -f(a) \end{aligned}$$

$\therefore f(x)$ is neither odd or even

iv. (2 marks)

$$\begin{aligned} f(x) &= |2x|+1 \\ f(a) &= |2a|+1 \\ &= 2|a|+1 \\ f(-a) &= |2(-a)|+1 \\ &= |-2a|+1 \\ &= 2|a|+1 \\ f(a) &= f(-a) \end{aligned}$$

$\therefore f(x)$ is even.

Again a graph would suffice. It is the absolute value graph $f(x) = |2x|$ shifted up by 1 unit.

(b) (3 marks)

$$|2x-1| > 6$$

$$|2x-1| = \begin{cases} 2x-1 & \text{if } 2x-1 > 0 \\ -(2x-1) & \text{if } 2x-1 < 0 \end{cases} \quad [1]$$

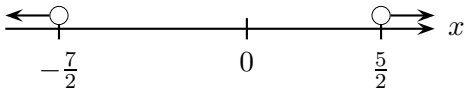
Case 1: $2x-1>0$

$$\begin{aligned} 2x - 1 &> 6 \\ {\scriptstyle +1} \quad &\quad {\scriptstyle +1} \\ 2x &> 7 \\ {\scriptstyle \div 2} \quad &\quad {\scriptstyle \div 2} \\ x &> \tfrac{7}{2} \quad [1] \end{aligned}$$

Case 2: $2x-1<0$

$$\begin{aligned} -(2x-1) &> 6 \\ -2x + 1 &> 6 \\ {\scriptstyle -1} \quad &\quad {\scriptstyle -1} \\ -2x &> 5 \\ {\scriptstyle \div -2} \quad &\quad {\scriptstyle \div -2} \\ x &< -\tfrac{5}{2} \end{aligned}$$

$\therefore x > \frac{7}{2} \quad x < -\frac{5}{2}$ [1]



Question 4

(a) i. (1 mark)

$$\begin{aligned} f(2) &= 2^2 - 3 \\ &= 1 \end{aligned}$$

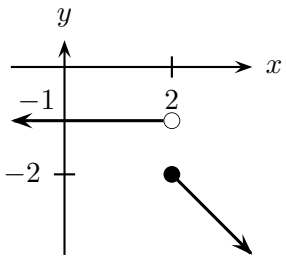
ii. (1 mark)

$$\begin{aligned} f(3) - f(0) &= (3^2-3) - (0-3) \\ &= 9 \end{aligned}$$

iii. (2 marks)

$$\begin{aligned} f(a+1) &= (a+1)^2 - 3 \quad [1] \\ &= a^2 + 2a + 1 - 3 \\ &= a^2 + 2a - 2 \quad [1] \end{aligned}$$

(b) (3 marks)



- [1] for sketching $f(x) = -x$ for $x \geq 2$.
- [1] for sketching $f(x) = -1$ for $x < 2$.
- [1] for correct open circle & closed circles at $x = 2$.

(c) i. (1 mark)

$$D = \{x : x \neq -2\}$$

ii. (1 mark)

$$\begin{aligned} y &= \frac{(x-2)\cancel{(x+2)}}{\cancel{(x+2)}} \\ &= x - 2 \qquad x \neq -2 \\ R &= \{y : y \neq -4\} \end{aligned}$$

iii. (2 marks)