NORTH SYDNEY BOYS HIGH SCHOOL

MATHEMATICS (PRELIMINARY COURSE)

2010 Assessment Task 2

General instructions

- Working time – 1 period.
- Commence each new question on a new sheet.
- Write using blue or black pen. Where diagrams are to be sketched, these may be done in pencil.
- Board approved calculators may be used.
- All necessary working should be shown in every question.
- Attempt **all** questions.

Class (please ✔)

◯ 11M2A – Mr Lowe

NAME: ..... **# PAGES USED**: .....

Marker's use only.

QUESTION	1	2	3	4	5	**Total**	%
MARKS	$\overline{6}$	$\overline{15}$	$\overline{9}$	$\overline{12}$	$\overline{11}$	$\overline{53}$	

Question 1 (6 Marks) Commence a NEW page. **Marks**

(a) Find the value of $\tan 26^\circ$ correct to 3 decimal places. **1**

(b) Find the size of the acute angle A in degrees and minutes (to the nearest minute) for $\sin A = \dfrac{5}{8}$. **1**

(c) Why would an error be generated when $\cos^{-1}\left(\dfrac{8.4}{7.6}\right)$ is entered on the calculator? **1**

(d) Find the exact value of the following expressions:

i. $\sin 45^\circ$. **1**

ii. $\cos 30^\circ$. **1**

iii. $\cot 210^\circ$. **1**

Question 2 (15 Marks) Commence a NEW page. **Marks**

(a) Solve for $0 \leq \theta < 360^\circ$:

i. $\sin\theta = \dfrac{1}{2}$. **2**

ii. $\cos 2\theta = \dfrac{\sqrt{3}}{2}$. **3**

iii. $\tan^2\theta = 1$. **3**

iv. $2\cos^2\theta - \cos\theta = 0$. **3**

(b) For the curve $y = 2\sin 3\theta$,

i. State the period. **1**

ii. State the amplitude. **1**

iii. Sketch the curve for $0^\circ \leq \theta \leq 360^\circ$. **2**

Question 3 (9 Marks) Commence a NEW page. **Marks**

(a) For $A(2,4)$, $B(5,6)$, find the

i. gradient of AB. **1**

ii. midpoint of AB. **2**

iii. distance between the points. **1**

iv. equation of AB. **2**

(b) Given the line $2x - 3y = 4$,

i. Express in gradient-intercept form. **1**

ii. State the gradient. **1**

iii. Find the angle the line makes with the positive x axis, correct to the nearest minute. **1**

Question 4 (12 Marks) Commence a NEW page. **Marks**

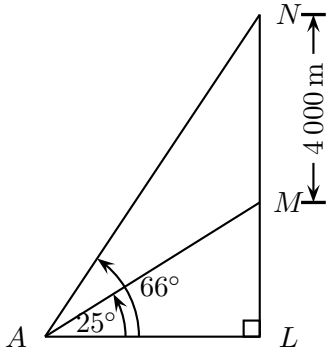
(a) In $\triangle ABC$, $\sin B = \frac{4}{5}$, $a = 6$ and $b = 9$. Find $\sin A$ as the simplest exact value. **2**

(b) Two sides of a triangular field are 60 m and 50 m with the included angle being 140°. By sketching a diagram, calculate correct to 3 decimal places:

i. the length of the third side. **2**

ii. the area of the field. **2**

(c) A rocket launched vertically from L is observed from A. Soon after the launch when at position M its angle of elevation is 25°. After it climbs 4 000 m to position N, its angle of elevation is 66°.



NOT TO SCALE

i. Find $\angle ANL$ and $\angle NAM$. **2**

ii. Use the sine rule to find AM. **2**

iii. Find how far the observer is from the launch pad correct to the nearest metre. **2**

Question 5 (11 Marks) Commence a NEW page. **Marks**

(a) Simplify $\cos^3\theta + \cos\theta\sin^2\theta$. **2**

(b) If $\tan\alpha = -\dfrac{5}{12}$ and $0° \leq \alpha \leq 360°$, find the exact value of $\sin\alpha$. **2**

(c) Prove $\dfrac{1}{1-\cos\theta} + \dfrac{1}{1+\cos\theta} = 2\operatorname{cosec}^2\theta$. **3**

(d) Two cars leave point A at the same time. One car averages 80 km/h along a straight road in the direction 025°T. The other car averages 90 km/h along a straight road in the direction 135°T. How far apart (to the nearest kilometre) are they after 3 hours? **3**

End of paper.

Suggested Solutions

Question 1

(a) (1 mark)

$$\tan 26^\circ = 0.488 \text{ (3 d.p.)}$$

(b) (1 mark)

$$A = \sin^{-1}\frac{5}{8} = 38^\circ 41'$$

(c) (1 mark)
As $-1 \leq \cos\theta \leq 1$, and from the expression, $\cos\theta > 1$, which is out of range.

(d) i. (1 mark)

$$\sin 45^\circ = \frac{1}{\sqrt{2}}$$

ii. (1 mark)

$$\cos 30^\circ = \frac{\sqrt{3}}{2}$$

iii. (1 mark)

$$\begin{aligned}\cot 210^\circ &= \frac{1}{\tan(180^\circ + 30^\circ)} \\ &= \frac{1}{\tan 30^\circ} = \frac{1}{\frac{1}{\sqrt{3}}} \\ &= \sqrt{3}\end{aligned}$$

Question 2

(a) i. (2 marks)

$$\begin{aligned}\sin\theta &= \frac{1}{2} \\ \theta &= 30^\circ, 150^\circ\end{aligned}$$

ii. (3 marks)

$$\begin{aligned}0^\circ \leq \theta &< 360^\circ \\ \therefore 0^\circ \leq 2\theta &< 720^\circ \\ \cos 2\theta &= \frac{\sqrt{3}}{2} \\ 2\theta &= 30^\circ, 330^\circ, 390^\circ, 690^\circ \\ \therefore \theta &= 15^\circ, 165^\circ, 195^\circ, 345^\circ\end{aligned}$$

iii. (3 marks)

$$\begin{aligned}\tan^2\theta &= 1 \\ \tan\theta &= \pm 1 \\ \therefore \theta &= 45^\circ, 135^\circ, 225^\circ, 315^\circ\end{aligned}$$

iv. (3 marks)

$$\begin{aligned}2\cos^2\theta - \cos\theta &= 0 \\ \cos\theta(2\cos\theta - 1) &= 0\end{aligned}$$

$\cos\theta = 0$	$\cos\theta = \frac{1}{2}$
$\therefore \theta = 90^\circ, 270^\circ$	$\therefore \theta = 60^\circ, 300^\circ$

$$\therefore \theta = 60^\circ, 90^\circ, 270^\circ, 300^\circ$$

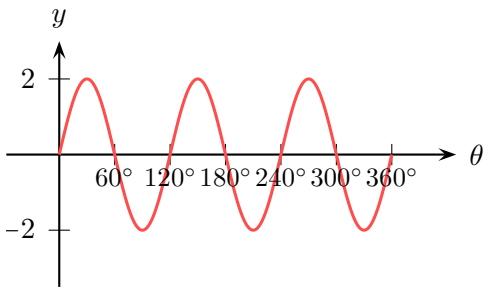
(b) i. (1 mark)

$$T = \frac{360^\circ}{3} = 120^\circ$$

ii. (1 mark)

$$a = 2$$

iii. (2 marks)



Question 3

(a) i. (1 mark)

$$\begin{aligned}&A(2,4) \quad B(5,6) \\ m &= \frac{y_2 - y_1}{x_2 - x_1} \\ &= \frac{6-4}{5-2} = \frac{2}{3}\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}(x, y) &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{5+2}{2}, \frac{6+4}{2}\right) \\ &= \left(\frac{7}{2}, 5\right)\end{aligned}$$

iii. (1 mark)

$$\begin{aligned}d &= \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2} \\ &= \sqrt{(5-2)^2 + (6-4)^2} \\ &= \sqrt{3^2 + 2^2} = \sqrt{13}\end{aligned}$$

iv. (2 marks)

$$\begin{aligned}\frac{y - y_1}{x - x_1} &= m \\ \underset{\times(x-5)}{\frac{y-6}{x-5}} &= \underset{\times(x-5)}{\frac{2}{3}} \\ y - \underset{+6}{6} &= \frac{2}{3}x - \underset{+6}{\frac{10}{3}} \\ y &= \frac{2}{3}x + \frac{8}{3}\end{aligned}$$

(b) i. (1 mark)

$$\begin{aligned}2x - \underset{-3y}{3y} &= \underset{-3y}{4} \\ 3y &= 2x - 4 \\ y &= \frac{2}{3}x - \frac{4}{3}\end{aligned}$$

ii. (1 mark)

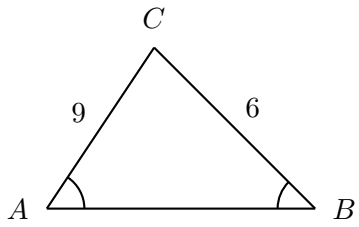
$$m = \frac{2}{3}$$

iii. (1 mark)

$$\begin{aligned}m &= \tan\theta \\ \tan\theta &= \frac{2}{3} \\ \therefore \theta &= \tan^{-1}\frac{2}{3} \\ &= 33.69^\circ = 33^\circ 41'\end{aligned}$$

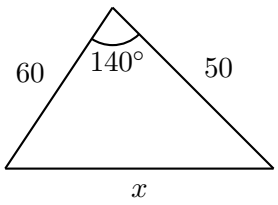
Question 4

(a) (2 marks)



$$\begin{aligned}\frac{\sin A}{\underset{\times 6}{6}} &= \frac{\sin B}{\underset{\times 6}{9}} \\ \sin A &= \frac{\cancel{6}^{2} \times \frac{4}{5}}{\cancel{3}} = \frac{8}{5} \times \frac{1}{3} \\ &= \frac{8}{15}\end{aligned}$$

(b) i. (2 marks)



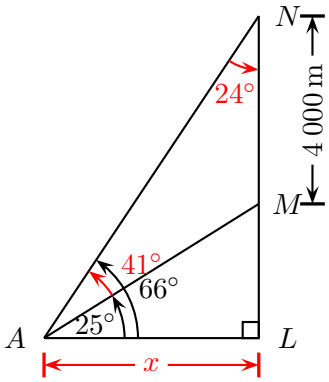
Using the cosine rule,

$$\begin{aligned}x^2 &= 60^2 + 50^2 - 2 \times 60 \times 50 \times \cos 140^\circ \\ &= 10696.267\cdots \\ x &= 103.423\,\text{m}\end{aligned}$$

ii. (2 marks)

$$\begin{aligned}A &= \frac{1}{2}ab\sin C \\ &= \frac{1}{2} \times 60 \times 50 \times \sin 140^\circ \\ &= 964.181\,\text{m}^2\end{aligned}$$

(c) i. (2 marks)



$$\angle ANL = 90° - 66° = 24°$$
$$\angle NAM = 66° - 25° = 41°$$

ii. (2 marks)

$$\frac{AM}{\sin 24°} = \frac{4\,000}{\sin 41°} \quad \times \sin 24°$$
$$AM = \frac{4\,000 \sin 24°}{\sin 41°}$$
$$= 2\,479.88 \text{ m}$$

iii. (2 marks)

$$\frac{x}{AM} = \cos 25°$$
$$x = AM \cos 25°$$
$$= \frac{4\,000 \sin 24°}{\sin 41°} \times \cos 25°$$
$$= 2\,247.53 \text{ m}$$
$$= 2\,248 \text{ m (nearest metre)}$$

Question 5

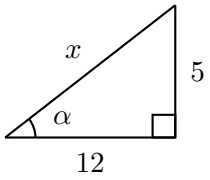
(a) (2 marks)

$$\cos^3\theta + \cos\theta \sin^2\theta$$
$$= \cos\theta(\cos^2\theta + \sin^2\theta) \quad {}^{1}$$
$$= \cos\theta$$

(b) (3 marks)

- ✓ [1] for locating α in the 2nd or 4th quadrants
- ✓ [1] for finding correctly the hypotenuse in the corresponding $\triangle$
- ✓ [1] for final correct answer

Alternatively, $[-1]$ for each mistake.



$$\tan\alpha = -\frac{5}{12}$$

$\therefore \alpha$ is in the second or fourth quadrants

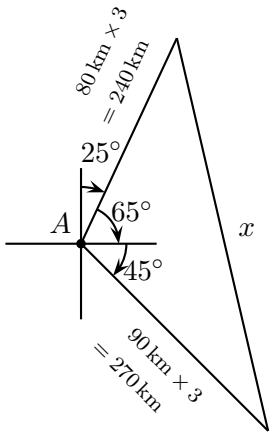
$$\therefore x = \sqrt{12^2 + 5^2} = 13$$
$$\therefore \sin\alpha = \pm\frac{5}{13}$$

(c) (3 marks)

$$\frac{1}{1-\cos\theta} + \frac{1}{1+\cos\theta}$$
$$= \frac{(1+\cos\theta) + (1-\cos\theta)}{(1-\cos\theta)(1+\cos\theta)}$$
$$= \frac{2}{1-\cos^2\theta}$$
$$= \frac{2}{\sin^2\theta}$$
$$= 2\,\text{cosec}^2\,\theta$$

(d) (3 marks)

- ✓ [1] for diagram
- ✓ [1] for attempt to use cosine rule
- ✓ [1] for final correct answer



Apply the cosine rule to the triangle,

$$x^2 = 240^2 + 270^2 - 2 \times 240 \times 270 \times \cos 110°$$
$$= 174\,825.81\cdots$$
$$x = 418.12 \text{ km}$$