Question 1 (23 marks)

(a) Simplify: **2**

i) $\sqrt{27}$

ii) $\sqrt{72}-\sqrt{50}$

(b) Expand and simplify: **2**

$(3\sqrt{2}+1)(\sqrt{2}-1)$

(c) Evaluate $\sqrt{\dfrac{4.56+86.7}{6.4^3\times 5.63}}$ correct to 3 significant figures. **2**

(d) Express $0.\dot{3}\dot{5}$ as a simple fraction. **3**

(e) Factorise completely:

i) $4x^2-36$ **2**

ii) $5y-15+xy-3x$ **2**

(f) Simplify:

i) $\dfrac{x-4}{2}+\dfrac{x+3}{3}$ **2**

ii) $\dfrac{a+2}{a^2-3a-4}\div\dfrac{a^2+4a+4}{a+1}$ **3**

(g) Solve the equation and leave the answer in simplest surd form: **3**

$n^2+10n+7=0$

(h) A number is increased by $7\frac{1}{2}\%$ to give 86. Find the number. **2**

Question 2 (21 marks)

(a) Solve $|2x-1|< 3$ and graph the answer on a number line. **3**

(b) Solve $2^x = 128$ **2**

(c) Solve simultaneously: **3**

$$3x + 2y = -6$$
$$x - 2y = -10$$

(d) State the domain of the function: **1**

$$y = \frac{4}{x-3}$$

(e) State the range of the function: **1**

$$y = 3^x + 3$$

(f) Determine $f(-x)$ for the function $f(x) = x^3 - 5x$ and hence state whether the function is **odd**, **even** or **neither**. Give a **reason** for your answer. **2**

(g) Simplify $8^{\frac{-2}{3}} \times 49^{\frac{1}{2}}$ **3**

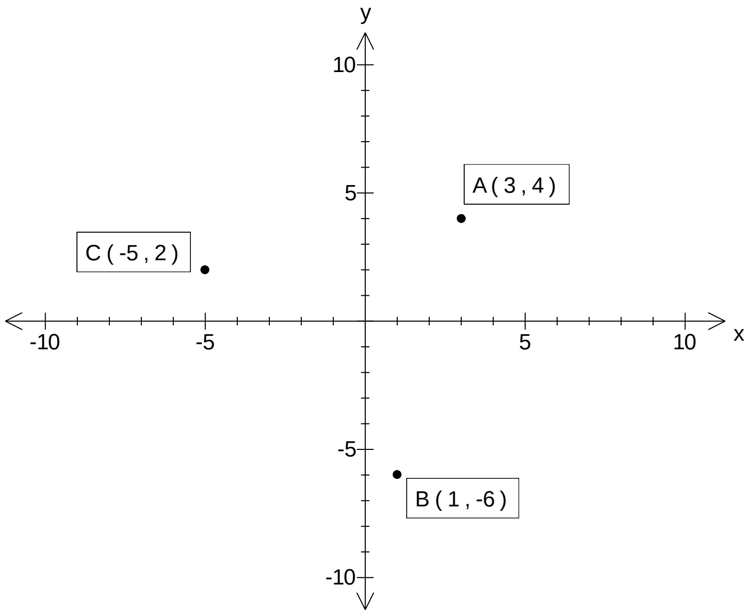
(h) Simplify $|-5| - |8|$. **2**

(i) i) Sketch the graphs of $y = 2x + 3$ and $y = x^2$ on the same Cartesian Plane. Clearly show the intercepts with the axes. **3**

iii) Shade the region of the plane defined by the inequations $y \geq x^2$ and $y \leq 2x + 3$. **1**

Question 3 (22 marks)

(a)



The points A(3,4), B(1,-6) and C(-5, 2) are the vertices of a triangle.

i) Show that the equation of the line *AC* is $x - 4y + 13 = 0$. **3**
Call this equation line *k*.

ii) P(2, -1) is the mid-point of AB. Find the gradient of BC and hence, find the equation of the line *l* through *P* parallel to *BC*. **3**

iii) Find the point of intersection *Q* of the lines *k* and *l*. **3**

iv) Show that *Q* is the mid-point of *AC*. **2**

v) Show that $PQ = \frac{1}{2}BC$ **4**

vi) Find the perpendicular distance from *B* to *AC*. **2**

(b) Find the **length of radius** and the **coordinates of the centre** of the circle: **2**

$$(x-4)^2 + (y-5)^2 = 16$$

(c) Find the equation of the line with *x*-intercept 4 that makes an angle of 45^o with the *x*-axis. **3**

Question 4 (15 marks)

(a) Solve $2\cos\theta = -1$ for $0^\circ \le \theta \le 360^\circ$ **3**

(b) Show that $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$ **3**

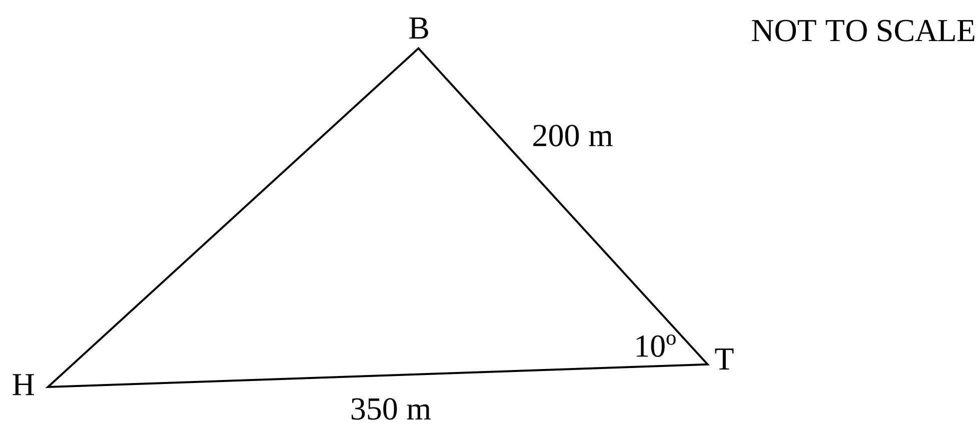
(c) Find the exact values of:

i) $\operatorname{cosec} 135^\circ$ **2**

ii) $\tan(-150^\circ)$ **2**

(d) If $\cos\alpha = \dfrac{2}{3}$, find the exact value of $\tan\alpha$ where α is acute. **2**

(e) On a golf course, the distance from a tee *T* to the hole *H* is 350 m. A golfer's ball comes to rest at point *B*, 200 m from *T*. Angle HTB is 10°, as shown in the diagram. How far is *B* from *H*? Give your answer correct to 2 decimal places. **3**



Question 5 (21 marks)

a) A series has n th term given by $T_n = n^3 - 5$. Find:

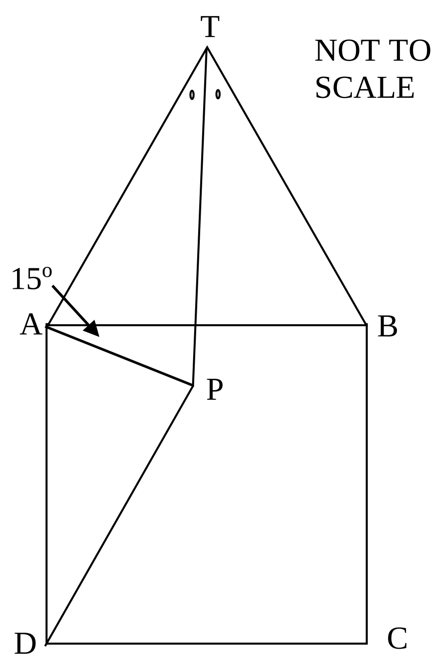
i) the 4th term **2**

ii) which term is 5827 **3**

b) The limiting sum of a geometric series is $-\frac{3}{10}$ and its first term is $-\frac{1}{2}$. Find the common ratio of the series. **3**

c) Find which term -370 is in the series 17 + 8 - 1 - … **3**

d) In the diagram, $ABCD$ is a square and ABT is an equilateral triangle. The line TP bisects $\angle ATB$, and $\angle PAB = 15^\circ$.



i) Copy the diagram onto your examination paper and explain why $\angle PAT = 75^\circ$ **3**

ii) Prove that $\Delta TAP \equiv \Delta DAP$ **3**

iii) Find $\angle APT$. **1**

iv) Prove that ΔDAP is isosceles. **3**

Q a) (i) $\sqrt{27} = \sqrt{3 \times 3 \times 3} = 3\sqrt{3}$ ✓

(ii) $\sqrt{72} - \sqrt{50} = 6\sqrt{2} - 5\sqrt{2}$
$= \sqrt{2}$ ✓

(b) $(3\sqrt{2}+1)(\sqrt{2}-1)$ ✓
$= 3.2 + \sqrt{2} - 3\sqrt{2} - 1$
$= 6 - 2\sqrt{2} - 1$
$= 5 - 2\sqrt{2}$ ✓ (2)

(c) $\sqrt{\dfrac{4.56 + 86.7}{6.4^3 \times 5.63}}$

$= 0.249$ ✓✓ (3 s.f.) (2)

d Let $x = 0.\dot{3}\dot{5}$
$100x = 35.3535$ ✓
$- \quad x = 0.3535$ ✓
$99x = 35$
$x = \dfrac{35}{99}$ ✓ (3)

e(i) $4x^2 - 36 = 4(x^2 - 9)$ ✓
$= 4(x-3)(x+3)$ ✓ (2)

(ii) $5y - 15 + xy - 3x$
$= 5(y-3) + x(y-3)$ ✓
$= (y-3)(5+x)$ ✓ (2)

(f) (i) $\dfrac{x-4}{2} + \dfrac{x+3}{3}$

$\dfrac{3(x-4)}{6} + \dfrac{2(x+3)}{6}$ ✓✓

$= \dfrac{3x - 12 + 2x + 6}{6}$

$= \dfrac{5x - 6}{6}$ ✓ (2)

f(ii) $\dfrac{a+2}{a^2-3a-4} \div \dfrac{a^2+4a+4}{a+1}$

$= \dfrac{\cancel{(a+2)}}{(\cancel{a+1})(a-4)} \times \dfrac{\cancel{(a+1)}}{(\cancel{a+2})(a+2)}$ ✓✓✓

$= \dfrac{1}{(a-4)(a+2)}$ ✓✓ (3)

(g) $n^2 + 10n + 7 = 0$
cannot factorise
∴ use formula

$a = 1$
$b = 10$
$c = 7$

$n = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-(10) \pm \sqrt{10^2 - 4(1)(7)}}{2(1)}$ ✓

$= \dfrac{-10 \pm \sqrt{100 - 28}}{2}$ ✓

$= \dfrac{-10 \pm \sqrt{72}}{2}$ ✓ (3)

∴ $n = \dfrac{-10 + \sqrt{72}}{2}$ ✓ or $n = \dfrac{-10 - \sqrt{72}}{2}$ ✓

$\Rightarrow -5 \pm 3\sqrt{2}$ ✓ (2)

(h) $x \times 1.075 = 86$
$\therefore x = \dfrac{86}{1.075}$ ✓
$= 80$ ✓ (2)

23

Q2

(a) $|2x-1| < 3$

$2x-1 < 3$ or $-(2x-1) < 3$

$2x < 4$ $\quad -2x+1 < 3$

$x < 2$ $\quad -2x < 2$

$x > -1$

$\therefore -1 < x < 2$

$$\begin{array}{cccccc} -1 & 0 & 1 & 2 & 3 & 4 \end{array}$$

(3)

(b) $2^x = 128$

$2^x = 2^7$

$x = 7$

(2)

(c) $3x + 2y = -6$ — (A)

$x - 2y = -10$ — (B)

A + B $\quad 4x = -16$

$x = -4$

Substitute $x = -4$ into (A)

$3(-4) + 2y = -6$

$2y = -6 + 12$

$2y = 6$

$y = 3$

(3)

$\therefore x = -4$ and $y = 3$

(d) $y = \dfrac{4}{x-3}$ $\quad x \neq 3$

$\therefore D = \{x : x \neq 3, \text{ all real values}\}$

(1)

(e) $R = \{y : y \text{ all real values}\}$

$y > 3$

(1)

(f) $f(x) = x^3 - 5x$

$\therefore f(-x) = (-x)^3 - 5(-x)$

$= -x^3 + 5x$

$= -(x^3 - 5x)$

$f(-x) = -f(x)$

$\therefore$ odd

(2)

(g) $8^{-\frac{2}{3}} \times 49^{\frac{1}{2}}$

$= \dfrac{1}{8^{2/3}} \times (7^2)^{1/2}$

$= \dfrac{1}{\sqrt[3]{8^2}} \times 7$

$= \dfrac{1}{\sqrt[3]{(2^3)^2}} \times 7$

$= \dfrac{1}{\sqrt[3]{2^6}} \times 7$

$= \dfrac{1}{2^2} \times 7$

$= \dfrac{1}{4} \times 7$

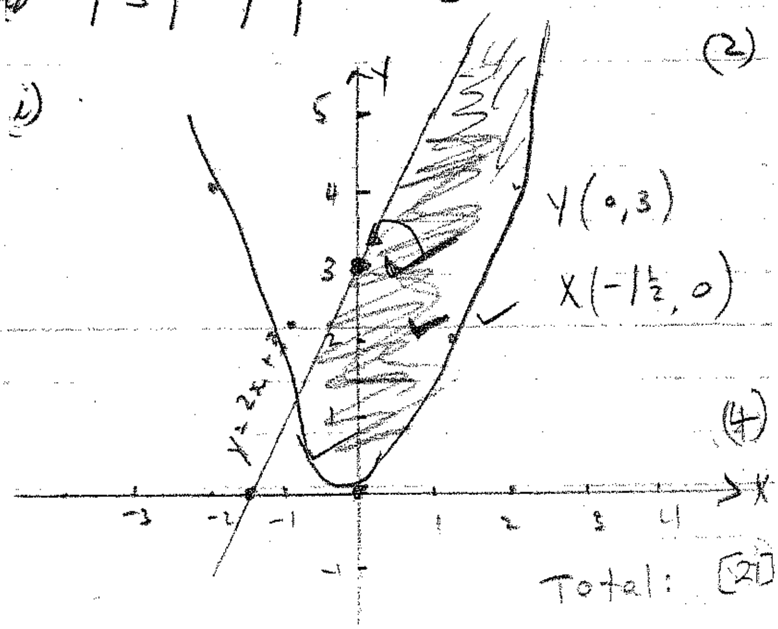
$= 7/4$

(3)

(h) $|-5| - |8| = 5 - 8 = -3$

(2)

i)



(4)

Total: 21

3 (i) $M = \frac{4-2}{3-(-5)} = \frac{2}{8} = \frac{1}{4}$ ✓

$\therefore y - 4 = \frac{1}{4}(x-3)$ ✓

$4y - 16 = x - 3$ ✓

$0 = x - 4y + 13$ (3)

(ii) $P(2,-1)$, // to BC

Gradient of $BC = \frac{-6-2}{1-(-5)}$

$= \frac{-8}{6}$ ✓

$= -4/3$

$\therefore$ Gradient of $l = -\frac{4}{3}$

$y - (-1) = -\frac{4}{3}(x - 2)$

$y + 1 = -\frac{4}{3}(x+2)$ ✓

$3y + 3 = -4x + 8$ (3)

$4x + 3y - 5 = 0$ ✓ or $y = -\frac{4}{3}x + 5/3$

(iii) $k: \quad x - 4y + 13 = 0$ — (A)

$l: \quad 4x + 3y - 15 = 0$ — (B) ✓

(A) × 4 $\quad 4x - 16y + 52 = 0$ — (C)

(B) − (C)

$19y - 57 = 0$

$19y = 57$ ✓

$y = 3$

$x - 4(3) + 13 = 0$ ✓

$x - 12 + 13 = 0$ (3)

$x = -1$ ✓

$\therefore Q(-1, 3)$

(iv) AC Midpoint $= \left(\frac{3 + -5}{2}, \frac{4+2}{2}\right)$

$= \left(\frac{-2}{2}, \frac{6}{2}\right)$

$= (-1, 3)$ ✓

$\therefore Q(-1,3)$ is Midpoint of AC ✓ (2)

(v) $PQ = \frac{1}{2} BC$

$P(2,-1)$ $\quad B(1,-6)$

$Q(-1,3)$ $\quad C(-5,2)$

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

$PQ_d = \sqrt{(-1-2)^2 + (3-(-1))^2}$ ✓

$= \sqrt{9 + 16}$ ✓

$= \sqrt{25}$

$= 5$ ✓

$BC_d = \sqrt{(-5-1)^2 + (2-(-6))^2}$ ✓

$= \sqrt{36 + 64}$

$= \sqrt{100}$ ✓

$= 10$ ✓ (4)

$\therefore PQ = \frac{1}{2} BC$

(vi) $x - 4y + 13 = 0$ (AC

$a = 1, b = -4, c = 13$ $\quad B(1,-6)$ $\quad x_1 \; y_1$

$d = \left| \frac{(1)(1) + (-4)(-6) + 13}{\sqrt{1^2 + (-4)^2}} \right|$ ✓ (2)

$= \frac{|1 + 24 + 13|}{\sqrt{17}}$ ✓ $= \frac{38}{\sqrt{17}}$ units ✓

Q3 (b) $(x-4)^2+(y-5)^2=16$

$r=\sqrt{16}$

$r=4$

Coordinates of centre $(4, 5)$ (2)

(c) $X(4, 0)$

$M = \tan 45^\circ$

$\therefore M = 1$

$\therefore y = 1x + b$

Substitute $X(4,0)$:

$0 = 1(4) + b$

$-4 = b$

$\therefore y = 1x - 4$ (2)

[Total: 22]

Q4

(a) $2\cos\theta = -1$

$\cos\theta = -\frac{1}{2}$

θ is in 2nd + 3rd quadrant

2nd quadrant

$180^\circ - 60^\circ$

$= 120^\circ$

3rd quadrant

$180^\circ + 60^\circ$

$= 240^\circ$ (3)

(b) $\sec\theta + \tan\theta = \dfrac{1+\sin\theta}{\cos\theta}$

$LHS = \sec\theta + \tan\theta$

$= \dfrac{1}{\cos\theta} + \dfrac{\sin\theta}{\cos\theta}$

$= \dfrac{1+\sin\theta}{\cos\theta}$ (3)

$= RHS$

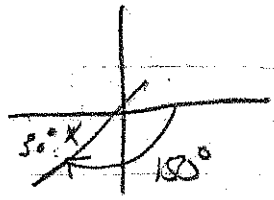
(c) (i) $\operatorname{cosec} 135^\circ = \dfrac{1}{\sin 135^\circ}$

$= \dfrac{1}{(\sin 180^\circ - 45^\circ)}$

$= \dfrac{1}{\sin 45^\circ}$

$= \dfrac{1}{\frac{1}{\sqrt{2}}} = \sqrt{2}$ (2)

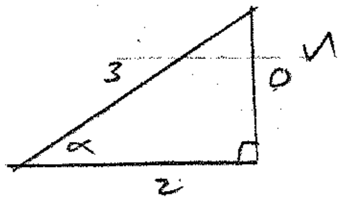
(ii) $\tan(-150^\circ)$



$= \tan 30^\circ$

$= \dfrac{1}{\sqrt{3}}$ (2)

(d) $\cos\alpha = \frac{2}{3} \left(\frac{a}{h}\right)$



$o^2 = 3^2 - 2^2$

$= 9 - 4$

$= 5$

$o = \sqrt{5}$

$\tan\alpha = \dfrac{\sqrt{5}}{2}$ (2)

(e) $t^2 = b^2 + h^2 - 2bh\cos T$

$= (350)^2 + (200)^2 - 2(350)(200)\cos 10^\circ$

$= 24626.9$

$t = 156.93$ m (3)

[Total: 15]

Q5

a i) $T_n = n^3 - 5$

$T_4 = 4^3 - 5$ ✓

$= 59$ ✓ (2)

(iii) $T_n = 5827 \quad n = ?$

$5827 = n^3 - 5$

$5832 = n^3$

$\sqrt[3]{5832} = n$

$18 = n$ (3)

(b) $S_\infty = \frac{a}{1-r} \qquad a = -\frac{1}{2}$

$S_\infty = -3/10$

$\frac{-3}{10} = \frac{-\frac{1}{2}}{1-r}$

$-3(1-r) = -5$

$-3 + 3r = -5$

$3r = -2$

$r = -2/3$ (3)

(c) $17 + 8 + -1 - \ldots \qquad T_n = -370$

$a = 17 \qquad d = -9$

$T_n = a + (n-1)d$

$-370 = 17 + (n-1)(-9)$

$-370 = 17 - 9n + 9$

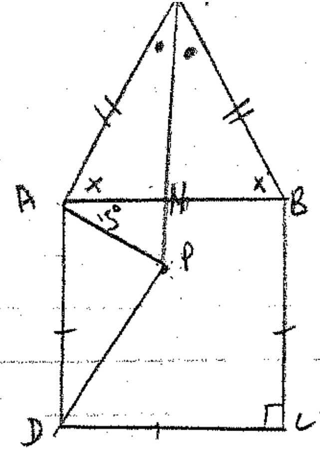
$-370 - 26 = -9n$

$-396 = -9n$

$\frac{396}{9} = n$

$44 = n$ (3)

(d)



(i) $\angle TAB = 60°$ (ΔTAB equilateral)

$\angle PAT = \angle TAB + \angle PAB$

$= 60° + 15°$

$= 75°$ (3)

(ii) $\angle DAP = 90° - 15°$ ($\angle DAB$ is $90°$)

$= 75°$

$TA = AB$ (sides of equilateral Δ)

but $AB = AD$ (sides of square)

$\therefore TA = AD$

In ΔTAP and ΔDAP

AP is common

$\angle PAT = \angle DAP$ (both $75°$)

$TA = DA$

$\therefore \Delta TAP \equiv \Delta DAP$ (SAS) (3)

(iii) $\angle ATP = 30°$ (Half of $\angle ATB$)

$\angle APT = 180° - (75° + 30°)$

$= 75°$ (2)

(iv) ΔATP is isosceles

($\angle PAT = \angle TAP = 75°$)

Since $\Delta ATP \equiv \Delta DAP$

ΔDAP is also isosceles (3)

[22]