Question 1

a) $\dfrac{1+\sqrt{7}}{3-\sqrt{7}} = a + b\sqrt{7}$

By rationalizing the denominator, find the values of a and b **(2)**

b) Factorise fully: $x^4 - ax^2 - 4x^2 + 4a$ **(2)**

c) $f(x) = \begin{cases} |x+1| & \text{for } x \le 0 \\ 4 - x^2 & \text{for } x > 0 \end{cases}$

Sketch the graph of $y = f(x)$ **(3)**

d) Find $\lim\limits_{h \to 0} \dfrac{f(x+h) - f(x)}{h}$ if $f(x) = x^2 + 1$ **(2)**

e) Find the largest possible domain of

$f(x) = \sqrt{x+2} - \sqrt{2-x}$ **(2)**

Question 2 (Start a new page) **Marks**

a) Expand $(x + y)^3$ **(1)**

(2)

b) Solve $\frac{x}{x+1} \geq 5$ **(3)**

c) For the curve $y = \frac{x^2}{1-x^2}$

(i) Write down the x and y intercepts **(1)**

(ii) Find whether the function is odd, even or neither **(1)**

(iii) Write down the equations of all asymptotes **(2)**

(iv) Hence sketch the curve $y = \frac{x^2}{1-x^2}$ **(2)**

(v) What is the range of the function? **(1)**

Question 3 (Start a new page)

a) Sketch $y = \cos x$ for $0^\circ \leq x \leq 360^\circ$ **(1)**

b) Find the exact value of $\cos 210^\circ$ **(2)**

c) Simplify: $\sin(180^\circ + \theta)\tan(90^\circ - \theta)$ **(2)**

d) Simplify: $(\cot^2\theta + 1)(1 - \cos^2\theta)$ **(2)**

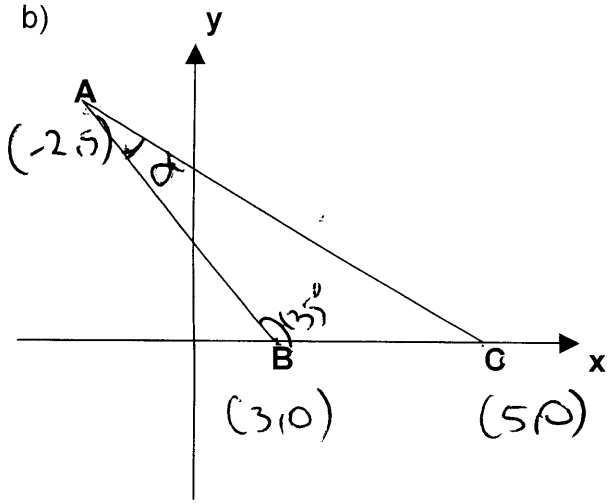
e) Solve: $\cos x = -\frac{1}{\sqrt{2}}$ for $0^\circ \leq x \leq 360^\circ$ **(2)**

f) If $\sec\theta = \frac{7}{3}$ and $\sin\theta < 0$ find the exact value of $\tan\theta$ **(2)**

Question 4 (Start a new page) **Marks**

a) Find the coordinates of the point which divides the interval LM in the ratio 2:3 given L is (-4,1) and M is (5,6) **(2)**

b)



A is the point (-2, 5)
B is the point (3, 0)
C is the point (5, 0)

Diagram not to scale

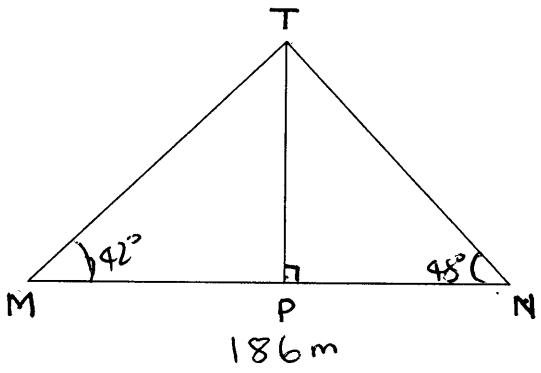
(i) Find the exact distance AC **(1)**

(ii) Find the gradient of AB **(1)**

(iii) Show that $\angle ABC = 135^\circ$ **(1)**

(iv) Hence find $\angle BAC$ to the nearest minute **(2)**

c)



The top, *T*, of a radio mast *TP* has an angle of elevation of 42° from a point *M* on the level ground on which it stands, and of 48° from a point *N* on the other side directly opposite *M*

If *MN* is 186 metres find the height of the radio mast *TP*. **(4)**

QUESTION 1

a) $\frac{1+\sqrt{7}}{3-\sqrt{7}} \times \frac{3+\sqrt{7}}{3+\sqrt{7}} = \frac{3+\sqrt{7}+3\sqrt{7}+7}{9-7}$

$= \frac{10+4\sqrt{7}}{2}$

$= 5+2\sqrt{7}$

$\therefore a = 5, \ b = 2$ — 2 marks

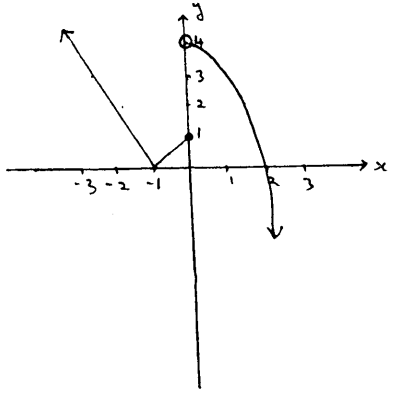
b) $x^4 - ax^2 - 4x^2 + 4a$

$= x^2(x^2-a) - 4(x^2-a)$ — 1 mark

$= (x^2-a)(x^2-4)$ — 1 mark

$= (x^2-a)(x-2)(x+2)$ — 1 mark

c)



3 marks

d) $\lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{[(x+h)^2+1]-[x^2+1]}{h}$

$= \lim_{h\to 0} \frac{x^2+2xh+h^2+1-x^2-1}{h}$ — 2 marks

$= \lim_{h\to 0} \frac{2xh+h^2}{h}$

$= \lim_{h\to 0} \frac{h(2x+h)}{h}$

$= \lim_{h\to 0} (2x+h)$

$= 2x$

e) $x+2 \geq 0$ $\qquad 2-x \geq 0$

$x \geq -2$ $\qquad -x \geq -2$

$x \leq 2$

Domain is: $-2 \leq x \leq 2$ — 2 marks

QUESTION 2

a) $(x+y)^3 = x^3 + 3x^2y + 3xy^2 + y^3$ — 1 mark

b) $\frac{x}{x+1} \geq 5$

$(x+1)^2 \frac{x}{x+1} \geq 5(x+1)^2 \qquad x \neq -1$

$x(x+1) \geq 5(x+1)^2$

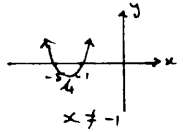
$0 \geq 5(x+1)^2 - x(x+1)$

$0 \geq (x+1)(5(x+1)-x)$

$0 \geq (x+1)(5x+5-x)$

$0 \geq (x+1)(4x+5)$ — 3 marks

$-\frac{5}{4} \leq x < -1$



$x \neq -1$

c) $y = \frac{x^2}{1-x^2}$

i) if $x=0$, $y=0$ $\therefore (0,0)$ — 1 mark

ii) $f(-x) = \frac{(-x)^2}{1-(-x)^2}$

$= \frac{x^2}{1-x^2}$

$\therefore$ even function — 1 mark

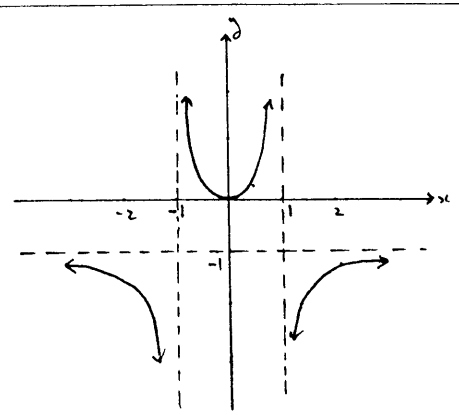
ii) $1-x^2 \neq 0$

$(1-x)(1+x) \neq 0$

vertical asymptotes: $x=1$, $x=-1$

$\lim_{x\to\infty} \frac{\frac{x^2}{x^2}}{\frac{1}{x^2}-\frac{x^2}{x^2}} = \frac{1}{0-1} = -1$

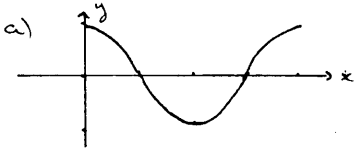
Horizontal asymptote: $y=-1$ — 2 marks



v) Range: $y \geq 0 \qquad y < -1$ — 2 marks

QUESTION 3

a)



1 mark

b) $\cos 210° = -\cos 30°$

$= -\frac{\sqrt{3}}{2}$ — 2 marks

c) $\sin(180°+\theta)\tan(90-\theta)$

$= -\sin\theta \cdot \cot\theta$

$= -\sin\theta \cdot \frac{\cos\theta}{\sin\theta}$

$= -\cos\theta$ — 2 marks

d) $(\cot^2\theta + 1)(1-\cos^2\theta)$

$= \operatorname{cosec}^2\theta \cdot \sin^2\theta$

$= \frac{1}{\sin^2\theta} \cdot \sin^2\theta$

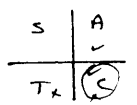
$= 1$ — 2 marks

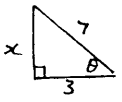
e) $\cos x = -\frac{1}{\sqrt{2}}$

$x = 180-45, \ 180+45$

$x = 135°, \ 225°$ — 2 marks

f)





4th quadrant

$x^2 = 7^2 - 3^2$

$= 40$

$x = \sqrt{40}$

$= 2\sqrt{10}$

$\therefore \tan\theta = -\frac{2\sqrt{10}}{3}$ — 2 marks

QUESTION 4

a) $k:l \qquad x_1, y_1 \qquad x_2, y_2$

$2:3 \qquad (-4,1) \qquad (5,6)$

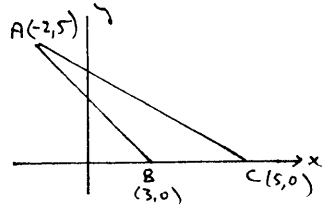
$\left(\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right)$

$= \left(\frac{2\times 5 + 3\times -4}{2+3}, \frac{2\times 6 + 3\times 1}{2+3}\right)$

$= \left(\frac{10-12}{5}, \frac{12+3}{5}\right)$

$= \left(-\frac{2}{5}, 3\right)$ — 2 marks

b)



i) $AC = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$

$= \sqrt{(5--2)^2 + (0-5)^2}$

$= \sqrt{7^2+5^2}$

$= \sqrt{74}$ — 2 marks

ii) $m = \frac{y_2 - y_1}{x_2 - x_1}$

$= \frac{0-5}{3--2}$

$= \frac{-5}{5}$

$= -1$ — 1 mark

iii) $m = \tan\theta$

$-1 = \tan\theta$

$\theta = 180 - 45$

$= 135^\circ$

$\therefore \angle ABC = 135^\circ$ — 1 mark

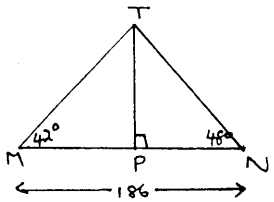
iv) $BC = 2$ units.

$\frac{\sin\angle BAC}{2} = \frac{\sin 135}{\sqrt{74}}$

$\sin\angle BAC = \frac{\sin 135^\circ}{\sqrt{74}} \times 2$

$\angle BAC = 9^\circ 28'$ — 2 marks

c)



$\angle MTP = 48^\circ$

$\angle NTP = 42^\circ$

$\therefore \angle MTN = 90^\circ$

$\cos 48^\circ = \frac{TN}{186}$

$TN = 124.458\,2928$

$\therefore$ In ΔTPN

$\sin 48 = \frac{TP}{124.458\,2928}$

$TP = 92.49$ metres — 4 marks

Marks

Marks