SYDNEY BOYS HIGH SCHOOL

MOORE PARK, SURRY HILLS

2004

YEAR 11 HALF-YEARLY EXAMINATION

Mathematics

General Instructions

- Reading Time – 5 Minutes
- Working time – One and a half hours
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators may be used.
- All necessary working should be shown in every question.

Total Marks – 72

- Attempt all questions.
- All questions are of equal value.
- Each section is to be answered in a separate booklet, labeled Section A (Questions 1, 2), Section B (Questions 3, 4) and so on.

Examiner: *A.M.Gainford*

Note: *This is an assessment task only and does not necessarily reflect the content or format of the Higher School Certificate.*

SectionA

Marks

Question 1 **6**

(a) Simplify $x(x-y)-y(y-x)$.

(b) Evaluate $\sqrt{\dfrac{3^2+12^2}{231-12^2}}$ correct to three significant figures.

(c) Express in simplest reduced form: $\left(\dfrac{2}{5}\right)^{10}\times\left(\dfrac{15}{4}\right)^{10}\times\left(\dfrac{2}{3}\right)^{9}$

Question 2 **6**

(a) Express $0\cdot\dot{7}\dot{2}$ as a common fraction in lowest terms.

(b) Simplify $2\sqrt{\dfrac{9}{4}}+\sqrt[3]{\dfrac{8}{125}}$ completely.

(c) State the value of the following, in exact terms:

(i) $\tan 60^0$

(ii) $\cos\dfrac{11\pi}{6}$

Section B

(Start a new booklet)

Question 3 **6**

(a) Solve for x: $x(x-4)=5$

(b) Express $\dfrac{1}{\sqrt{3}-2}$ with rational denominator.

(c) Find the value of x if $\sqrt{x}=\sqrt{50}-\sqrt{18}$.

Question 4 **6**

(a) Expand and simplify: $\sqrt{(a-4)(a+4)+16}, \quad a>0.$

(b) Three legs of a triangular sailing course have lengths 8 km, 10 km, and 16 km.

(i) Draw a sketch showing this information.

(ii) Calculate the size of the smallest angle, correct to the nearest minute.

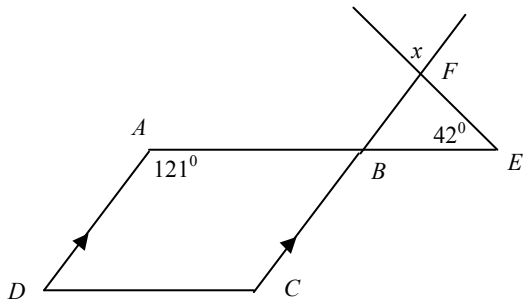
(c) On a number line sketch the solution of $4-x \geq 3.$

Section C
(Start a new booklet.)

Question 5 **6**

(a) In the diagram $AD \parallel BC$.

Copy the diagram onto your worksheet and find the value of x, giving reasons.



(b) Sketch the graph of $y=2\sin(90^0-x)$ in the domain $-90^0 \leq x \leq 360^0$.

Question 6 **6**

(a) Find the shortest distance between the parallel lines $y=2x$ and $4x-2y+7=0$.

(b) Factorise completely each of the following:

(i) $5x^2-9xy-2y^2$

(ii) $x^2+4x+4-y^2$

(c) Solve $|5-2x|=3$.

Section D

(Start a new booklet.)

Question 7 **6**

(a) Solve the following system of simultaneous equations:

$$2(x-y)=-7$$
$$6x+8y=35$$

(b) Draw a neat sketch of the graph of the function $y=\sqrt{9-x}$.

State the domain and range of this function.

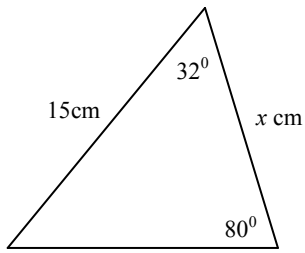
(c) Show that $f(x)=\dfrac{4^x+4^{-x}}{2}$ is an even function.

Question 8 **6**

(a) Find the least value of the quadratic expression $x^2-4x+10$, and state the x-value at which it occurs.

(b) Find the values of k for which the quadratic equation $3x^2+2x+k=0$ has no real roots.

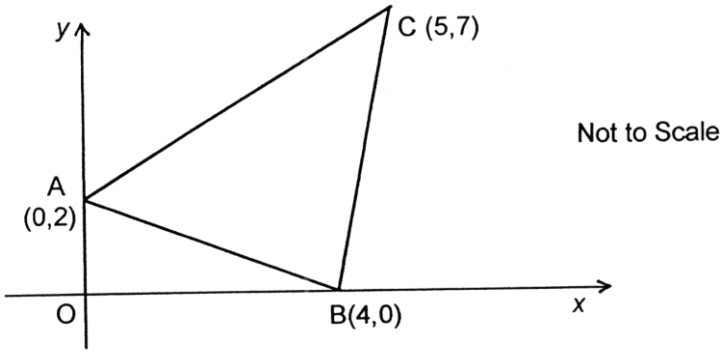
(c) Find the value of x correct to two decimal places.



Section E

(Start a new booklet.)

Question 9 **6**



The diagram shows the points $A(0, 2)$, $B(4, 0)$, and $C(5, 7)$.

Copy the diagram onto your work sheet.

(a) Find the co-ordinates of M, the mid-point of AB.

(b) Show that the gradient of AB is $-\frac{1}{2}$.

(c) Find the equation of the perpendicular bisector of AB.

(d) Show that the perpendicular bisector of AB passes through C.

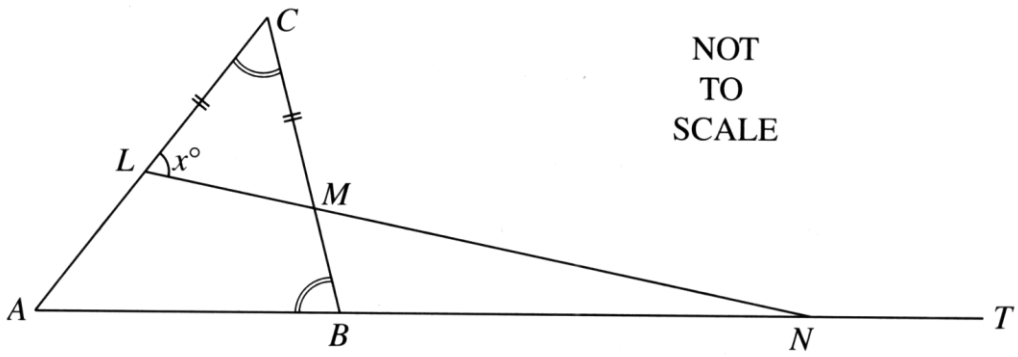
(e) What type of triangle is ABC ? (Give a reason for your answer.)

Question 10 **6**

(a) Sketch on the number plane the region in which all three of the following inequalities are satisfied:

$$x - y + 2 > 0$$
$$2x + y \geq -2$$
$$x - 3 \leq 0$$

(b)



In the diagram ABC is an isosceles triangle with $\angle ABC = \angle ACB$. The line LMN is drawn as shown so that $CL=CM$, and $\angle CLM = x^0$.

Copy or trace the diagram to your booklet.

(i) Show that $\angle ABC = (180 - 2x)^0$.

(ii) Hence show that $\angle TNL = 3x^0$.

This is the end of the paper.

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YEAR 11

HALF YEARLY EXAMINATION

Mathematics

Sample Solutions

Section	Marker
A	Mr Fuller
B	Mr Dowdell
C	Mr Boros
D	Ms Nesbitt
E	Ms Opferkuch

Section A

Question 1

(a) $x(x-y)-y(y-x)$

$= x^2 - xy - y^2 + xy$

$= x^2 - y^2$

(b) $\sqrt{\frac{3^2+12^2}{231-12^2}}$

$= \sqrt{\frac{9+144}{231-144}}$

$= \sqrt{\frac{153}{87}}$

$= 1.33$

(c) $\left(\frac{2}{5}\right)^{10} \times \left(\frac{15}{4}\right)^{10} \times \left(\frac{2}{3}\right)^{9}$

$= \left(\frac{3}{2}\right)^{10} \times \left(\frac{2}{3}\right)^{9}$

$= \left(\frac{3}{2}\right)^{9} \times \left(\frac{2}{3}\right)^{9} \times \left(\frac{3}{2}\right)$

$= \frac{3}{2}$

Question 2

(a) let $x = 0.727272...$

$100x = 72.727272....$

$99x = 72$

$x = \frac{72}{99}$

$x = \frac{8}{11}$

(b) $2\sqrt{\frac{9}{4}} + 3\sqrt{\frac{8}{125}}$

$= 2 \times \frac{3}{2} + \frac{2}{5}$

$= 3 + \frac{2}{5}$

$= 3\frac{2}{5}$

(c) (i) $\tan 60^\circ = \sqrt{3}$

(ii) $\cos \frac{11\pi}{6} = \cos \frac{\pi}{6}$

$= \frac{\sqrt{3}}{2}$

Q3 (a) $x(x-4) = 5$

$x^2 - 4x - 5 = 0$

$(x-5)(x+1) = 0$

$x = 5$ or $x = -1$ (2)

(b) $\frac{1}{\sqrt{3}-2} \times \frac{\sqrt{3}+2}{\sqrt{3}+2}$

$= \frac{\sqrt{3}+2}{3-4}$

$= -(\sqrt{3}+2)$ (2)

(c) $\sqrt{x} = \sqrt{50} - \sqrt{18}$

$= 5\sqrt{2} - 3\sqrt{2}$

$= 2\sqrt{2}$

$\therefore x = 8$ (2)

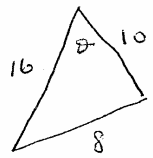
4 (a) $\sqrt{(a-4)(a+4)+16}$

$= \sqrt{a^2 - 16 + 16}$

$= \sqrt{a^2}$

$= a$ (as $a > 0$) (2)

(b)



$\cos\theta = \frac{16^2 + 10^2 - 8^2}{2 \times 16 \times 10}$

$= \frac{292}{320}$

$= \frac{73}{80}$

$\therefore \theta = 24.146\ldots$

$\approx 24° 9'$ (2)

(c) $4 - x \geq 3$

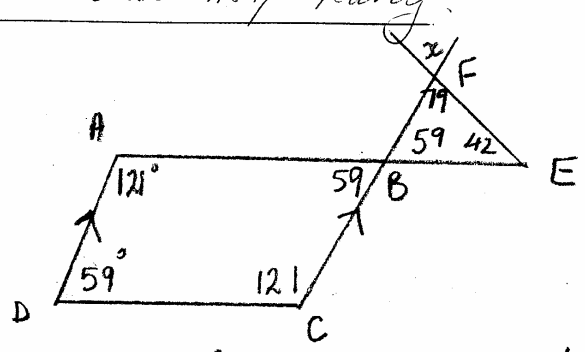
$-x \geq -1$

$x \leq 1$

(2)

Section C

(5) (a)



(1)

$A\hat{B}C = 180 - 121 = 59^\circ$ co interior angles

$A\hat{B}C = F\hat{B}E = 59^\circ$ vert. opp.

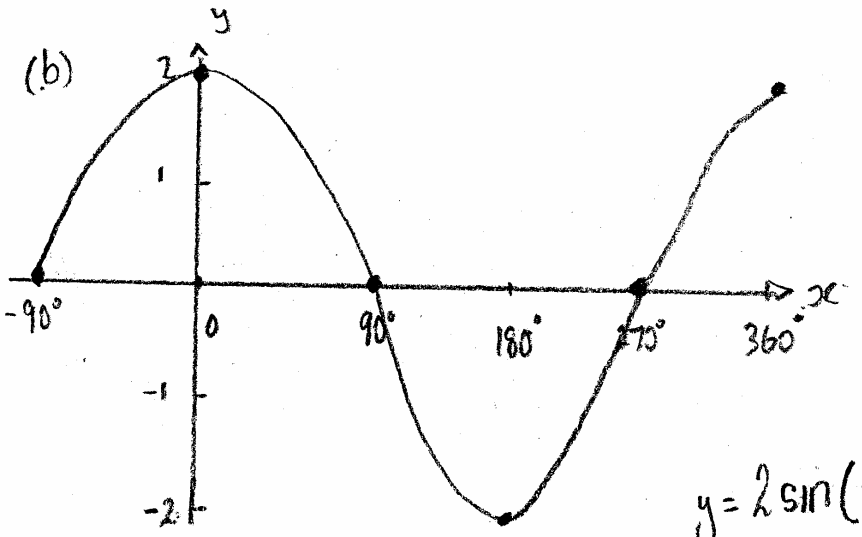
$B\hat{F}E = 180 - (59 + 42)$

$= 79^\circ$

$\therefore x = B\hat{F}E = 79^\circ$ vert. opp.

(2)

(b)



(3)

$y = 2\sin(90 - x)$

$= 2\cos x$

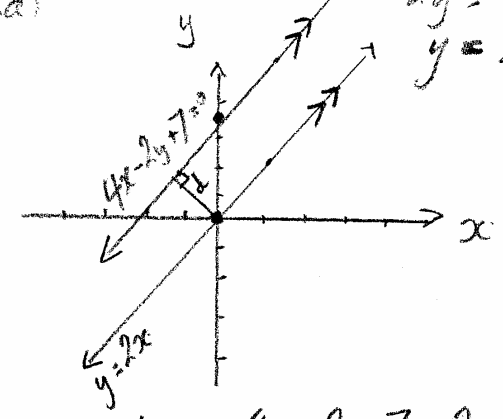
since $\sin(90 - x) = \cos x$

(6) /6

(a) $y = 2x$ and $4x - 2y + 7 = 0$

$2y = 4x + 7$

$y = 2x + 3\frac{1}{2}$



Use $(0,0)$ on $4x - 2y + 7 = 0$

$$d = \left| \frac{7}{\sqrt{16+4}} \right|$$

$$= \frac{7}{\sqrt{20}} = \frac{7}{2\sqrt{5}} \text{ or } (\div 1.56)$$

$$\frac{7}{2\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{7\sqrt{5}}{10} \text{ units} \quad (2)$$

(b) (i) $5x^2 - 9xy - 2y^2$

$= (5x + y)(x - 2y)$ (1)

(ii) $x^2 + 4x + 4 - y^2$

$= (x+2)^2 - y$

$= (x + 2 - y)(x + 2 + y)$ (2)

(c) $|5 - 2x| = 3$

$5 - 2x = 3$ $\quad$ $5 - 2x = -3$

$-2x = -2$ $\quad$ $-2x = -8$

$x = 1$ $\quad$ $x = 4$

(1)

Q7(a) $2x-2y=-7$ (1)

$6x+8y=35$ (2)

$8x-8y=-28$ (1)×4

$14x = 7$

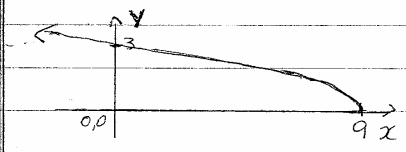
$x = \frac{1}{2}$

$2\times\frac{1}{2}-2y=-7$ (1)

$-2y=-8$

$y=4 \quad x=\frac{1}{2}$ (3)

(b)



Domain: $x \leq 9$

Range: $y \geq 0$ (2)

(c) $f(-x) = \frac{4^{-x}+4^{x}}{2} = f(x)$

$\therefore f(x)$ is an even function (1)

Q8(a) least value at centre axis

$x = -\frac{b}{2a}$

$x = 2$

least value $= 6$

OT

$x^2-4x+10 = (x-2)^2+6$

vertex $(2,6)$

least value $= 6$ when $x=2$ (1)

(b) No real root when $\Delta<0$

$b^2-4ac<0$

$2^2-4\times3\times k<0$

$-12k < -4$

$k > \frac{1}{3}$ (2)

(c) Third angle $= 180-80-32$

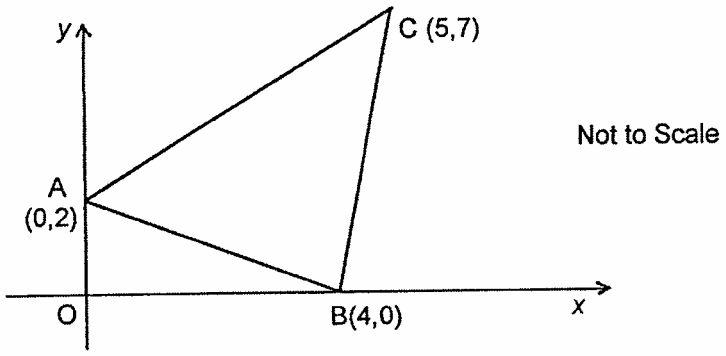
$= 68°$

$\frac{x}{\sin 68} = \frac{15}{\sin 80}$

$x = 15\,\frac{\sin 68}{\sin 80}$

$x = 14.12$ (2dp) (3)

Question 9



(a) $\text{M}: x=\frac{4+0}{2}, y=\frac{2+0}{2}$ (1)

$=2 \qquad =1$

$\therefore$ M (2,1)

(b) $m_{AB}=\frac{0-2}{4-0}$ (1)

$=-\frac{1}{2}$

$\therefore m_{AB}=-\frac{1}{2}$

(c) $m_1 \times m_2 = -1$ (1)

$-\frac{1}{2}\times m_2 = -1$

$m_2 = 2$

Using $y-y_1=m(x-x_1)$, where $x=2, y=1, m_2=2$

$y-1=2(x-2)$

$\therefore 2x-y-3=0$

(d) Using $2x-y-3=0$, where $x=5, y=7$ (1)

$2\times5-7-3=0$

$\therefore LHS = RHS$

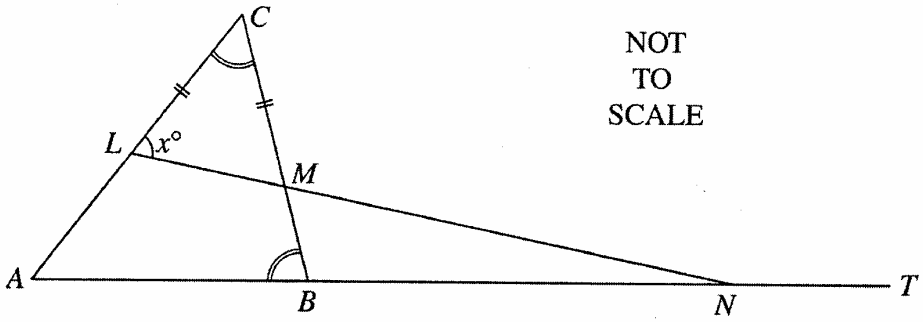
(e) In ΔABC (2)

$d_{AC} = d_{BC} = \sqrt{50}$

$d_{AB} = \sqrt{20}$

$\therefore \Delta ABC$ is isosceles

Question 10



(b) (i) In ΔCLM (1)

$\angle CLM = x^0$ (data)

$\angle CML = x^0$ (base $\angle$, isos. Δ)

$\therefore \angle ACB = 180 - 2x$ ($\angle$ sum of Δ)

In ΔABC

$\angle ACB = \angle ABC$ (data)

$\therefore \angle ABC = 180 - 2x$

(ii) In ΔMBN (2)

$\angle BMN = \angle CML$ (vert. opp. $\angle$s)

$= x^0$

$\angle MBN = 180 - \angle ABC$ (supp. $\angle$s)

$= 180 - (180 - 2x)$

$= 2x$

$\therefore \angle TNL = \angle BMN + \angle MBN$ (ext. $\angle$ = sum of two opp. int. $\angle s$)

$= x + 2x$

$= 3x$

(a)