SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2006
YEAR 11 MATHEMATICS
HALF YEARLY EXAM

Mathematics (2 unit)

General Instructions

- Reading Time – 5 Minutes
- Working time – 90 Minutes
- Write using black or blue pen. Pencil may be used for diagrams.
- Board approved calculators maybe used.
- Marks may **NOT** be awarded for messy or badly arranged work.
- All necessary working should be shown in every question.

Total Marks – 65

- Attempt questions 1-6

Examiner: *P.Bigelow*

Question 1 (10 Marks) Marks

(a) If $f(x) = 1 + x + x^2$, find $f(2)$. [1]

(b) Expand and simplify $(x+4)(x-6)$ [2]

(c) Express $\frac{1}{\sqrt{2}}$ with a rational denominator [1]

(d) Factorise [3]

(i) $a^2 + 3a + 2$

(ii) $49 - y^2$

(iii) $x^3 + y^3$

(e) Solve $7y - 4 = 2y + 11$ [1]

(f) Evaluate $\frac{7.93}{8.22 - 3.47}$ correct to 3 significant figures. [1]

(g) If $\sqrt{45} + \sqrt{80} = a\sqrt{5}$, find a. [1]

Question 2 (10 Marks)

(a) Write down the domain of the function $f(x)=\sqrt{1-x}$. [1]

(b) Sketch the line $2x-y-4=0$. [1]

(c) Sketch the parabola $y=(x+1)(x-3)$. [1]

(d) $y=\sqrt{4-x^2}$ is the equation of a semi-circle. [3]

(i) Sketch the semi-circle

(ii) State the domain and range of the function.

(e) Classify the functions as ODD, EVEN or NEITHER. [3]

(i) $f(x)=x$

(ii) $f(x)=x^2-1$

(iii) $f(x)=\dfrac{1}{1+x^2}$

(f) Solve $|x+1|=2$ [1]

Question 3 (11 marks)

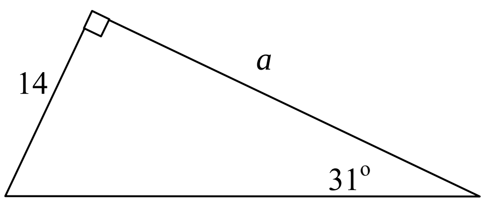
(a) Solve the following pair of equations simultaneously [2]

$$x - 2y = 4$$
$$2x + y = 3$$

(b) Find the equation of the line parallel to $x + y - 1 = 0$ and passing through $(4,-6)$. [2]

(c) Find the angle sum of a regular 10 sided polygon. [1]

(d) Find the value of a (correct to 1 decimal place). [2]



(e) Find the value of θ (correct to the nearest minute) where $\sin\theta = \frac{4}{7}$. [2]

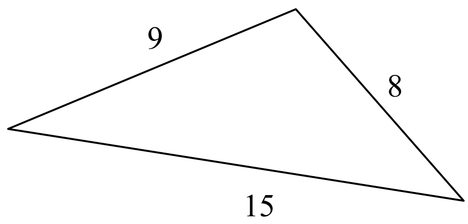
(f) Find the exact value of [2]

(i) $\sin 60°$

(ii) $\tan 300°$

Question 4 (10 Marks)

(a) Find the largest angle in the triangle (correct to the nearest minute). [2]



(b) Sketch $y = \cos x$ for $0° \le x \le 360°$ [2]

(c) Show that (–4, –5), (2, 7) and (5, 13) are collinear. [2]

(d) Find the shortest distance between the line $4x - 3y + 15 = 0$ and the origin. [2]

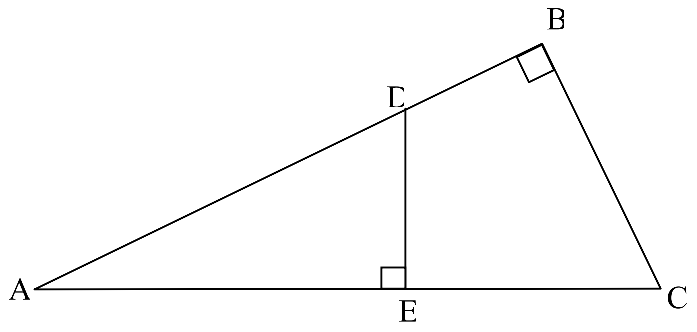
(e) Find, without a calculator, $0.\dot{1}\dot{5}$ as a fraction in simplest form. [2]

Question 5 (12 Marks)

(a) Write $\sqrt[3]{x^4}$ in index form. [1]

(b) Write down the coordinates of the focus and equation of the directrix for the parabola $x^2 = 20y$. [2]

(c) Show that ΔABC is similar to ΔAED. [2]



(d) Given that α and β are roots of the equation $x^2 - 3x + 1 = 0$, find [4]

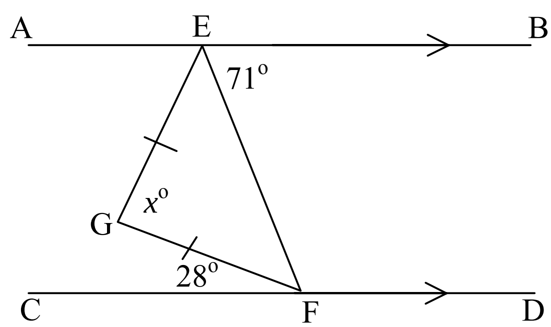
(i) $\alpha + \beta$

(ii) $\alpha\beta$

(iii) $\frac{1}{\alpha} + \frac{1}{\beta}$

(iv) $\alpha^2 + \beta^2$

(e) AB is parallel to CD. GE=GF. Find the value of x. [3]



Question 6 (12 Marks)

(a) Find the centre and radius of the circle [2]

$$x^2 - 2x + y^2 + 4y - 4 = 0$$

(b) Find the equation of the line, passing through the intersection of the lines $2x - y = 1 = 0$ and $3x + y - 6 = 0$ and containing the point (–2, 3). [3]

(c) A parabola $y = ax^2 + bx + c$ passes through A(–1, 4), B(0, 7) and C(1, 8). Determine the values of a, b and c. [2]

(d) Given the quadratic expression [2]

$$x^2 + (k - 3)x + k\ .$$

For what values of k, is the expression positive for all values of x?

(e) In ΔABC, $\hat{A} = 38°21'$, $b = 11.6\text{m}$ and $a = 7.9\text{m}$. Find the size of the angle B (correct to the nearest degree). [3]

END OF PAPER

2006 Mathematics Continuers: **Solutions Part A**

Question 1 (10 Marks)

(a) Evaluate $8^{2\cdot 1}$ to 4 significant figures, [2]

Solution: By calculator, $78{\cdot}79324245 \approx 78{\cdot}79$ to 4 sig. fig.

(b) Write $0{\cdot}\dot{1}\dot{3}$ as a simplified fraction. [1]

Solution:
$$\begin{aligned} \text{Let } x &= 0{\cdot}\dot{1}\dot{3}, \\ 100x &= 13{\cdot}\dot{1}\dot{3}, \\ 99x &= 13, \\ \therefore\ x &= \frac{13}{99}. \end{aligned}$$

(c) Simplify

(i) $3x - (4 - x)$ [1]

Solution: $3x - 4 + x = 4x - 4$

(ii) $\dfrac{x+1}{3} + \dfrac{2x}{5}$ [1]

Solution: $\dfrac{5x + 5 + 6x}{15} = \dfrac{11x + 5}{15}$

(d) Convert 270° to radians in exact form. [1]

Solution: $270^\circ \times \dfrac{\pi}{180^\circ} = \dfrac{3\pi}{2}$

(e) Factorise

(i) $x^2 - 9$ [1]

Solution: $(x+3)(x-3)$

(ii) $64 + x^3$ [1]

Solution: $4^3 + x^3 = (4 + x)(16 - 4x + x^2)$

(f) Given that

$$f(x) = \begin{cases} 6 - x^2 & \text{if } x \geq 0, \\ |x| & \text{if } x < 0, \end{cases}$$

evaluate

(i) $f(-2)$ [1]

Solution: $|-2| = 2$

(ii) $f(0)$ [1]

Solution: $6 - 0^2 = 6$

Question 2 (11 Marks)

(a) Solve $|2x + 6| = 10$. [2]

Solution:

$$\begin{aligned} 2x + 6 &= 10, \\ 2x &= 4, \\ \therefore\ x &= 2. \end{aligned} \quad \text{or} \quad \begin{aligned} -2x - 6 &= 10, \\ -2x &= 16, \\ x &= -8. \end{aligned}$$

(b) Simplify $\dfrac{1}{\sec^2\theta} + \dfrac{1}{\operatorname{cosec}^2\theta}$ [2]

Solution: $\cos^2\theta + \sin^2\theta = 1$

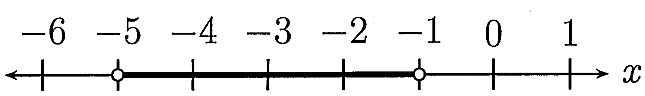
(c) (i) Solve the inequation $|x + 3| < 2$. [2]

Solution:

$$\begin{aligned} -2 < {} & x + 3 < 2, \\ -5 < {} & x < -1. \end{aligned}$$

(ii) Hence graph the solution on a number line. [1]

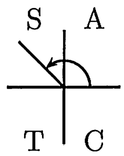
Solution:



(d) Find the equation of the circle with centre $(-4, 6)$ and radius $\sqrt{5}$. [2]

Solution: $(x + 4)^2 + (y - 6)^2 = 5$

(e) Find the exact value of $\tan \dfrac{3\pi}{4}$. [2]

Solution: 

$$-\tan \frac{\pi}{4} = -1$$

Question 3 (9 Marks)

(a) (i) Write down the expansion for $\sin(A + B)$. [1]

Solution: $\sin A \cos B + \cos A \sin B$

(ii) Hence find the exact value of $\sin 75°$. [2]

Solution:
$$\begin{aligned}\sin(45° + 30°) &= \sin 45° \cos 30° + \cos 45° \sin 30°, \\ &= \frac{1}{\sqrt{2}} \times \frac{\sqrt{3}}{2} + \frac{1}{\sqrt{2}} \times \frac{1}{2}, \\ &= \frac{1 + \sqrt{3}}{2\sqrt{2}} \text{ or } \frac{\sqrt{2} + \sqrt{6}}{4}.\end{aligned}$$

(b) If α and β are the roots of $2x^2 + 3x + 4 = 0$, find the value of

(i) $\alpha\beta$ [1]

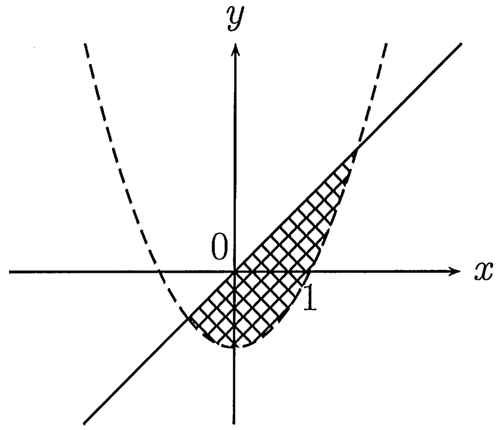
Solution: $\dfrac{4}{2} = 2$

(ii) $\dfrac{1}{\alpha} + \dfrac{1}{\beta}$ [2]

Solution:
$$\begin{aligned}\frac{\alpha + \beta}{\alpha\beta} &= \frac{-3}{2} \times \frac{1}{2}, \\ &= -\frac{3}{4}.\end{aligned}$$

(c) Sketch the intersection of the regions $y > x^2 - 1$ and $y \leq x$. [3]

Solution:



Question 4 (11 Marks)

(a) State the domain and range of $g(x) = \sqrt{x+4}$. [2]

Solution: Domain: $x \geq -4$,
Range: $g(x) \geq 0$.

(b) Show that $f(x) = x^3 + 3x$ is an odd function. [2]

Solution:
$$\begin{aligned} f(-x) &= (-x)^3 + 3(-x), \\ &= -x^3 - 3x, \\ &= -(x^3 + 3x), \\ &= -f(x). \end{aligned}$$
$\therefore\ f(x)$ is odd.

(c) Find correct to the nearest minute the angle between $y = 6x - 7$ and $y = x + 3$. [3]

Solution: $m_1 = 6$, $m_2 = 1$.
$$\begin{aligned} \tan\alpha &= \left|\frac{6-1}{1+6\times 1}\right|, \\ &= \frac{5}{7}. \end{aligned}$$
$\therefore\ \alpha = 35°32'$.

(d) Find the values of A and B if $2(x-1)^2 \equiv A(x^2+1)+Bx$. [2]

Solution: Let $x = 0$, $2 = A$.
Let $x = 1$, $0 = 4 + B$,
$B = -4$.

(e) For $A(5,\ 1)$ and $B(-3,\ 7)$ find the coördinates of the point that divides the interval AB internally in the ratio $3:1$. [2]

Solution: $\left(\dfrac{3\times(-3)+1\times 5}{3+1},\ \dfrac{3\times 7+1\times 1}{4}\right) = (-1,\ 5\tfrac{1}{2})$

7II CONTINUERS

SECTION B

5 (a) $x^2 - 6x + y^2 + 10y + 9 = 0$

$(x-3)^2 + (y+5)^2 = -9 + 9 + 25$

$= 25$.

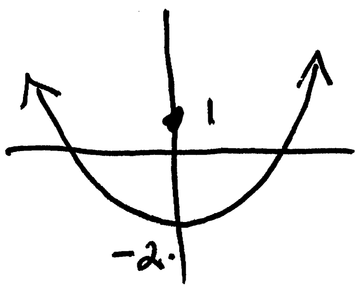
(i) The circle doesn't pass the vertical line test for a function.

OR. There are points (ie two) where the x-value is the same.

OR. It is not a 1-1 correspondence or mapping

(ii) Centre $(3,-5)$ radius 5.

(b)



NB $a = 3$.

$\therefore (x-0)^2 = 4 \times 3 (y+2)$

$x^2 = 12(y+2)$

(c) $\frac{x-3}{x} < 0$.

$x(x-3) < 0$ (ie multiply both sides by x^2)

$\therefore 0 < x < 3$

(d) $x^4 - 10x^2 + 9 = 0$ let $u = x^2$

$u^2 - 10u + 9 = 0$

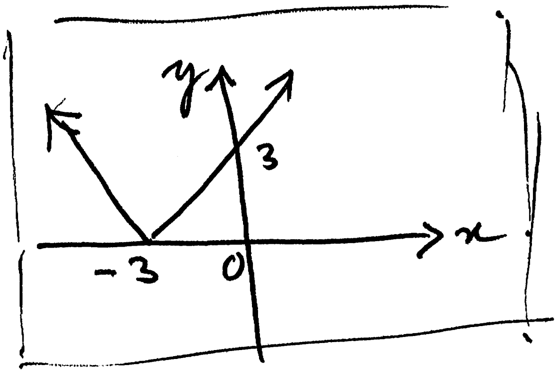
$(u-9)(u-1) = 0$

$u = 9, 1$

ie $x^2 = 9, 1$

$x = \pm 3, \pm 1$

6 (a)



✓✓

(b) $y = -a(x-0)(x-4)$

$\therefore y = -ax(x-4)$

now $(2, +4)$ lies on it.

$\therefore +4 = -2a \cdot -2$

$\therefore \underline{a = 1}$

$\therefore \boxed{y = -x(x-4)}$ ✓✓

(c) For real roots $\Delta \geq 0$

$\therefore p^2 - 64 \geq 0$

$(p-8)(p+8) \geq 0$

$\therefore \boxed{p \geq 8,\ p \leq -8}$ ✓✓

(d) $\tan\theta = \sqrt{3}$, $0 < \theta < \frac{\pi}{2}$

$\therefore \theta = \frac{\pi}{3}$

$\therefore \boxed{\cos 2\theta = \cos\frac{2\pi}{3} = -\frac{1}{2}}$ ✓✓✓

Q7

(a) $4x-3y-8=0$ and point $(-2,3)$

$$d = \left| \frac{4\times -2 - 3\times 3 - 8}{\sqrt{4^2+(-3)^2}} \right|$$

$$= \left| \frac{-8-9-8}{5} \right|$$

$$= \left| \frac{-25}{5} \right|$$

$$= 5$$

✓ ✓

(b) Find the intersection of

$2x - y = 3 \quad — ①$

$3x - y = -2 \quad — ②$

② − ①

$x = -5$

Sub in ①

$-10 - y = 3$

$y = -13$

$\therefore (-5, -13)$

∴ Equation of line passing through $(3,-1)$ and $(-5,-13)$

$$\frac{y+1}{x-3} = \frac{-13+1}{-5-3} = \frac{-12}{-8}$$

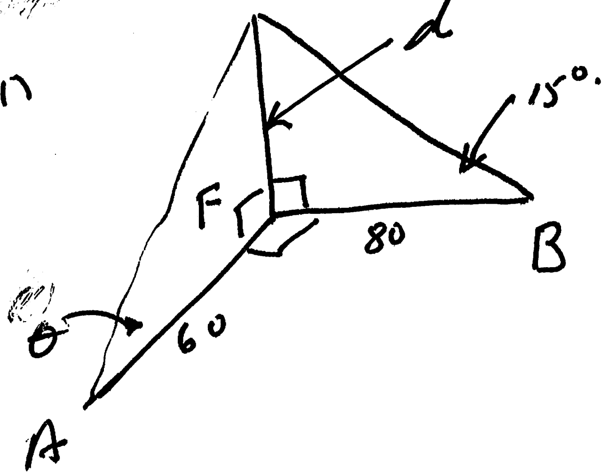
$$= \frac{3}{2}$$

$\therefore 2y+2 = 3x-9$

$3x - 2y - 11 = 0$

✓ ✓ ✓

(c) (i)



(ii) $\frac{d}{80} = \tan 15^\circ$

$\therefore d = 80\tan 15^\circ$

now $\tan\theta = \frac{d}{60}$

$= \frac{80\tan 15^\circ}{60}$

$= 0.3573$

$\therefore \theta = 19^\circ 40'$ (OR 20°)

(d) $2x + y - z = -3$ —— ①

$-x + 3y - 2z = 1$ —— ②

$x - y + 5z = 12$ —— ③

① + ③

$3x + 4z = 9$ —— ④

③ × 3

$3x - 3y + 15z = 36$ (3a)

(3a) + ②

$2x + 13z = 37$ ⑤

④ × 2

$6x + 8z = 18$ —— (4a)

⑤ × −3

$-6x - 39z = -111$ (5a)

(4a) + (5a)

$-31z = -93$

$z = 3$

Sub in ④

$3x + 12 = 9$

$3x = -3$

$x = -1$

Sub in ①

$-2 + y - 3 = -3$

$y = 2$

$\therefore (-1, 2, 3)$

Q8

(a) $x^2 + Lx + M = 0$ has roots $\alpha, 2\alpha$.

(i) now $\alpha + 2\alpha = -L$ & $\alpha \times 2\alpha = M$

$\therefore 3\alpha = -L$ — ① $2\alpha^2 = M$ — ②

$\alpha = -\frac{L}{3}$

From ① & ②

$2 \times \left(\frac{-L}{3}\right)^2 = M$

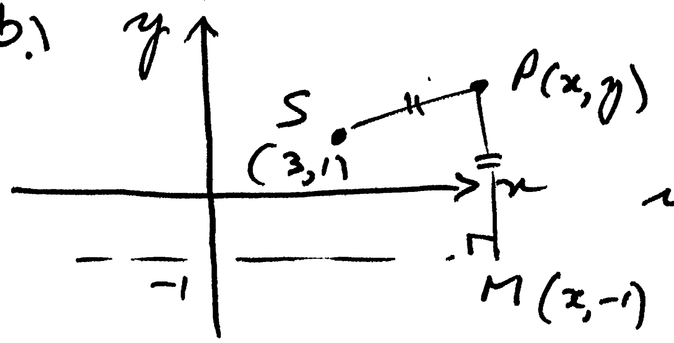
$\frac{2L^2}{9} = M$

$\boxed{2\alpha^2 = 9M}$

(ii) If L is rational then $-\frac{L}{3} = \alpha$ is rational

& hence $2\alpha = -\frac{2L}{3}$ is also rational

(b)



now $SP = PM$

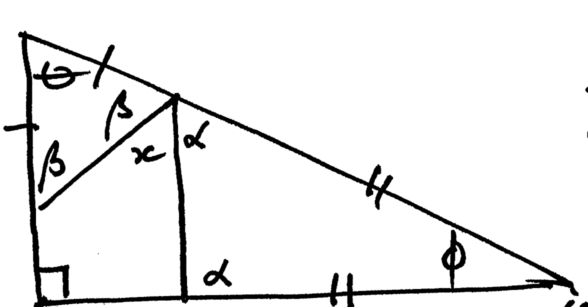
ie $\sqrt{(x-3)^2 + (y-1)^2} = y+1$

$(x-3)^2 + (y-1)^2 = (y+1)^2$

$x^2 - 6x + 9 + y^2 - 2y + 1 = y^2 + 2y + 1$

$\boxed{(x-3)^2 = 4y}$

(c)



now

$\alpha + \beta + x = 180°$ — ①

$\theta + \phi = 90°$

ie $(180 - 2\beta) + (180 - 2\alpha) = 90°$

ie $2\alpha + 2\beta = 270°$

$\alpha + \beta = 135°$ — ②

From ① & ② $\boxed{x = 45°}$