SYDNEY BOYS HIGH SCHOOL
MOORE PARK, SURRY HILLS

2008

YEAR 11
Mathematics (2U)
Half Yearly Examination

Mathematics

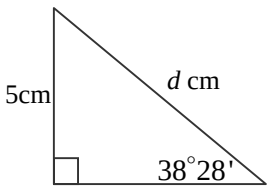
General Instructions

- Write using black or blue pen
- Board approved calculators may be used.
- All necessary working should be shown in every question if full marks are to be awarded.
- Marks may not be awarded for messy or badly arranged work.

Reading time: 5 minutes
Working time: 90 minutes
Total Marks – 70

- Attempt All Questions.
- All questions are NOT of equal value.

Examiner: *A. Ward*

Number	Question	Marks
1.	Find 0.009738148 correct to three significant figures.	1
2.	Write down the supplement of $126^\circ 52'$.	1
3.	Simplify $-5p^4 + 3p^2 + 2p^4$.	1
4.	Simplify $2\frac{1}{2} \times 3\frac{7}{8} \div 5\frac{5}{9}$.	1
5.	Simplify $\sqrt{27} + \sqrt{48}$.	1
6.	Expand and simplify $(2x-3)(3x+4)$.	1
7.	Solve $\frac{6t-1}{t} = 4$.	1
8.	Factorise $x^2 - 13x + 30$.	1
9.	Calculate: $\frac{\sqrt{4.7-3.2}}{9.4-1.6}$ correct to 3 decimal places.	1
10.	Express $0.\dot{2}\dot{1}$ as a rational number.	1
11.	Solve $x^2 - 4x = 0$.	1
12.	Find the value of d to 3 decimal places. 	1
13.	Write down the exact value of $\cos 210^\circ$.	1
14.	Solve $(x-4)^2 = 17$. Leave your answer in surd form.	1
15.	Express $\frac{7\pi}{15}$ radians in degrees.	1
16.	Simplify $\frac{5x}{x^2-4} - \frac{x+2}{10x^2}$.	2
17.	Express in $\sqrt{1452}$ the form $a\sqrt{b}$ where $\sqrt{b}$ is irrational.	2
18.	Find c in terms of a, b and d: $$d = \sqrt{b^2 - 4ac}$$	2

19. Factorise fully:

a) $rs-15+3s-5r$ 2

b) $2x^2+x-21$ 2

c) x^4-a^4 2

d) x^3-a^3 2

20. Solve $2x^2+8x+16=0$ by completing the square. 2

21. If $\cos\theta=\dfrac{-7}{25}$ and $90^\circ\le\theta\le180^\circ$, find $\tan\theta$. 2

22. Solve $3(2+y)-4(2y-7)\le 9$. 2

23. Express with a rational denominator:

$$\frac{2\sqrt{5}+1}{2\sqrt{5}-1}$$ 2

24. Solve: $|2x-5|=7$ 2

25. Simplify $(x+a)^2+(x-a)^2$ 2

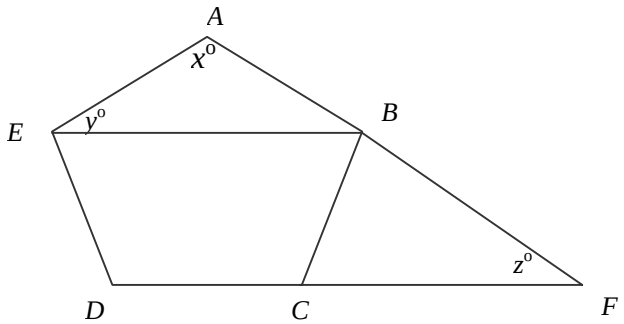
26. Solve $x+\dfrac{2}{x}=-3$. 2

27. Solve the simultaneous equations:

$$x+y+z=7$$
$$2x+3y-z=0$$
$$3x+4y+2z=17$$ 3

28. A triangle PQR whose area is 3cm^2 has sides $PQ=4\,\text{cm}$ and $QR=\sqrt{3}\,\text{cm}$. Find $\angle PQR$. 3

29. ABCDE is a regular pentagon, with AB and DC produced to F. Find the values of *x*, *y* and *z*. (Do not give reasons)



3

30. In triangle *RPT*, $r=15$, $t=8$ and $\angle RPT=2$ radians. Find side *p* to 2 decimal places. 3

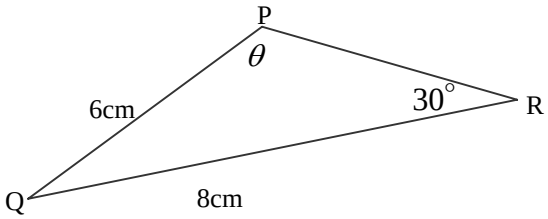
31. Solve the inequality:

$$|x+3| \leq 2x-2$$

3

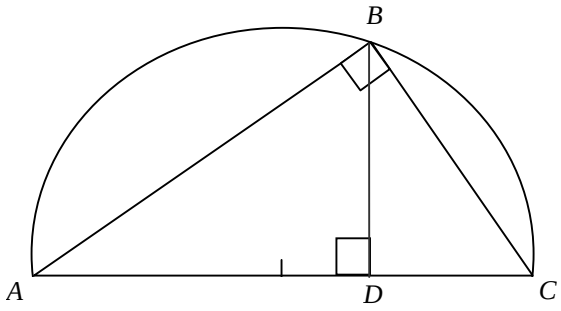
32. If $p = \dfrac{\cos\theta}{2}$ and $q = \sin^3\theta$, find $\sin^2\theta + \cos^2\theta$ in terms of p and q. 3

33. In triangle PQR, the obtuse angle $\angle QPR = \theta$. The angle $\angle PRQ = 30^\circ$ and the lengths PQ and QR are 6cm and 8cm respectively. Find the value of $\cos\theta$ correct to 3 significant figures.



4

34.



In the diagram AC is the diameter of a circle radius r and BD is perpendicular to AC.

(a) Prove that triangles ABC, ADB and BDC are similar. 2

(b) Given that $BC = \dfrac{1}{2}r$ show that $AD:DC = 15:1$ 3

END OF EXAMINATION

Year 11 2 unit ½ yearly 2008

1. 0.009738148 3 s.f

$= 0.00974$ (1)

2. Supp of $126°52'$

$= 180° - 126°52'$

$= 53°8'$ (1)

3. $-5p^4 + 3p^2 + 2p^4$

$= 3p^2 - 3p^4$ (1)

4. $2\frac{1}{2} \times 3\frac{7}{8} \div 5\frac{5}{9}$

$= 1\frac{119}{160}$ OR $\frac{279}{160}$ (1)

5. $\sqrt{27} + \sqrt{48}$

$= \sqrt{9}\cdot\sqrt{3} + \sqrt{16}\cdot\sqrt{3}$

$= 3\sqrt{3} + 4\sqrt{3}$

$= 7\sqrt{3}$ (1)

6. $(2x-3)(3x+4)$

$= 6x^2 + 8x - 9x - 12$

$= 6x^2 - x - 12$ (1)

7. $\frac{6t-1}{t} = 4$

$6t - 1 = 4t$

$2t = 1$

$t = \frac{1}{2}$ (1)

8. $x^2 - 13x + 30$

$= (x-3)(x-10)$ (1)

9. $\frac{\sqrt{4.7} - 3.2}{9.4 - 1.6}$

$= 0.157018573$

$= 0.157$ 3dp (1)

10. $x = 0.\dot{2}\dot{1}$

$100x = 21.\dot{2}\dot{1}$

$99x = 21$

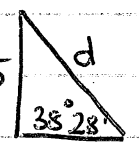
$x = \frac{21}{99} = \frac{7}{33}$ (1)

11. $x^2 - 4x = 0$

$x(x-4) = 0$

$x = 0$, $x - 4 = 0$

(½) $x = 4$ (½)

12. 

$\frac{d}{\sin 90} = \frac{5}{\sin 38°28'}$

$d = 8.037819828$

$= 8.038$ 3dp (1)

13. $\cos 210° = -\sqrt{3}/2$ (1)

14. $(x-4)^2 = 17$

$x - 4 = \pm\sqrt{17}$

$x = 4 \pm \sqrt{17}$ (1)

15. $\frac{7\pi}{15}$ in degrees

$\frac{7\pi}{15} \times \frac{180}{\pi} = 84°$ (1)

16. $\frac{5x}{x^2-4} - \frac{x+2}{10x^2}$

$= \frac{5x}{(x+2)(x-2)} - \frac{x+2}{10x^2}$

$= \frac{50x^3 - (x+2)^2(x+2)}{10x^2(x+2)(x-2)}$ ①

$= \frac{50x^3 - (x^2+4x+4)(x-2)}{10x^2(x+2)(x-2)}$

$= \frac{50x^3 - (x^3-2x^2+4x^2-8x+4x-8)}{10x^2(x+2)(x-2)}$

$= \frac{50x^3 - x^3 - 2x^2 + 4x + 8}{10x^2(x+2)(x-2)}$

$= \frac{49x^3 - 2x^2 + 4x + 8}{10x^2(x+2)(x-2)}$ ①

17. $\sqrt{1452} = a\sqrt{b}$

$\sqrt{4} \cdot \sqrt{121} \cdot \sqrt{3} = a\sqrt{b}$

$2 \cdot 11 \cdot \sqrt{3} = a\sqrt{b}$

$22\sqrt{3} = a\sqrt{b}$

$\therefore a = 22$ ① $\quad b = 3$ ①

18. $d = \sqrt{b^2 - 4ac}$

$d^2 = b^2 - 4ac$ ①

$4ac = b^2 - d^2$

$c = \frac{b^2 - d^2}{4a}$ ①

19. a) $rs - 15 + 3s - 5r$

$s(r+3) - 5(r+3)$ ①

$= (r+3)(s-5)$ ①

b) $2x^2 + x - 21$

$\frac{(2x+7)(2x-6)}{2}$

$= \frac{2(x-3)(2x+7)}{2}$

$= (x-3)(2x+7)$ ②

c) $x^4 - a^4$

$= (x^2-a^2)(x^2+a^2)$ ①

$= (x^2+a^2)(x+a)(x-a)$ ①

d) $x^3 - a^3$

$= (x-a)(x^2+ax+a^2)$ ②

20. $2x^2 + 8x + 16 = 0$

$x^2 + 4x + 8 = 0$

(complete the square)

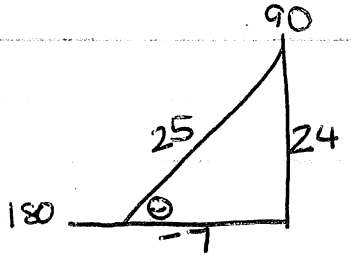
$x^2 + 4x + 4 = -8 + 4$

$(x+2)^2 = -4$

$x + 2 = \sqrt{-4}$

$\therefore$ no solutions ②

21. $\cos\theta = -7/25$



①

$\tan\theta = \frac{-24}{7}$ ①

22. $3(2+y) - 4(2y-7) \leq 9$

$6 + 3y - 8y + 28 \leq 9$ ①

$34 - 5y \leq 9$

$-5y \leq -25$

$y \geq 5$ ①

23. $\frac{2\sqrt{5}+1}{2\sqrt{5}-1} \times \frac{2\sqrt{5}+1}{2\sqrt{5}+1}$ ①

$= \frac{20 + 4\sqrt{5} + 1}{20 - 1}$

$= \frac{21 + 4\sqrt{5}}{19}$ ①

24. $|2x - 5| = 7$

$2x - 5 = 7$ ①

$2x = 12$

$x = 6$

$2x - 5 = -7$ ①

$2x = -2$

$x = -1$ ①

25. $(x+a)^2 + (x-a)^2$

$x^2 + 2xa + a^2 + x^2 - 2xa + a^2$ ①

$= 2x^2 + 2a^2$ ①

$= 2(x^2 + a^2)$

26. $x + \frac{2}{x} = -3$

$x^2 + 2 = -3x$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2, -1$ ① ①

27. $x + y + 3 = 7$ ①

$2x + 3y - 3 = 0$ ②

$3x + 4y + 23 = 17$ ③

② × 2

$4x + 6y - 23 = 0$ ④

④ + ③ $7x + 10y = 17$ ⑤

① + ② $3x + 4y = 7$ ⑥

⑤ × 3 $21x + 30y = 51$ ⑦

⑥ × 7 $21x + 28y = 49$ ⑧

⑦ − ⑧

$2y = 2$

$y = 1$ ①

into ⑥ $3x + 4 = 7$

$3x = 3$

$x = 1$ ①

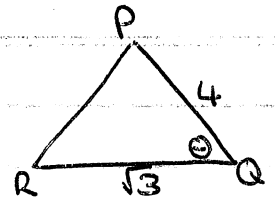
into ①

$1 + 1 + 3 = 7$

$3 = 5$ ①

test in ②

$2 + 3 - 5 = 0$ ✓

28. 

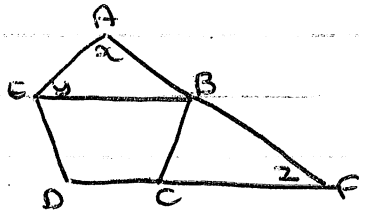
$A = 3cm^2$

$A = \frac{1}{2}ab\sin C$ ①

$3 = \frac{1}{2} \times 4 \times \sqrt{3} \times \sin\theta$

$\sin\theta = \frac{6}{4\sqrt{3}}$ ①

$\theta = 60°$ ①

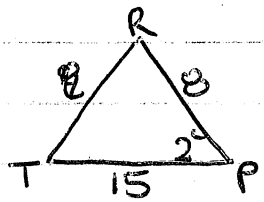
29. 

regular pentagon $= 540°$

$\therefore x = 108°$ ①

$y = 36°$ ①

$3 = 36°$ ①

30. 

$p^2 = 8^2 + 15^2 - 2 \times 8 \times 15 \times \cos 2°$

$= 289 - 240\cos 2°$

$p = \sqrt{289 - 240\cos 2°}$

$= 13.75226379$

$= 13.75$ ③ (1½)

deg 1.01

31. $|x+3| \leq 2x - 2$

$x + 3 \leq 2x - 2$ $\quad -x - 3 \geqslant 2x - 2$

$5 \leq x$ $\quad -1 \geqslant 3x$

$x \geqslant 5$ $\quad x \leqslant -\frac{1}{3}$

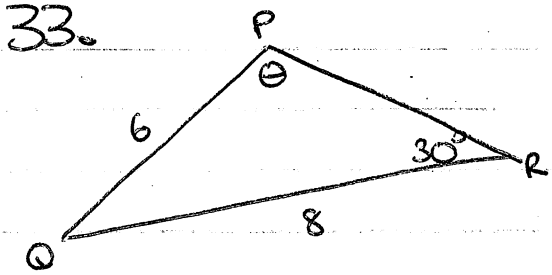
(1½) (1½)

32. $p = \frac{\cos\theta}{2}$, $q = \sin^3\theta$

$2p = \cos\theta$ $\quad q = \sin^2\theta \cdot \sin\theta$

$4p^2 = \cos^2\theta$ ① $\quad \sin^2\theta = \frac{q}{\sin\theta}$ ①

$\cos^2\theta + \sin^2\theta = 4p^2 + q^2$ ①

33. 

$\frac{\sin 30}{6} = \frac{\sin\theta}{8}$ ①

$\sin\theta = \frac{8\sin 30}{6}$

$\sin\theta = \frac{2}{3}$ (1½)

$\theta = 41.8103149$ ②

$\cos\theta = 0.745355992$

$= 0.745$ (3 s.f) (1½)

34. a) let $\angle A = x$.

$\triangle ABC$ & $\triangle ABD$

$\angle ABC = 90° = \angle ADB$

$\angle A$ common.

$\therefore \angle ACB = 90 - \angle A = \angle ABD$

$\therefore$ $\triangle ABC$ & $\triangle ABD$ are similar

$\triangle ABD$ & $\triangle BDC$

$\angle ADB = 90° = \angle BDC$

$\angle C = 90° - \angle A$ (from above)

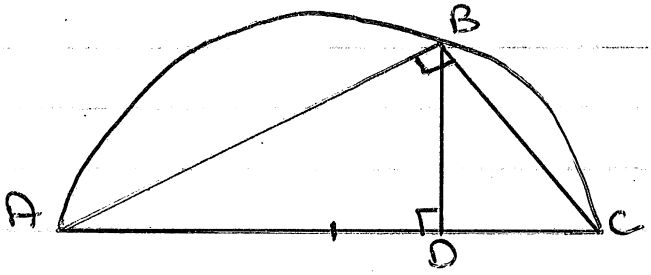
& $\angle ABD = 90° - \angle A$

$\therefore \angle BAD = \angle CBD$

$\therefore \triangle ABD \,|||\, \triangle BDC$

$\therefore \triangle ABC \,|||\, \triangle ADB \,|||\, \triangle BDC$ (2)

b)



radius = r (given)

$BC = \frac{1}{2}r$

$AC = 2r$ (diameter)

$\therefore (2r)^2 = \left(\frac{r}{2}\right)^2 + AB^2$

$AB^2 = 4r^2 - r^2/4$

$= \frac{15}{4}r^2$

$AB = \frac{\sqrt{15}}{2}r$

$AC : AB = AB : AD$

(similar $\triangle$'s sides same ratio)

$2r : \frac{\sqrt{15}}{2}r = \frac{\sqrt{15}}{2}r : AD.$

$2 : \frac{\sqrt{15}}{2} = \frac{\sqrt{15}}{2}r : AD$

$1 : \frac{\sqrt{15}}{4} = \frac{\sqrt{15}}{2}r : AD$

$\therefore AD = \frac{\sqrt{15}}{4} \div \frac{\sqrt{15}}{2}r$

$\boxed{AD = \frac{15}{8}r}$

In $\triangle ABD$

$AB^2 = AD^2 + BD^2$

$\frac{15}{4}r^2 = \frac{225}{64}r^2 + BD^2$

$BD^2 = \frac{15}{64}r^2$

$BD = \frac{\sqrt{15}}{8}r$

In $\triangle BCD$

$BC^2 = BD^2 + DC^2$

$\frac{r^2}{4} = \frac{15}{64}r^2 + DC^2$

$DC^2 = \frac{1}{64}r^2$

$\boxed{DC = \frac{1}{8}r}$

$\therefore AD : DC$

$\frac{15}{8}r : \frac{1}{8}r$

$= 15 : 1$ (3)

(I'm sure there is an easier way to do this!)