Question Seven (20 Marks)

(a) The Holden Car Company offers a loan of $50000 on any of their cars purchased before 31st May, 2003. The loan attracts an interest of just $\frac{1}{2}\%$ per month, and to celebrate Holden's 75 years in Australia the company also offers an interest free period for the first six months. However, the first repayment is due at the end of the first month.

A customer takes out the loan and agrees to repay the loan over ten years by making 120 equal monthly repayments of M. Let A_n be the amount owing at the end of the n^{th} repayment (in $), then:

(i) Show that $A_6 = 50000 - 6M$ 1

(ii) Show that $A_8 = (50000 - 6M) \times 1.005^2 - M(1.005 + 1)$ 2

(iii) Hence, show that $A_{120} = (50000 - 6M) \times 1.005^{114} - M \times \frac{(1.005^{114} - 1)}{1.005 - 1}$ 2

(iv) Hence, show that $M = \frac{50000 \times 1.005^{114}}{6 \times 1.005^{114} + \frac{1.005^{114} - 1}{0.005}}$ 1

(v) Finally, find the value of the monthly repayments to the nearest cent. 1

(b) How many terms of the series $23 + 19 + 15 + ...$ must be added to give a sum of 50? 3

(c) The first term of an arithmetic series is 3 and the twentieth term is 136. Find the common difference and the sum of 20 terms. 3

(d) The series $\frac{1}{3} - \frac{1}{6} + \frac{1}{12} - .....$ is a geometric series. 7

(i) What is the common ratio?

(ii) Find the forth term.

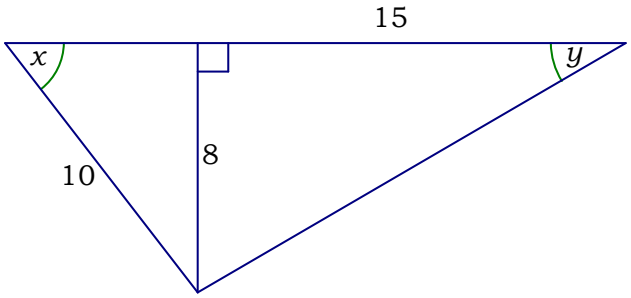
(iii) Find the sum of the first 8 terms.

End of Question Seven

Start a new answer booklet

Question Eight (20 Marks)

(a) From the diagram, write down the value of: 6



(i) $\cos y$

(ii) $\sin x$

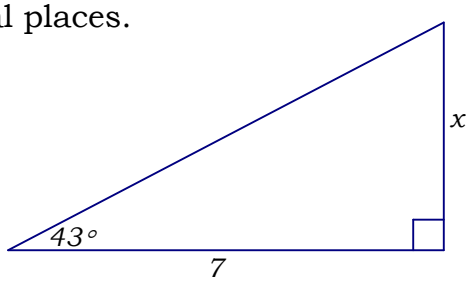
(iii) $\cot x$

(iv) $\text{cosec}\, y$

(v) $\sec x$

(vi) $\cot y$

(b) Find x to 2 decimal places. 1



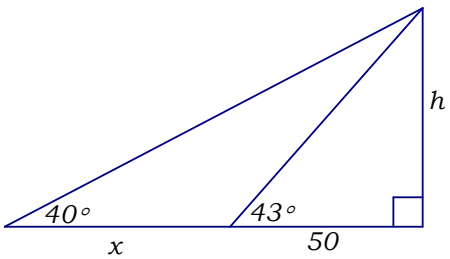
(c) Suppose in a right angled triangle that α is an acute angle and $\sec\alpha = \frac{\sqrt{11}}{3}$. Find the exact value of: 6

(i) $\text{cosec}\,\alpha$

(ii) $\cot\alpha$

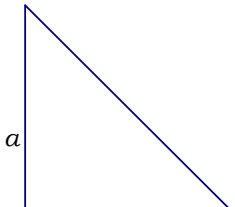
(iii) Show that $\text{cosec}^2\alpha - \cot^2\alpha = 1$

(d) 2



Find the value of h and x to 2 decimal places.

(e) Show that 5



(i) $a = b\tan\alpha$

(ii) $\sin^2\alpha = \dfrac{a^2}{a^2+b^2}$

End of Question Eight
End of Examination.

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Q1)

a) $-2+5=3$.

b) $\frac{3}{\sqrt{5}} \times \frac{\sqrt{5}}{\sqrt{5}} = \frac{3\sqrt{5}}{5}$.

c) $\frac{17}{99}$.

d) $45 \times 1.12 = 50.4$.

e) $|x+1| = 4$.

$x+1 = 4 \quad x+1 = -4$

$x = 3 \text{ or } x = -5$

f) $\frac{3}{a-b} + \frac{1}{b-a}$.

$= \frac{3}{a-b} - \frac{1}{a-b}$.

$= \frac{2}{a-b}$.

g) $(2x+3)(x-4) = 0$.

$2x = -3 \text{ or } x = 4$

$x = -3/2 \text{ or } x = 4$.

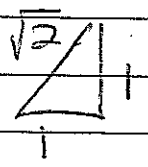
h) $2x = -7(500 - x)$

$2x = -7(500) + 7x$

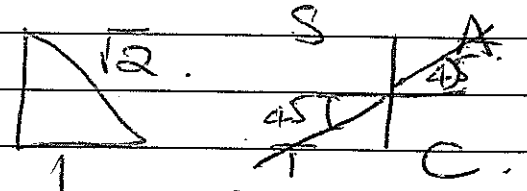
$3500 = 5x$

$x = 700$.

i) $\cos 45^\circ = \frac{1}{\sqrt{2}}$.



j) $\tan x = 1$.



$x = 45^\circ, 180^\circ + 45^\circ$

$x = 45^\circ$ or 225°.

k) $\frac{x}{4} + \frac{3x-1}{3}$.

$= \frac{x(3) + 4(3x-1)}{12}$

$= \frac{3x + 12x - 4}{12}$.

$= \frac{15x - 4}{12}$.

l) $\frac{16^{1/3}}{4^3} = \frac{4}{4^3} = \frac{1}{4^2} = \frac{1}{16}$.

m) $4^x = 8$

$2^{2x} = 2^3$ $\quad x = 3/2$

$2x = 3$

n) $m_1 = 34$ (A) $m_2 = 7$ (B) $M = 53$ (C) $g = 9.8$.

$$\left(\frac{m_1 - m_2}{M + m_1 m_2}\right) g$$

$$= \left(\frac{34 - 7}{53 - 338}\right) 9.8.$$

$$= 0.9093 \text{ (4 sig fig)}.$$

$$\begin{array}{r} 34 \\ 27 \\ \hline 138 \end{array}$$

o) $7 - 4x > 12$.

$-4x > 5$.

$x < -\frac{5}{4}$.

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Q2)

a) $x + 3y = 13$. ← (1)

$2x + 5y = 21$. ← (2).

(1) x2.

$2x + 6y = 26$ ← (3).

(3) − (2). $2x + 5y = 21$.

$y = 5$.

SUBS $y=5$ $x + 3(5) = 13$.

$x = -2$. (3)

(b). $Rx + 4 = 0$.

$R(-5) + 4 = 0$.

$-5R = 4$

$R = \frac{4}{-5}$ (2)

$R = -\frac{4}{5}$.

c) i) $|7| + |-3| = 7 + 3 = 10$. (1)

ii) $|-7| - |-15| = 7 - 15 = -8$. (1)

d) $x \times 1.075 = 86$.

$x = \frac{86}{1.075}$ $x = 80$. (2)

e) $|x+5| = 2x - 1$.

$x + 5 = 2x - 1$. $x + 5 = -(2x - 1)$

$6 = x$ $x + 5 = -2x + 1$. (2)

$x = 6$. $3x = -4$.

$x = -4/3$.

f) $\frac{a^2+a}{a+1} = \frac{a(a+1)}{a+1} = a.$ (2)

g) i) $(2\sqrt{2}-1)(3\sqrt{2}+3)$

$= (2\sqrt{2})(3\sqrt{2}) - 3\sqrt{2} + 6\sqrt{2} - (3)$

$= 12 + 3\sqrt{2} - 3.$ (2)

$= 9 + 3\sqrt{2}$

$\sqrt{a}(\sqrt{a} - \sqrt{b})$

$a - \sqrt{ab}.$

h) $x = \frac{5-\sqrt{2}}{5+\sqrt{2}}.$

$x + \frac{1}{x} = \frac{5-\sqrt{2}}{5+\sqrt{2}} + \frac{5+\sqrt{2}}{5-\sqrt{2}}$

$= \frac{(5-\sqrt{2})(5-\sqrt{2}) + (5+\sqrt{2})(5+\sqrt{2})}{(5+\sqrt{2})(5-\sqrt{2})}$

$= 25 - 10\sqrt{2} + 2 + 25 + 10\sqrt{2} + 2$

$= 50 + 4.$

$= 54.$

i) $x^2 + 6x + 1 = 0.$

$x = \frac{-6 \pm \sqrt{36 - 4(1)(1)}}{2(1)}.$ (5)

$= \frac{-6 \pm \sqrt{32}}{2}$

$= \frac{-6 \pm \sqrt{16 \times 2}}{2}.$

$= \frac{-6 \pm 4\sqrt{2}}{2} = \pm 2\sqrt{2} - 3.$

so $2\sqrt{2} - 3$ is a soln.

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Q3

a) i) $px + 4p + ax + 4a$

$= p(x+4) + a(x+4)$

$= (p+a)(x+4)$.

ii) $a^3 + 3a^2b + ab^2 + 3b^3$

$= a^2(a+3b) + b^2(a+3b)$

$= (a^2+b^2)(a+3b)$

iii) $9a^2 - 4b^2$

$= (3a-2b)(3a+2b)$

iv) $\frac{x^2}{81} - y^2 = \left(\frac{x}{9} - y\right)\left(\frac{x}{9} + y\right)$

v) $z^3 + 1^3 = (z+1)(z^2 + z + 1)$

vi) $m^3p^3 - 1 = (mp)^3 - 1^3$

$= (mp-1)(m^2p^2 + mp + 1)$

vii) $2(x-y)^3 + 54$

$= 2((x-y)^3 + 27)$

$= 2((x-y)^3 + 3^3)$

$= 2(x-y+3)((x-y)^2 - 3(x-y) + 9)$

$= 2(x-y+3)(x^2 - 2xy + y^2 - 3x + 3y + 9)$

$= 2(x-y+3)(x^2 - 3x - 2xy + 3y + y^2 + 9)$

viii) $x^2 + 6x - 7$

$= (x+7)(x-1)$

ix) $x^2 + 14x + 33$

$(x+11)(x+3)$

x) $8x^2 + 2x - 3$

$= 8x^2 + 6x - 4x - 3$

$= 2x(4x+3) - (4x+3) = (2x-1)(4x+3)$

b(ii)

$$\frac{2x+2y}{x^3-y^3} \times \frac{x^2-2xy+y^2}{x^2-y^2}$$

$$= \frac{2(x+y)}{(x-y)(x^2+xy+y^2)} \times \frac{(x-y)^2}{(x+y)(x-y)} \quad (3)$$

$$= \frac{2}{x^2+xy+y^2}$$

iii) $$\frac{x^3-(x-y)^3}{x^2-(x-y)^2}$$

$$= \frac{(x-(x-y))(x^2+(x)(x-y)+(x-y)^2)}{(x+(x-y))(x+(x-y))}$$

$$= \frac{x^2+x^2-xy+x^2-2xy+y^2}{(2x-y)} \quad (3)$$

$$= \frac{3x^2-3xy+y^2}{(2x-y)}$$

Q3(c)(i) $\frac{m}{n} - \frac{n}{m} = \frac{m^2-n^2}{nm} = \frac{(m-n)(m+n)}{nm}$ 2

c)(ii) $\frac{1}{a+5} + \frac{2}{3} = \frac{3+2(a+5)}{3(a+5)} = \frac{2a+13}{3(a+5)}$ 2

(iii) $\frac{1}{x} + \frac{3}{x} - \frac{1}{x^2} = \frac{x+3x-1}{x^2} = \frac{4x-1}{x^2}$ 2

Q3 (b). i). $\frac{4x^3y - 16xy}{x^2+2x-8}$.

$= \frac{4xy(x^2-4)}{(x+4)(x-2)}$.

$= \frac{4xy(x-2)(x+2)}{(x+4)(x-2)}$

$= \frac{4xy(x+2)}{x+4}$.

b(ii). NEXT PAGE.

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Q4.

a) $\frac{1}{x+5} + \frac{1}{x-2} = \frac{1}{(x+5)(x-2)}$

$\frac{x-2+x+5}{(x+5)(x-2)} = \frac{1}{(x+5)(x-2)}$

$2x+3=1$

$2x=-2$

$x=-1$

(b) $-3 \leq \frac{2x-1}{3} < 3$

$-9 \leq 2x-1 < 9$

$-8 \leq 2x < 10$

$-4 \leq x < 5$

c) i) $\frac{\sqrt{x^2}}{|x|}$ $\quad x>0 \quad \frac{\sqrt{x^2}}{x} = 1$

$x<0 \quad \frac{\sqrt{x^2}}{-x} = -1$

ii) $\sqrt{9-6x+x^2}$

$= \sqrt{(x-3)^2}$

$= (x-3)$

d) $2x^2+6x-5=0$

$x^2+3x-5/2=0$

$x^2+3x+(3/2)^2 = \frac{5}{2}+(\frac{3}{2})^2$

$(x+\frac{3}{2})^2 = \frac{19}{4}$

$x+3/2 = \pm\sqrt{19/4}$

$x = \frac{-3 \pm \sqrt{19}}{2}$

4e) $13^2 = (a+7)^2 + a^2$

$169 = a^2 + 14a + 49 + a^2$

$169 = 2a^2 + 14a + 49$

$2a^2 + 14a - 120 = 0$

$a^2 + 7a - 60 = 0$

$(a+12)(a-5) = 0$

$a = -12$ or $a = 5$

DISCARD -VE. $a = 5$.

f) $x(x+1) = 72$.

$x^2 + x - 72 = 0$

$(x+9)(x-8) = 0$

$x = -9$ or $+8$.

g) $x^2 + 4x = 60$

$x^2 + 4x - 60 = 0$

$(x+10)(x-6) = 0$

$x = -10$ or $+6$

DISCARD -VE 10 $x = 6$.

h) $y = 3x - 2$ $y = x^2$

EQUATE

$x^2 = 3x - 2$

$x^2 - 3x + 2 = 0$

$(x-2)(x-1) = 0$

$x = 2$ $y = 4$

or

$x = 1$ $y = 1$.

4i) $x + 2y = -8$ — ①

$xy = 8$ — ②

$x = \frac{8}{y}$ — ③

SUB ③ in ①.

$\frac{8}{y} + 2y = -8$

$8 + 2y^2 = -8y$

$2y^2 + 8y + 8 = 0$

$y^2 + 4y + 4 = 0$

$(x+2)^2 = 0$

$y = -2$ $x = -4$.

x y

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Q5.

a) $\angle DCH = \angle EBC = 53^\circ$ (corresponding $\angle$s & FE // DC)

$\angle EBC = \angle ABF = 53^\circ$ (vertically opposite)

b) i) If ED = FD and AB = BD.

as ABCD is a square.

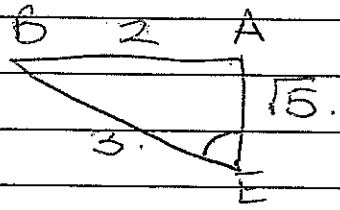
then. AE = CF.

AB = BC. (ABCD is a SQUARE)

$\therefore$ BE = BF (Pythag)

$\therefore$ $\triangle BAE \equiv \triangle BCF$ (S.S.S.)

ii) $\frac{BE}{BA} = \frac{3}{2}$.



$\tan \angle AEB = \frac{2}{\sqrt{5}}$.

Q5)

c) (i) $\angle BAC$ is common.

$$\frac{AB}{AD} = \frac{25}{40} = \frac{5}{8}.$$

$$\frac{AC}{AE} = \frac{20}{32} = \frac{5}{8}.$$

$\triangle ABC \,|||\, \triangle ADE$ (included angle equal and surrounding ratio equal).

(ii) BC // DE BECAUSE.

$\angle ABC = \angle ADE$. (similar $\triangle$'s $\triangle ABC \,|||\, \triangle ADE$)

∴ corresponding angles are equal. and BC // DE.

(iii) $DE = 20$.

$$\frac{BC}{DE} = \frac{5}{8}. \quad \text{(similar } \triangle\text{'s } \triangle ABC \,|||\, \triangle ADE\text{)}$$

$$\frac{BC}{20} = \frac{5}{8}.$$

$$BC = \frac{100}{8} = 12.5.$$

d) i) $\angle CAB = 180 - y - \beta$. ($\angle$ SUM $\triangle$)

$$\angle AFE = \frac{180 - (180 - y - \beta)}{2}$$

$$= \frac{\beta + y}{2}$$

$$= \frac{1}{2}(\beta + y).$$

1

ii) $\angle DCA = 180 - \beta$ ($\angle$ on straight line)

$$\angle DAC = 180 - (180 - \beta) - 2\alpha$$

$$= \beta - 2\alpha.$$

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Q6

a) i) $A = P(1+r)^n$. (2)

$A = 2000(1+0.08)^6$.

$A = 3173.75$.

ii) $I = A - P$

$= A - 2000 = 1173.75$. (2)

b). $I = PRT$.

$I = 1300 \times 0.12 \times 5$. (2)

$= .780$.

c). $A_1 = 11500(1.075)$

~~$A_2 = A_1(1.075) + 11500$~~

~~$= 11500 + 11500(1.075)$~~

$A_2 = (A_1 + 11500)(1.075)$.

$= [11500(1.075) + 11500](1.075)$

$= 11500(1.075)^2 + 11500(1.075)$.

$= 11500(1.075)[1.075 + 1]$.

$A_3 = (A_2 + 11500)(1.075)$.

$= (11500(1.075)(1.075+1) + 11500)(1.075)$.

$= 11500(1.075)^2(1.075+1) + 11500(1.075)$.

$=$

(c) i)

$$A_3 = 11500(1.075)\left[(1.075)(1+1.075)+1\right]$$

$$= 11500(1.075)\left[1.075+1.075^2+1\right]$$

$$= 11500(1.075)\left[1+1.075+1.075^2\right]$$

$$A_{20} = 11500(1.075)\left[1+1.075+1.075^2+\ldots+1.075^{19}\right]$$

$$= 11500(1.075)\left[\frac{1(1.075^{20}-1)}{1.075-1}\right]$$

$$= 11500(1.075)\left(\frac{1.075^{20}-1}{0.075}\right)$$

$$= 535,354.12(3)\ (2\text{ dp})$$ (3)

(ii) $X = (A_{20} - 60,000)(1-0.18) + 60,000$

$= (A_{20} - 60,000)(0.82) + 60,000$ (2)

$= 449,790.38$

(d) x = amt invested now.

$n = 10$.

$r =$ 6% for 6 8% for 4 yrs.

$A = 200000$.

$$200000 = \left(x(1.06)^6\right)\left((1.08)^4\right)$$

$$\frac{200000}{(1.06)^6(1.08)^4} = x$$

$x = \$103,633.4085$.

(4)

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Q7 (9)

(i) $P = 50{,}000$. $r = 0.005$.

6 mnth int free.

$A_1 = 50{,}000 - M$.

$A_2 = A_1 - M$.

$= 50000 - M - M$.

$= 50000 - 2M$.

$\therefore A_6 = 50000 - 6M$.

(ii) $A_7 = (50000 - 6M)(1.005) - M$.

$A_8 = A_7(1.005) - M$.

$= (50000 - 6M)(1.005) - M)(1.005) - M$.

$= (50000 - 6M)(1.005)^2 - 1.005M - M$.

✓✓ $= (50000 - 6M)(1.005)^2 - M(1 + 1.005^1)$.

(iii) $A_{120} = (50000 - 6M)(1.005)^{114} - M(1 + 1.005 + \ldots 1.005^{113})$

$= (50000 - 6M)(1.005)^{114} - M\left(\frac{1.(1.005^{114} - 1)}{1.005 - 1}\right)$

$= (50000 - 6M)(1.005) - \frac{M(1.005^{114} - 1)}{0.005}$.

Q7(a)

(iv) $A_{120} = 0$ (as loan paid off).

$$0 = (50000 - 6M)(1.005^{114}) - M\left(\frac{1.005^{114} - 1}{0.005}\right)$$

$$M\left(\frac{1.005^{114} - 1}{0.005}\right) = (50000 - 6M)(1.005^{114})$$

$$M\left(\frac{1.005^{114} - 1}{0.005}\right) = 50000(1.005^{114}) - 6M(1.005^{114})$$

$$6M(1.005^{114}) + M\left(\frac{1.005^{114} - 1}{0.005}\right) = 50000(1.005^{114})$$

$$M\left(6(1.005^{114}) + \left(\frac{1.005^{114} - 1}{0.005}\right)\right) = 50000(1.005^{114})$$

$$M = \frac{50000(1.005^{114})}{6(1.005^{114}) + \left(\frac{1.005^{114} - 1}{0.005}\right)}$$ ✓✓

(v)

$$\frac{88,287.87107}{10.595 + 153.151}$$

MA1023 Tutorial

539.176

Q7) (b)

$23 + 19 + 15$. $a = 23$ $d = -4$ $n = ?$

$$S_n = 50 = \frac{n}{2}(2a + (n-1)d)$$

$$= \frac{n}{2}(46 + (n-1)(-4))$$

$$100 = n(46 + (n-1)(-4))$$

$$100 = 46n - 4n^2 + 4n$$

$$100 = 50n - 4n^2$$

$$50 = 25n - 2n^2$$

$$2n^2 - 25n + 50 = 0$$

$$2n^2 - 20n - 5n + 50 = 0$$

$$2n(n-10) - 5(n-10) = 0$$

$$(2n-5)(n-10) = 0$$

$$n = \frac{+5}{2} \text{ or } n = 10$$

as n must be an integer $n = 10$

(c) $a = 3$

$T_{20} = 136$

$$T_{20} = a + (n-1)d = 136$$

$$= 3 + 19d = 136$$

$$19d = 133$$

$$d = 7$$

$$S_{20} = \frac{n}{2}(a + l) = 10(3 + 136) = 1390$$

7d i) common ratio $r = -\frac{1}{2}$.

ii) $T_n = ar^{n-1}$.

$T_4 = \frac{1}{3}\left(-\frac{1}{2}\right)^3$.

$= -\frac{1}{24}$.

iii) $S_n = \frac{a(1-r^n)}{1-r}$.

$S_8 = \frac{\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^8\right)}{1-\left(-\frac{1}{2}\right)}$.

$= \frac{\frac{1}{3}\left(1-\left(-\frac{1}{2}\right)^8\right)}{\frac{3}{2}}$.

$= \frac{85}{384}$.

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Q8

(a) i) $\cos y = \frac{15}{17}$.

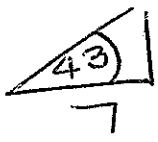
ii) $\sin x = \frac{8}{10} = \frac{4}{5}$.

iii) $\cot x = \frac{adj}{opp} = \frac{6}{8} = \frac{3}{4}$.

iv) $\text{cosec } y = \frac{hyp}{opp} = \frac{17}{8}$.

v) $\sec x = \frac{hyp}{adj} = \frac{10}{6} = \frac{5}{3}$.

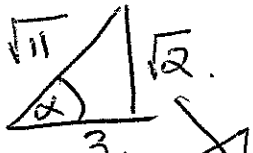
vi) $\cot y = \frac{adj}{opp} = \frac{15}{8}$.

(b) 

$\tan 43 = \frac{x}{7}$.

$x = 7 \tan 43 = 6.527$

$= 6.53 \text{ (2dp)}$.

(c) 

$\sec \alpha = \frac{hyp}{adj} = \frac{\sqrt{11}}{3}$.

$\sqrt{(\sqrt{11})^2 - 3^2} = \sqrt{11-9} = \sqrt{2}$.

i) $\text{cosec}\,\alpha = \frac{hyp}{opp} = \frac{\sqrt{11}}{\sqrt{2}}$.

ii) $\cot \alpha = \frac{adj}{opp} = \frac{3}{\sqrt{2}}$.

iii) $\text{cosec}^2 \alpha - \cot^2 \alpha = \frac{11}{2} - \frac{9}{2} = \frac{2}{2} = 1$.

8 d) $\tan 43 = \frac{h}{50}$.

$h = 50 \tan 43$.

$h = 46.63$ (2dp)

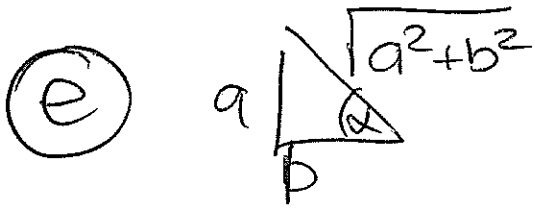
$\tan 40 = \frac{h}{x+50}$.

$x+50 = \frac{h}{\tan 40}$.

$x+50 = 55.5664$.

$x = 5.57$ (2dp)

e)



$\tan \alpha = \frac{a}{b}$.

$a = b \tan \alpha$.

$\sin \alpha = \frac{a}{\sqrt{a^2+b^2}}$.

$\sin^2 \alpha = \frac{a^2}{a^2+b^2}$.

$(\sin \alpha)^2 = \frac{a^2}{(\sqrt{a^2+b^2})^2} \Rightarrow$