SYDNEY GRAMMAR SCHOOL HALF-YEARLY EXAMINATION 2003

FORM V MATHEMATICS

Time allowed: 2 hours **Exam date:** 14th May 2003

Instructions:

All questions may be attempted.
All questions are of equal value.
All necessary working must be shown.
Marks may not be awarded for careless or badly arranged work.
Approved calculators and templates may be used.

Collection:

Collect writing booklets in one bundle.
Start each question in a new writing booklet.
If you use a second booklet for a question, place it inside the first. Don't staple.
Write your name, class, and master's initials on each writing booklet:

5P: JNC 5Q: REN 5R: BDD

Checklist:

Folded A4 writing booklets required — 6 per boy.
Candidature: 44 boys

QUESTION ONE (Start a new writing booklet)

(a) Solve $5y - 1 = 14$.

(b) Expand and simplify $(2 + \sqrt{5})^2$.

(c) Simplify $\sqrt{98}$.

(d) Write down the exact value of $\tan 120°$.

(e) Write down the gradient of the line with equation $3x + y + 7 = 0$.

(f) Express $\dfrac{6}{\sqrt{3}}$ with a rational denominator.

(g) Evaluate $|2| - |-7|$.

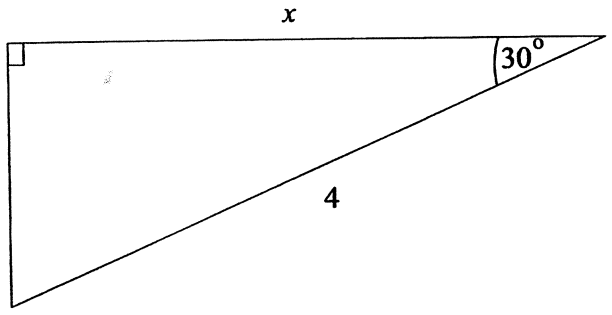
(h) Solve the inequation $2x - 1 \leq 5$ and graph the solution on a number line.

(i) Simplify $4\sqrt{3} + \sqrt{27}$.

(j) Factorise $2x^2 + 7x - 15$.

(k) Find the value of $f(2)$ if $f(x) = x^2 + 3x - 1$.

(l)



In the diagram above, find the exact value of x.

Exam continues next page ...

QUESTION TWO (Start a new writing booklet)

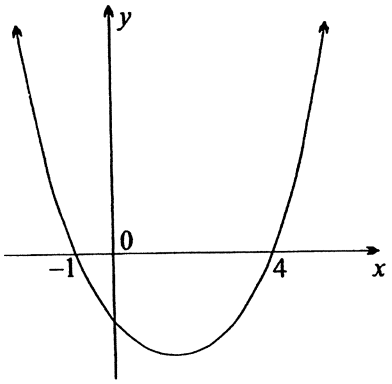
(a) Solve the equation $|x-4|=3$.

(b) Express $\dfrac{12}{\sqrt{5}-2}$ with a rational denominator.

(c) Sketch the graph of $y=x+1$ on a number plane.

(d) Find rational numbers x and y such that $x+3\sqrt{y}=\sqrt{45}-7$.

(e)



In the diagram above, the graph of $y=x^2-3x-4$ is drawn. Use the graph to solve the inequation $x^2-3x-4\geq 0$.

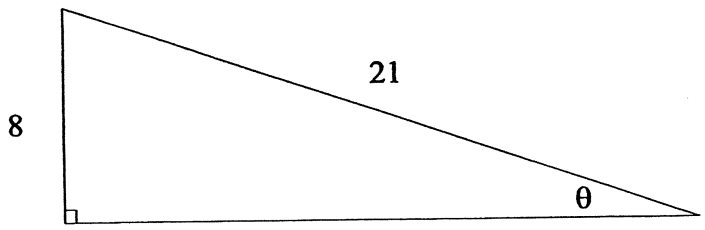
(f) Solve the inequation $|x+2|<4$.

(g) State the domain and range of the function $f(x)=x^2$.

(h) Shade the region in the number plane in which $y>3-x$.

QUESTION THREE (Start a new writing booklet)

(a)



In the diagram above, find the value of θ, correct to the nearest minute.

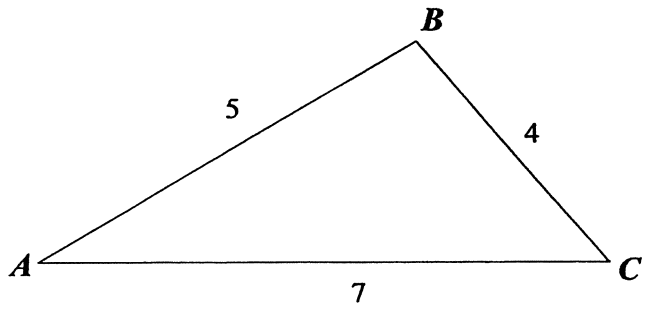
(b) Solve $\cos x=-\dfrac{\sqrt{3}}{2}$, for $0^\circ\leq x\leq 360^\circ$.

Exam continues overleaf ...

(c) Simplify $\dfrac{\tan\alpha}{\sec\alpha}$.

(d) Given that $\sin\theta = \dfrac{1}{3}$ and that θ is obtuse, find the exact value of $\tan\theta$.

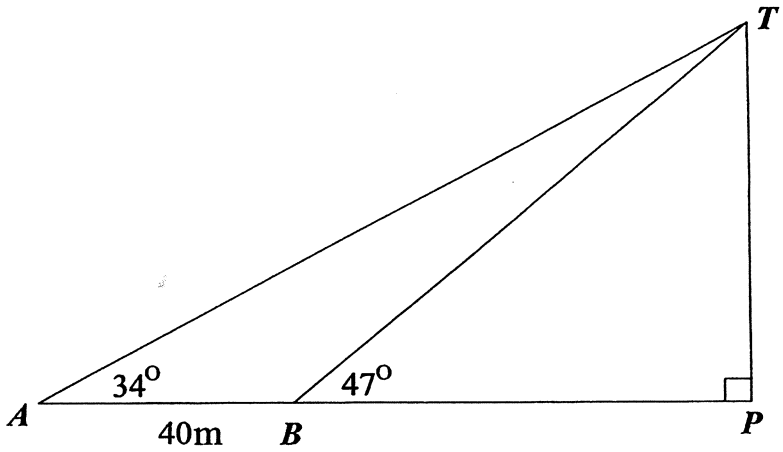
(e) (i)



In the diagram above, use the cosine rule to find the size of $\angle ABC$, correct to the nearest minute.

(ii) Find the area of $\triangle ABC$, correct to one decimal place.

(f)



In the diagram above, the angle of elevation of the top of the tower PT from A is 34°. From a point B 40 metres closer to the tower, the angle of elevation is 47°.

(i) Show that $\angle ATB = 13°$.

(ii) Use the sine rule to find the distance BT, correct to the nearest metre.

(iii) Hence find the height of the tower PT, correct to the nearest metre.

Exam continues next page ...

QUESTION FOUR (Start a new writing booklet)

(a) Write down the gradient of the line that makes an angle of $30°$ with the positive direction of the x-axis.

(b) The point A has coordinates $(-2, 1)$ and the point B has coordinates $(-1, -1)$.

(i) Find the distance AB.

(ii) Find the midpoint of the interval AB.

(iii) Find the gradient of the interval AB.

(iv) Find the equation of the line that passes through the points A and B.

(c) Find, in general form, the equation of the line that has a gradient of $-\frac{2}{3}$ and passes through the point $(-2, 1)$.

(d) Find the equation of the line that passes through the origin and is perpendicular to the line $2x - y + 9 = 0$.

(e) Show that the point $(-1, 5)$ lies on the line $3x + 2y - 7 = 0$.

(f) Find the perpendicular distance from the point $(3, 5)$ to the line $2x - y + 4 = 0$.

QUESTION FIVE (Start a new writing booklet)

(a) Sketch the following functions on SEPARATE number plane graphs showing intercepts, if any, and all other important features:

(i) $3x - y + 6 = 0$,

(ii) $y = (x - 3)(1 - x)$, (you do not need to give the coordinates of the vertex),

(iii) $y = |x + 2|$,

(iv) $y = -\dfrac{1}{x}$.

(b) Consider the function $f(x) = \sqrt{1 - x^2}$.

(i) Sketch the graph of $y = f(x)$.

(ii) State the domain of $f(x)$.

(iii) Explain why $f(x)$ is an even function. (You may do this by referring to your graph or by giving an algebraic explanation.)

(c) The equation of a circle is $(x - 3)^2 + (y + 2)^2 = 16$.

(i) Write down the coordinates of the centre of the circle and its radius.

(ii) Find the perpendicular distance from the line $y = 2x + 3$ to the centre of the circle.

(iii) Explain why the line $y = 2x + 3$ does not intersect the circle.

Exam continues overleaf ...

QUESTION SIX (Start a new writing booklet)

(a) Solve $\sin 2x = -\frac{1}{2}$, for $0° \leq x \leq 180°$.

(b) (i) Sketch the graphs of $y = |3x - 1|$ and $y = 1 - x$ on the same number plane.

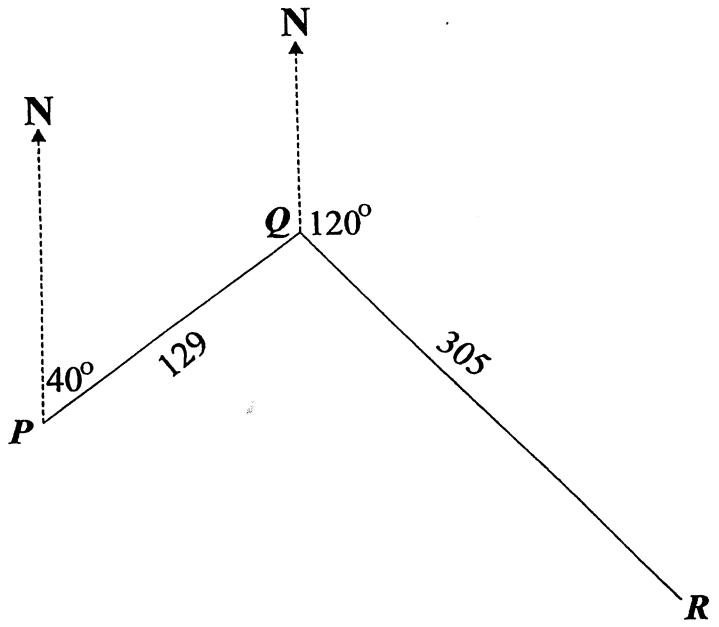
(ii) Solve $|3x - 1| = 1 - x$.

(iii) Hence or otherwise, solve the inequation $|3x - 1| > 1 - x$.

(c) Prove the identity $\frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta} = 2\sec^2\theta$.

(Start your proof: LHS $= \frac{1}{1+\sin\theta} + \frac{1}{1-\sin\theta}$)

(d)



In the diagram above, a ship leaves Port P and sails 129 kilometres in a direction of 040° to Port Q. From Port Q the ship changes direction and sails 305 kilometres in a direction of 120° to Port R.

(i) Explain why $\angle PQR = 100°$.

(ii) How far is the ship then from Port P? (Give your answer correct to the nearest kilometre.)

(iii) What is the bearing then of Port P from the ship? (Give your answer correct to the nearest degree.)

JNC

FORM V Half Yearly 2003

2 UNIT

Question 1

(a) $y = 3$ ✓✓

(b) $4 + 4\sqrt{5} + 5 = 9 + 4\sqrt{5}$ ✓✓

(c) $7\sqrt{2}$ ✓

(d) $-\sqrt{3}$ ✓

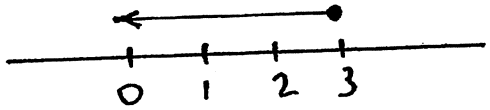
(e) -3 ✓

(f) $\frac{6\sqrt{3}}{3} = 2\sqrt{3}$ ✓✓

(g) $2 - 7 = -5$ ✓

(h) $2x \leq 6$

$x \leq 3$ ✓✓



✓

(i) $4\sqrt{3} + 3\sqrt{3}$ ✓

$= 7\sqrt{3}$

(j) $(2x - 3)(x + 5)$ ✓

(k) $f(2) = 2^2 + 3 \times 2 - 1$ ✓

$= 9$

(l) $\cos 30^\circ = \frac{x}{4}$ ✓

$x = 4 \times \frac{\sqrt{3}}{2}$

$= 2\sqrt{3}$ ✓

Question 2

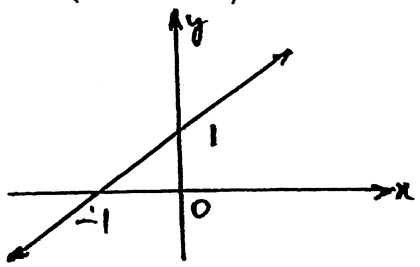
(a) $x - 4 = 3$ or $-(x-4) = 3$

$\therefore x = 7$ or 1 ✓✓ (1 each)

(b) $\frac{12}{\sqrt{5}-2} \times \frac{\sqrt{5}+2}{\sqrt{5}+2}$ ✓

$= 12(\sqrt{5}+2)$ ✓

(c)



✓✓

(d) $x = -7$, and ✓

$3\sqrt{y} = \sqrt{45}$ ✓

$3\sqrt{y} = 3\sqrt{5}$

$\therefore y = 5$ ✓

(e) $x \leq -1$ or $x \geq 4$ ✓✓ (1 each)

(f) $x + 2 < 4$ $\quad -(x+2) < 4$

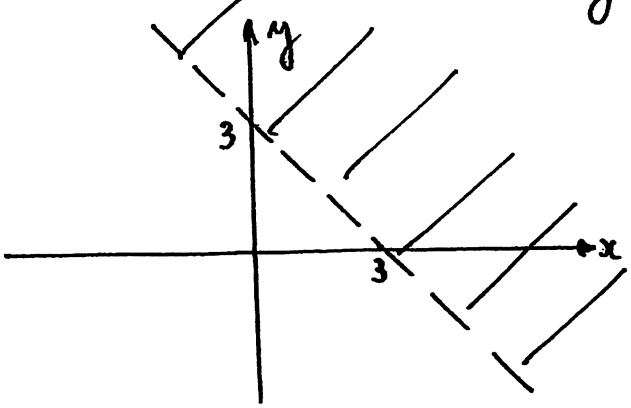
$x < 2$ ✓ $\quad x + 2 > -4$

$x > -6$ ✓✓

ie $-6 < x < 2$ (not needed)

(g) Domain is all real x and range is $y \geq 0$. ✓✓ (1 each)

(h)



✓✓✓

1 line with intercepts

1 dotted

1 shading correct side

Question 3

(a) $\sin\theta = \frac{8}{21}$ ✓

$\theta = 22^\circ 24'$ ✓

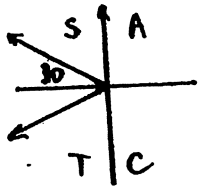
(b) $\cos^2 30^\circ + \sin^2 30^\circ$

$= \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2$

$= \frac{3}{4} + \frac{1}{4}$

$= 1$

(b) Related $\angle = 30^\circ$ ✓

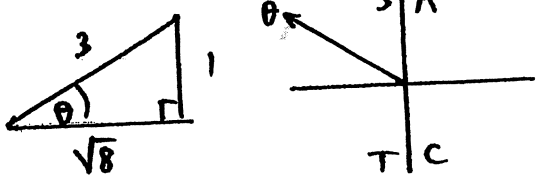


$x = 150^\circ$ or 210° ✓✓

(c) $\frac{\frac{\sin\alpha}{\cos\alpha}}{\frac{1}{\cos\alpha}} = \frac{\sin\alpha}{\cos\alpha} \times \frac{\cos\alpha}{1}$ ✓✓

$= \sin\alpha$

(d)



$\tan\theta = -\frac{1}{\sqrt{8}}$ ✓✓ 1 for $\sqrt{8}$, 1 for −ve

$\left(= -\frac{\sqrt{2}}{4}\right)$

(e) (i) $\cos B = \frac{4^2 + 5^2 - 7^2}{2 \times 4 \times 5}$ ✓ (Wrong formula and correct from there and correct rounding 1/3)

$= -\frac{1}{5}$ ✓

$\therefore \angle ABC = 101^\circ 32'$ ✓ (must be nearest minute for this mark)

(ii) Area $\Delta = \frac{1}{2} \times 4 \times 5 \times \sin 101^\circ 32$ ✓ (1/2 for wrong formula and no subsequent error)

$= 9.7979\ldots$

$\doteq 9.8$ ✓ (don't penalise rounding)

(f)

(i) $\angle ATB + 34 = 47$ (exterior $\angle$ theorem) ✓

$\therefore \angle ATB = 13^\circ$

(ii) $\frac{BT}{\sin 34^\circ} = \frac{40}{\sin 13^\circ}$ ✓

$BT = 99.4337\ldots$

$\doteq 99$ m ✓

(iii) $\sin 47 = \frac{PT}{99}$

$\therefore PT = 99 \times \sin 47$ ✓

$= 72.404\ldots$

$= 72$ m ✓

or 73.

Question 4

(a) $m = \tan 30^\circ$ ✓

$= \frac{1}{\sqrt{3}}$ ✓

(b) (i) distance $= \sqrt{1^2 + 2^2}$

$= \sqrt{5}$ ✓✓

(ii) Midpoint $= \left(-\frac{3}{2}, 0\right)$ ✓✓

(iii) $m = -\frac{2}{1}$

$= -2$ ✓✓

(iv) $y - 1 = -2(x + 2)$ ✓

$y = -2x - 3$ ✓ (any form)

(c) $y - 1 = -\frac{2}{3}(x + 2)$ ✓

$3y - 3 = -2x - 4$ ✓

$2x + 3y + 1 = 0$ ✓ (penalise if not in general form)

(d) gradient of line is 2. So gradient of required line is $-\frac{1}{2}$. ✓

Eqtn is:

$y - 0 = -\frac{1}{2}(x - 0)$ ✓

$2y = -x$

$x + 2y = 0$. ✓

(e) $3x + 2y - 7$

$= 3x - 1 + 2 \times 5 - 7$

$= -3 + 10 - 7$ ✓

$= 0$

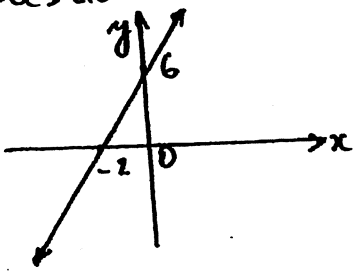
So point lies on line.

(f) $d = \frac{|6 - 5 + 4|}{\sqrt{2^2 + 1^2}}$ ✓

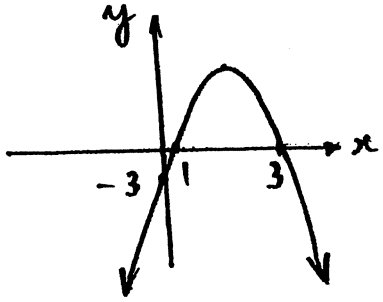
$= \frac{5}{\sqrt{5}} = \sqrt{5}$ ✓ (no need to rationalise)

Question 5

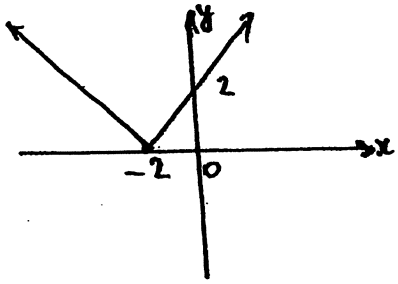
a)



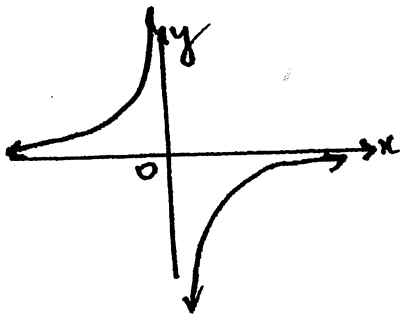
✓✓



✓✓

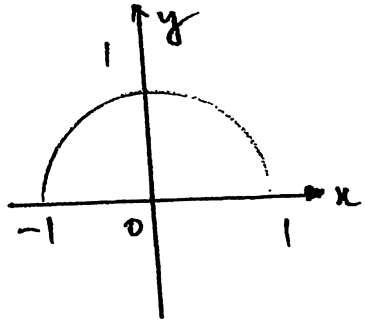


✓✓



✓✓

(b)(i)



✓✓ intercepts must be marked

(ii) domain is $-1 \leq x \leq 1$ ✓✓

(iii) symmetric in y axis, or

$f(a) = \sqrt{1-a^2}$

$f(-a) = \sqrt{1-(-a)^2}$

$= \sqrt{1-a^2} = f(a)$ ✓✓

(c) (i) centre is $(3, -2)$ ✓

$r = 4$ ✓

(ii) $d = \dfrac{|3 \times 2 + (-2) \times (-1) + 3|}{\sqrt{1^2 + 2^2}}$ ✓

$= \dfrac{11}{\sqrt{5}}$ ✓

$= \dfrac{11\sqrt{5}}{5}$

(iii) $\dfrac{11\sqrt{5}}{5} \doteq 4.9$ and so the distance from the line to the centre of the circle is greater than the radius. ✓

Graphs: −1 per error of omission of intercepts

0 marks if wrong shape

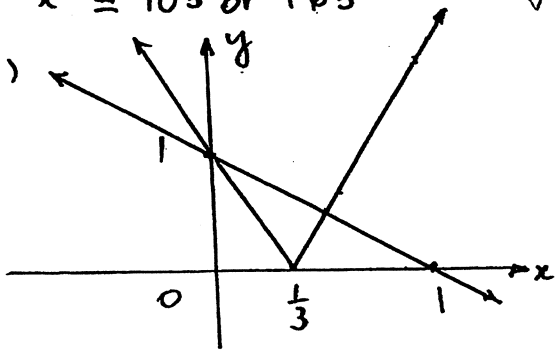
Question 6

(a) Related $\angle = 30^\circ$ ✓

$2x = 210$ or 330 ✓

$x = 105^\circ$ or 165° ✓

(b) (i)



✓ — line

✓✓ — absolute value graph

(ii) $|3x-1| = 1-x$

$3x-1 = 1-x$ or $-(3x-1) = 1-x$

$4x = 2$ ✓✓ $\qquad 2x = 0$ ✓✓

$x = \frac{1}{2}$ $\qquad x = 0$

(iii) $x < 0$ or $x > \frac{1}{2}$ ✓

(c) $LHS = \dfrac{1}{1+\sin\theta} + \dfrac{1}{1-\sin\theta}$

$= \dfrac{1-\sin\theta+1+\sin\theta}{1-\sin^2\theta}$ ✓

$= \dfrac{2}{\cos^2\theta}$

$= 2\sec^2\theta$ ✓

(d) (i) $\angle PQN = 140^\circ$ (co interior $\angle$, $\parallel$ lines)

$\angle PQR + 120 + 140 = 360$ ($\angle$ in revolution)

$\therefore \angle PQR = 100^\circ$ ✓✓

(ii) $PR^2 = 129^2 + 305^2 - 2\times 129\times 305\times \cos 100$ ✓

$= 123330.3751\ldots$

$\doteq 351$ ✓

(iii) $\dfrac{\sin R}{129} = \dfrac{\sin 100}{351}$

$\sin R = 0.3619\ldots$

$\therefore R \doteq 21^\circ$ ✓

So bearing is $270 + (90-60-21) = 279^\circ$ ✓